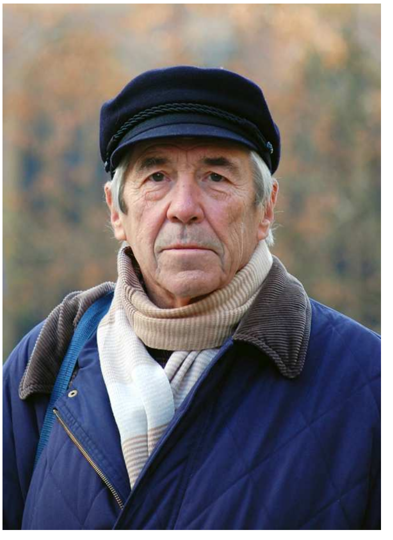
Russian Math. Surveys **67**:3 573–577
Uspekhi Mat. Nauk **67**:3 175–178 DOI 10.1070/RM2012v067n03ABEH004798

Leonid Pavlovich Shil'nikov (obituary)



A remarkable mathematician, one of the most prominent specialists in the theory of dynamical systems and bifurcation theory, a laureate of the Lyapunov Prize of the Russian Academy of Sciences and of the Lavrent'ev Prize of the National Academy of Sciences of Ukraine, a Humboldt Professor, Head of the Department of Differential Equations of the Research Institute of Applied Mathematics and Cybernetics of Nizhnii Novgorod University, Professor Leonid Pavlovich Shil'nikov passed away on 26 December 2011.

Leonid Pavlovich was born on 17 December 1934 in the town of Kotel'nich of Kirov Province, in a family of workers. After graduating from secondary school in 1952 he entered the Physics-Mathematics Faculty of Gor'kii (now Nizhnii Novgorod) State University, from which he graduated in 1957. After completing his post-graduate studies (1957–1960), in 1962 he defended his Ph.D. dissertation, "Birth of periodic motions from singular trajectories", where he generalized to the multidimensional case the non-local bifurcations that had been studied for systems on the plane by A. A. Andronov and E. A. Leontovich.

Soon after that he began studying the theory of systems with complex (chaotic) dynamics, which was just then coming into existence. By that time, the example of Smale's horseshoe had already appeared (1961), and D. V. Anosov's note on geodesic flows had been published (1962), where the notion of hyperbolicity was formulated and its primary significance was revealed. In developing the results of his dissertation Shil'nikov discovered (*Dokl. Akad. Nauk SSSR*, 1965) that infinitely many Smale horseshoes, and hence a complex dynamics, exist in a neighbourhood of a homoclinic loop of a saddle-focus. Today we know that a Shil'nikov loop determines the chaotic dynamics of a wide range of models in various areas of science. But at that time the result that a complex dynamics can arise near such a simple and familiar object as a separatrix loop was totally unexpected. Shil'nikov realized that the study of homoclinic bifurcations gave him a unique opportunity for studying complex dynamics of multidimensional systems, and this became a central subject of his scientific activities.

AMS 2010 *Mathematics Subject Classification.* Primary 01A70.

Already in 1965–1967 he published a complete solution of the classical problem, going back to Poincaré and Birkhoff, of the structure of a neighbourhood of a structurally stable homoclinic curve of a saddle periodic motion. Shil'nikov considered this result to be especially important and tirelessly emphasized that a structurally stable homoclinic is a basic 'building block' of chaos theory. He did not dwell only on the case of a homoclinic to a periodic trajectory, but immediately published a paper on the structure of a neighbourhood of a family of trajectories that are homoclinic to an invariant torus (in particular, to a quasi-periodic motion). Later he studied the general non-autonomous and infinite-dimensional cases (jointly with L. M. Lerman). For solving these problems Shil'nikov developed the 'cross-map method', which in addition turned out to be very convenient for obtaining asymptotic representations of solutions of a system in a neighbourhood of non-linear saddles and became a foundation for research in the theory of homoclinic bifurcations.

