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Leonid Pavlovich Shilnikov. On His 75^{th} Birthday

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L. P. Shilnikov. Luxembourg. Autumn 2007.

On December 17, 2009 we all raised our glasses to celebrate the 75th anniversary of Leonid Pavlovich Shilnikov, our dear friend, mentor and fellow researcher, creator of the homoclinic bifurcation theory for high-dimensional dynamical systems. His works greatly influenced the overall development of mathematical theory of dynamical systems, as well as nonlinear dynamics in general. Shilnikov's findings have became the classics, and been included in the most text- and reference books which are used worldwide by mathematics students and nonlinear dynamists to study the qualitative theory of dynamical systems and chaos. The elegance and completeness of his results let them reach to "the heart of the matter," and provide applied researchers with in-depth mathematical understanding of outcomes of natural experiments. No doubt that this popularity is due the status of "a living classic" that Professor Shilnikov has attained over several decades for his continuous hard works on the bifurcation theory of multi-dimensional dynamical systems, mathematical chaos theory and theory of strange attractors. Out of his so many fundamental achievements, here we will brief on a few, the key ones in our view.

We will begin with his works on the theory of global bifurcations in multi-dimensional dynamical systems, the works which had built the foundation for the theory. The basics of bifurcations for systems in a plane had originally been discovered and studied by A. A. Andronov and E. A. Leontovich as early as in the 1930s. Among them, of special, for this story, interest are two nonlocal bifurcations that occur in the system with a homoclinic loop of either a saddle or a saddle-node equilibrium state. In the late 1950s and early 1960s L. P. Shilnikov studied high-dimensional versions of these bifurcations, and he identified the cases, for which the breakdown of the homoclinic loop would lead to the emergence of a singe periodic trajectory. This research direction was held yet along the traditional lines drawn by the Andronov School in Gorky.

L. P. Shilnikov, a Lavrentiev award recipient, and A. N. Sharkovsky. Kiev, Ukraine, 2005.
Leonid talks about the Bogulyubov-Mitropolsky averaging method at June 2009 meeting of the Nizniy Novgorod Mathematical Society. He was its first president (*from the family album*).

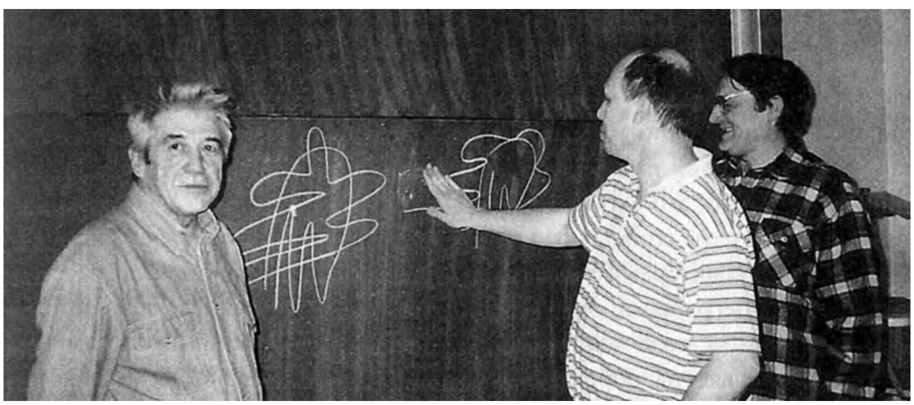
Soon after, in 1965, Leonid made his first ground-breaking discovery that had fundamentally changed the views on dynamics of 3D and up systems: a codimension-1 bifurcation of a homoclinic loop of a saddle-focus (satisfying to what is called nowadays Shilnikov conditions) generates infinitely many periodic trajectories in the phase space. The essence of the proof was that the corresponding Poincaré map near the homoclinic loop possesses infinitely many Smale horseshoes, which is a proof for chaotic dynamics in the system. Typically, a system with the Shilnikov saddle-focus exhibits "spiral chaos," a phenomenon which is now recognized as ubiquitously present in high-dimensional systems, regardless of their origin whether is math, physics or biology. In 1965 no one suspected that a simple bifurcation may lead to chaos, so that was the astonishingly unexpected discovery. For Leonid it was a starting point in his quest for other examples of this phenomenon. Soon he found that the disseverance of a *saddle-saddle*, or a Shilnikov saddle-node, with several homoclinic loops (still codimension-1 bifurcation) could lead to the appearance of a complex chaotic invariant set in the phase space of the system. This was genuinely the first example of "homoclinic Ω-explosion" for instant chaotic dynamics at crossing the boundary of Morse–Smale systems.

The 1960s are also marked by Leonid' work to solve the famous Poincaré–Birkhoff problem on the structure of the set of trajectories in a neighborhood of transverse homoclinic orbit to a periodic motion. As he had proved, this invariant and hyperbolic (locally maximal) set is conjugate to a suspension of the Bernoulli subshift on two symbols. This result has shown that the existence of a transverse Poincaré homoclinic orbit is a universal criterion of chaos. Western authors more frequently refer this criterion to S. Smale's result published slightly earlier. However, it is fair to say that Shilnikov had given a complete description of the neighborhood of the transverse homoclinic, whereas Smale had described only a certain subset, with the use of extra conditions (that may not hold in resonant cases) on linearization properties. To overcome characteristic obstacles of resonant cases, Shilnikov had introduced a novel technique for "boundary problem" which is referred today to as the Shilnikov coordinates. This approach was further perfected by his students to solve several other hard problems. In particular, the technique was employed by Leonid to study the structure of a neighborhood of a homoclinic tube to an invariant torus, and with L. M. Lerman to examine the structure of trajectories close to homoclinic one for infinitely dimensional and nonautonomous systems.

In the 1970s Shilnikov focused his research interests on the following question: what bifurcations, or bifurcation boundaries may separate a class of Morse–Smale systems with simple behavior from

A. D. Morozov, M. I. Malkin, L. P. Shilnikov and L. M. Lerman
at the banquet of a conference in Nuzhnyi Novgorod.

a class of systems with the complex dynamics in the Banach space of dynamical systems? In part, the answers to this question would help one figure out solutions to a broader problem on typical bifurcation scenarios of transitions, or routes to chaos. Leonid's research in this direction had brought up many interesting results, though we will touch only several of them here.



How to untangle a homoclinic tangle? L. P. Shilnikov, D. V. Turaev and S. V. Gonchenko. Berlin, 2004.

Basic properties of the transitions to chaos through homoclinic tangencies were originally studied in the classical works by Leonid with N. Gavrilov, dated back to the early 1970s. Later on in the 1980s, Shilnikov proceeded the trend with his new co-authors: S. Gonchenko and D. Turaev. Their new shocking findings, along with S. Newhouse's theory on dense structural instability, have become the foundations for new "homoclinic chaos" theory. The curious reader is referred to a recently published collection of papers "Homoclinic Tangencies," R&C Dynamics Publ. 2007. Needless to say that this trinity keeps this research trend up so far.

Another bifurcation of multi-dimensional systems, studied jointly with V.S. Afraimovich in 1974, describes the disappearance of a saddle-node periodic orbit with the unstable manifold returning to the orbit through its stable region. This bifurcation may seem alike a resonant torus bifurcation. However, they found a new exciting dynamical phenomenon at the breakdown of the torus, namely, the onset of chaotic dynamics. This bifurcation has proved to be a typical mechanism of route to the torus breakdown which has been reported in various dynamical systems and applied models, which experience this "torus-chaos" transition in a new terminology. Later in 1978, Leonid and

Table discussion: the speakers are V. S. Afraimovich, Ya. Umansky, L. P. Shilnikov, G. M. Zaslavsky, L. A. Bunimovich (host). The chairman is surrounded by wife Lyudmila and grand-daughter Lukiya. Atlanta, USA, 2007.