Then there was a series of papers, both by Shil'nikov himself and jointly with his students, which in essence created a new area in the theory of dynamical systems — the theory of global bifurcations. All such bifurcations were divided into three large classes: bifurcations that do not lead out of the class of Morse–Smale systems; bifurcations from Morse–Smale systems to systems with complex dynamics; bifurcations within the class of systems with complex dynamics. The basic non-local bifurcations of the first type in the multidimensional case were studied by Shil'nikov himself. He also discovered and described non-local bifurcations of the second type: in particular, historically the first example of a bifurcation from Morse–Smale systems to systems with complex dynamics was a bifurcation of a bouquet of homoclinic loops of an equilibrium state of saddle-saddle type (1969). Later he and his first student N. K. Gavrilov began studying homoclinic tangencies, and he and V. S. Afraimovich investigated periodic perturbations of autonomous systems with a separatrix loop, breakdown of invariant tori (that is, phenomena of transition from two-frequency regimes to chaos), and bifurcations of homoclinic trajectories of saddle-node type (some of the latter results with V. I. Luk'yanov). In these papers various types of chaotic behaviour were investigated, and it was discovered that complex dynamics is not described by hyperbolic sets only — chaotic regimes are very often neighbours to stable ones. Subsequently Shil'nikov proposed the concept of a 'quasi-attractor' (an attracting set that, along with hyperbolic subsets, may also contain stable periodic trajectories with large periods) as the most adequate mathematical image of a dynamical chaos observed in applied problems.

At that time physicists and scientists in other areas were becoming interested in systems with complex dynamics. The question of the extent to which the dynamical chaos discovered by mathematicians is needed for the sciences was quite widely discussed, and Shil'nikov took an active part in these discussions. A turning point, both here and in the West (in essence, a proof that dynamical chaos is one of the fundamental phenomena in nature), was the discovery of a strange attractor in the Lorenz system. Unlike hyperbolic attractors, the theory of which had already been well developed, the Lorenz attractor does not preserve its structure under variation of the parameters: it undergoes bifurcations. Shil'nikov recalled that as soon as he learned about the Lorenz attractor, it became clear to him that the methods of bifurcation theory needed here had already been created by him and his school. He started a series of remarkable investigations on the Lorenz attractor, jointly with

Afraimovich and V. V. Bykov. The resulting papers (1977–1983) strikingly showed his deep understanding of dynamics and bifurcations and his ability to create adequate mathematical models. Without this, it would have been impossible to analyse in such detail the overall picture of the appearance and disappearance of an attractor, and of the changes in its structure under variation of the parameters. Up till now the Afraimovich–Bykov–Shil'nikov theory remains the most complete and convenient theory for practical analysis of the structure and evolution of Lorenz-type attractors in various models.

In 1982 Shil'nikov replaced Evgeniya Aleksandrovna Leontovich-Andronova as head of the Department of Differential Equations of the Research Institute of Applied Mathematics and Cybernetics. He led the work of a united team, which was informally joined by a solidary circle of students and colleagues. The weekly seminar headed by Shil'nikov was always a centre of lively and intense discussions: it was a great educational experience to give a talk at this seminar. Shil'nikov was often able to find the correct formulations of problems even in areas of knowledge that were far from his own. He was distinguished by his special intuition — many of his colleagues have said that contacts with him have had an enormous influence on their whole scientific career. He attached great importance to promoting new ideas and discoveries in the theory of dynamical systems among specialists in radio physics, biophysics, hydrodynamics, and meteorology. He was among the organizers of several conferences and schools on non-linear topics, and he himself actively participated in such conferences. Specialists in quite diverse areas of science maintained and valued close contacts with Shil'nikov and his school. He had warm relations with the Moscow and Kiev mathematicians V. M. Alekseev, D. V. Anosov, Yu. S. Ilyashenko, A. B. Katok, E. F. Mishchenko, A. M. Samoilenko, Ya. G. Sinai, A. N. Sharkovskii, and many others. When opportunities to travel abroad appeared after 1990, Shil'nikov began receiving invitations to give talks at numerous international conferences (including the International Congress of Mathematicians in Beijing) and at universities in the USA, Belgium, France, Germany, Italy, and Israel. He also became a member of the editorial boards of several international journals. For many years he continued his collaboration with the Nobel Prize laureate I. R. Prigozhin and his colleagues, which started from the conference "Homoclinic chaos" (Brussels, 1991) devoted to Shil'nikov's work.

He was especially interested in the problem of a mathematical description of bifurcation scenarios concerning the onset of turbulence. He published several papers on this topic, in which he showed how initial local bifurcations of a regular dynamical regime give rise to the formation of geometric structures responsible for subsequent global bifurcations and chaos. Another area of activity, coming from physics, was the study of spatial chaos. Here, in a major series of joint papers with researchers in his department and with Moscow physicists from the group of V. M. Eleonskii, various local and global bifurcations in conservative and Hamiltonian systems were investigated, localized solutions with non-trivial structure were constructed, and a new type of homoclinic trajectories — homoclinics to homoclinic loops — was discovered and studied.

The principal subject of Shil'nikov's research always remained global bifurcations and strange attractors. Together with his son Andrei and D. V. Turaev he continued his studies of the Lorenz attractor. A new class of multidimensional strange attractors was discovered — wild hyperbolic attractors, that is, attracting sets that include homoclinic tangencies but do not contain stable trajectories. The study of homoclinic tangencies was continued in a cycle of papers with S. V. Gonchenko and Turaev. The main discovery, as Shil'nikov himself reckoned, was that bifurcations of systems with quadratic tangencies give rise to homoclinic tangencies of arbitrarily high order. Thus, it was explicitly shown that the traditional logic of analysis of bifurcations by the increase of codimension, coming from singularity theory, does not work in the study of many systems with complex dynamics. In particular, if there is a homoclinic tangency, then it is impossible in principle to obtain a complete description of all changes in the system!

Shil'nikov's papers on the so-called blue sky catastrophe became popular among specialists in non-linear dynamics. This type of bifurcations for flows on two-dimensional surfaces was discovered earlier by V. S. Medvedev. In the investigations of Shil'nikov (jointly with Turaev, A. L. Shil'nikov, and Gavrilov) new multidimensional examples were constructed and a general theory of such bifurcations was developed. As it turned out, these bifurcations may also give rise to the birth of non-trivial hyperbolic attractors. From the applied viewpoint, it is especially interesting that the blue sky catastrophe plays an important role in the dynamics of fast-slow systems, in particular, in the dynamics of neurons.

Altogether Shil'nikov published more than 160 papers and several books, including the two volumes of *Methods of qualitative theory in non-linear dynamics*, which were co-authored with Turaev, L. Chua, and A. L. Shil'nikov and have appeared in English, Russian, and Chinese. He continued to work and do research right up to his death. In his last papers he again returned to his favourite subject — the Lorenz attractor and its multidimensional generalizations.

Shil'nikov was one of the most outstanding specialists in the theory of dynamical systems, a founder of the theory of global bifurcations. He had many students who themselves became well-known specialists in the theory of dynamical systems, and they in turn have had students. He was a supervisor of sixteen Ph.D. dissertations, and four of his students became doctors of the sciences. A very important quality of his was scientific courage. When solving important but difficult problems he did not succumb to the temptation to follow a popular or fashionable path, but went his own way, creating new methods of investigation.

He was the indisputable leader of mathematical life in Nizhnii Novgorod, one of the initiators of the creation of the Nizhnii Novgorod Mathematical Society (1995) and its first president. His principal place of employment was the Institute of Applied Mathematics and Cybernetics of Nizhnii Novgorod University (before it was organized in 1963 he worked in the Mathematics Department of the Gor'kii Research Institute for Physics and Technology), where he was successively a senior research fellow, head of the laboratory (from 1967), and head of the department from 1982 almost up to the end of his life. He also taught at Nizhnii Novgorod University, giving both regular (undergraduate) and special (postgraduate) courses. Right up to his last year he taught the course "Bifurcations of multidimensional dynamical systems", which was very popular among university students. He always attracted the strong students; others simply would not have been able to work with him. Upon entering his circle, people immediately found themselves in an extraordinary atmosphere of keen interest in research.

Shil'nikov was a lively and interesting person, and a good family man. As a university student he met his future wife Lyudmila Ivanovna, and having lived with her for 55 years, he was invariably her faithful partner and friend, and for children and grandchildren he always remained an attentive and loving head of family, worthy of respect and emulation. He took great interest in history, especially the history of science, read avidly on such subjects, and liked football very much and was a connoisseur of the sport. Fishing at his dacha in the summers was a special passion of his.

For us Leonid Pavlovich will remain a great scientist, teacher, and a remarkable person. We shall always remember him, develop his ideas, and go further in research. May his memory live forever, and our profuse thanks to him for all that he accomplished.

D. V. Anosov, V. S. Afraimovich, L. A. Bunimovich, S. V. Gonchenko,
V. Z. Grines, Yu. S. Ilyashenko, A. B. Katok, S. A. Kashchenko,
V. V. Kozlov, L. M. Lerman, A. D. Morozov, A. I. Neishtadt,
Ya. B. Pesin, A. M. Samoilenko, Ya. G. Sinai, D. V. Treschev,
D. V. Turaev, A. N. Sharkovskii, and A. L. Shil'nikov

Translated by E. KHUKHRO