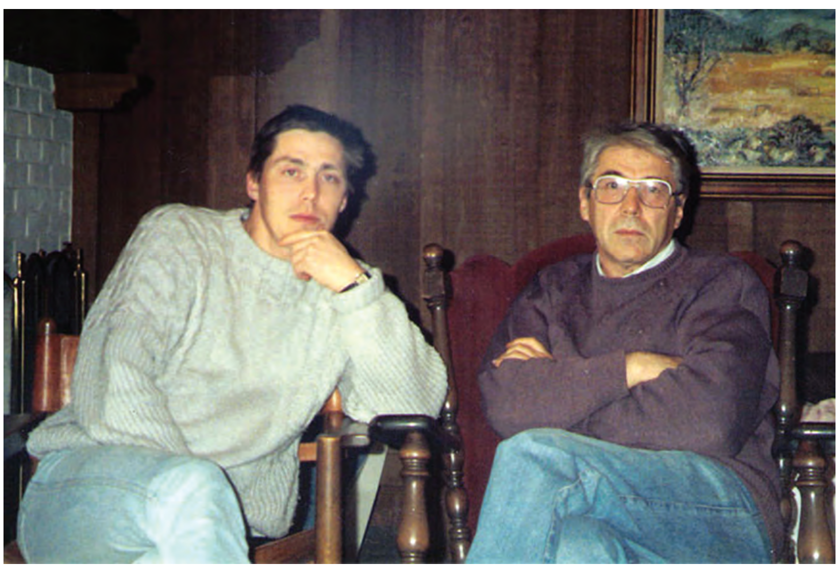
his student V. I. Lukyanov examined a similar bifurcation of a saddle-node periodic orbit whose unstable manifold comes back transversally to its strongly stable one. This result provided, in particular, a rigorous theoretical explanation to the phenomenon known as "transition to chaos through intermittency" which had been observed frequently in many applications as well. Leonid's joint work with D. Turaev (1995) brought in a generalization of both global constructions above. They discovered several novel kinds of bifurcations referred today to by a stunning term "blue sky catastrophe." It was shown by them that codimension-1 homoclinic saddle-node bifurcations of periodic orbits can lead to the formation of strange hyperbolic attractors. In addition they discovered a new bifurcation in charge for stability loss of periodic orbits, number 7 in the list of such bifurcations known up to date.

A key principle of the Andronov school of nonlinear oscillations in Gorky (formerly) and Nizhniy Novgorod today is its permanent interest in applied problems. The principle has been carried over by Professor Shilnikov as well: mathematics-wise this means solving a theoretical problem with a reasonably minimal number of initial constrains and conjectures. This would allow other researches to use transparently the sound mathematical methods developed for solving various applied problems too. One such problem being popular in the mid 1970s and remaining popular yet, was on the structure of the strange attractor in the famous Lorenz model. Its equations are intended to describe rotations of some convection rolls formed in a horizontal layer of water heated from below. Leonid was persuaded by the idea that examination of this model would eventually make a further breakthrough for in-debts understanding of the origin and the nature of deterministic chaos. Doing so, he managed a research team to achieve this purpose.

This time, his phenomenological constructions for bifurcations in the Lorenz model were to be supplemented with and verified by intense numerical simulations. Eventually this led to an abstract mathematical model describing adequately the properties of the Lorenz model. The former is called the Lorenz geometric model. The theoretical and numerical findings of the team were reported in two papers in 1977 and 1982 joint with V. S. Afraimovich and V. V. Bykov. The papers describe in detail the bifurcations and the structure of the Lorenz attractor, which is a genuinely strange attractor, i. e., without stable obits, but structurally unstable in contrast to the attractors provided by the hyperbolic theory. The research of the team was on the very frontier of dynamical systems field at that time, as almost simultaneously, there came out numerous papers on the Lorenz attractors by R. Williams, J. Guckenheimer, J. Kaplan, J. Yorke, O. Langford, et al. The theory of Afraimovich–Bykov–Shilnikov has remained the most complete and practical for other models and applications; it allowed one to make feasible predications of evolutions of the Lorenz-like attractors including the existence of lacunae, detection of the existence regions of the Lorenz-like attractors in different models, as well as effective computations of various metric properties of the attractors.

Leonid Shilnikov and his students have established a great number of research trends. The list includes the theory of pseudo-hyperbolic strange attractors in cooperation with D. V. Turaev, which


Leonid Shilnikov is a student of FizMat Department at Gorky State University.
Codimension-2 homoclinic bifurcations, or Lorenz attractor? — Elementary! Golden era for bursting research in the Gorky School led by Leonid in the 1970–80s (*from the family album*).



During the work on the book; with son Andrey in Berkeley, USA, 1993 (*from the family album*).

can be regarded as an essential generalization of the theory of Lorenz attractor and its bifurcations; this trend the father continued with his son Andrey on examination of complex chaooic dynamics in normal forms.

For several years Professor Shilnikov had run the laboratory at the Department of Differential Equations headed by E. Leontovich–Andronova at Research Institute for Applied Mathematics and Cybernetics. Since 1984 he became the Head of the Department. Leonid is a thoughtful and caring mentor for his students including N. K. Gavrilov, V. S. Afraimovich, A. D. Morozov, L. M. Lerman, L. A. Belyakov, V. V. Bykov, V. I. Lukyanov, S. V. Gonchenko, M. I. Malkin, N. D. V. Turaev,

Cherishing a precious moment: fishing has remained Leonid's favorite hobby besides science.
At the Lake Lanier in Atlanta, 2007.
LP with his five Lyudmila Ivanovna on a spring break on Mexican Golf coast in Florida, 2007
(*from the family album*).

A. L. Shilnikov, M. V. Shashkov, O. V. Stenkin, V. S. Gonchenko, I. Ovsyannikov and V. Biragov, and his lab fellows: I. Belykh, A. Bautin, G. Polotovskii, N. Roschin, Ya. Umaskii, S. Grines and S. Zhuzhoma, who became actively and independently working researchers.

Leonid has published nearly two hundred research papers and co-authored a few books including Bifurcation Theory and Catastrophe Theory with V. I. Arnold, V. S. Afraimovich, Yu. S. Ilyashenko, in Russian (1986) and English (1994), and Methods of Qualitative Theory in Nonlinear Dynamics. Parts I, II with A. L. Shilnikov, D. V. Turaev, and L. Chua (1998, 2001), and its recent translations in Russian (2003, 2009) and Chinese (2010).

Professor Shilnikov's scientific achievements were acknowledged by several awards including A. M. Lyapunov Award of Russian Academy of Science (1998) and M. A. Lavrentiev Award of National Academy of Science of Ukraine (2005), Professorship of Alexander von Humboldt Foundation of Germany in 2002. He has served on editorial boards of many international journals. He has been an invited speaker at large number of meetings held in Russia and all over the world, invited to visit and speak at leading research universities in USA, Belgium, France, Israel, Germany, Italy, China.

For each and every of us, Leonid Pavlovich is Teacher, extraordinary Expert and mythic Prophet in mathematics and nonlinear dynamics. He has his own, Shilnikov "nonaxiomatic" style: the conditions of his theorems are meant to be verified with ease. Perhaps, because of that Leonid became a global attractor for many colleagues and research fellows from various fields of science, besides mathematics: physics, biology, neurophysiology, chemistry and engineering, and he stays that way always. Many scientists acknowledge that Shilnikov's ideas and charisma have greatly influenced their own development, both professional and personal.

We wish LP, as we call him amongst ourselves, very good health, new ideas in science, true disciples and best of luck!

V. S. Afraimovich, San Luis Potosi University, Mexico

L. M. Lerman, Nizhny Novgorod State University, Russia

S. V. Gonchenko, Nizhny Novgorod Research Institute for Applied Mathematics and Cybernetics, Russia