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Editorial

Leonid Pavlovich Shilnikov

This special issue presents a selection of papers from the conference "Dynamics, Bifurcations and Strange Attractors" dedicated to the memory of Leonid Pavlovich Shilnikov (1934–2011) to commemorate his contributions to the theory of dynamical systems and bifurcations. The conference was held at the Lobachevsky State University of Nizhny Novgorod, Russia, on 1–5 July 2013.

The conference was attended by 155 participants from all over the world, who contributed to the three focal topics: bifurcations and strange attractors; dynamical systems with additional structures (Hamiltonian, time-reversible, etc.); applications of dynamical systems. The topics were chosen in confluence with pivotal contributions by L. P. Shilnikov to the fields. The speakers presented their current research and outlined future directions in both theory and frontier applications.

The organizers of the conference are grateful to its sponsors: Russian Foundation of Basic Research, D. Zimin's Russian Charitable Foundation "Dynasty," R&D company Mera-NN, and K. V. Kirsenko (Russia), as well as Office of Naval Research (USA) and its officers, Drs. M. Harper (UK) and M. Shlesinger (USA).

We thank the Editors-in-Chief of the *International Journal of Bifurcations and Chaos*: Ron Chen and Leon Chua for having the proceedings published here. L. P. Shilnikov served on the Editorial Board of the journal from the time it was founded.

Our dear friend, mentor and fellow researcher, L. P. Shilnikov conceptualized the theory of global bifurcations of high-dimensional systems and was one of the founders of the mathematical theory of dynamical chaos. He built a profound research school in the city of Nizhny Novgorod (Gorky formerly) — the Shilnikov School that continues to this day.

His works greatly influenced the overall development of the mathematical theory of dynamical systems as well as nonlinear dynamics, in general. Shilnikov's findings have been included in most text- and reference books, and are used worldwide by mathematics students and nonlinear dynamists to study the qualitative theory of dynamical systems and chaos. The elegance and completeness of his results let them reach "the heart of the matter," and provide applied researchers with an in-depth mathematical understanding of the outcomes of natural experiments. The popularity and appreciation are due to the "living classic" status attained by Professor Shilnikov over several decades of his life through continuous hard work on bifurcation theory of multidimensional dynamical systems, mathematical chaos theory and theory of strange attractors.

L. P. Shilnikov was born in Kotelnich, Kirov region of Russia on December 17, 1934. After graduating from a local high school in 1952, he became a student in the Department of Physics and Mathematics at Gorky State University. After graduation in 1957, he continued his PhD studies at the same university. He defended his PhD thesis "On birth of stable periodic orbits from singular trajectories" in 1962, it focused on the multidimensional generalization of basic homoclinic bifurcations, which were originally discovered and studied for systems on a plane by A. A. Andronov and E. A. Leontovich in the early 1930s.

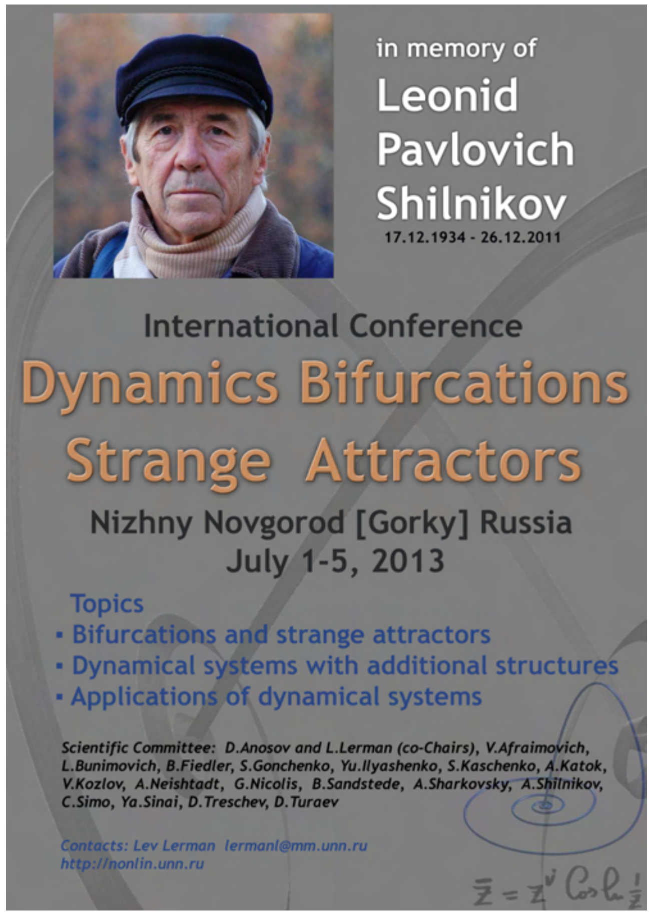

Fig. 1. Poster of the L. P. Shilnikov memorial conference in 2013.

Of very special interest were two nonlocal bifurcations that occur in systems with a homoclinic loop of either a saddle or a saddle-node equilibrium state. In the late 1950s and early 1960s, L. P. Shilnikov studied high-dimensional versions of these bifurcations. He identified the cases where the breakdown of the homoclinic loop would lead to the emergence of a single (stable) periodic orbit. This research direction went along the traditional lines drawn by the Andronov School in Gorky. Shortly after, in 1965, Leonid made his first groundbreaking discovery that fundamentally changed the views on the dynamics in systems of three or more dimensions — A homoclinic loop of a saddle-focus (satisfying what is now called the Shilnikov condition) that generates infinitely many periodic trajectories in the phase space. He showed that the corresponding Poincaré map near the homoclinic loop possesses infinitely many Smale horseshoes — a *de-facto* proof of chaotic dynamics in a system. Typically, a system with the Shilnikov saddle-focus exhibits "spiral chaos," a phenomenon that is now recognized as ubiquitously present in various high-dimensional systems, ranging in diverse applications from mathematics and physics to biology and finance. In 1965, a simple bifurcation leading to dynamically deterministic chaos was an astonishingly unexpected discovery. For Leonid, it was the starting point in a life-long quest — the study of systems with complex dynamics, which became the keynote theme of his research, and the research of his future students as well.

He soon found that the bifurcation of a saddle-saddle, or a Shilnikov saddle-node, with several homoclinic loops (still a codimension-1 bifurcation) gives rise to a nontrivial hyperbolic invariant set in the phase space of the system. This was the first genuine example

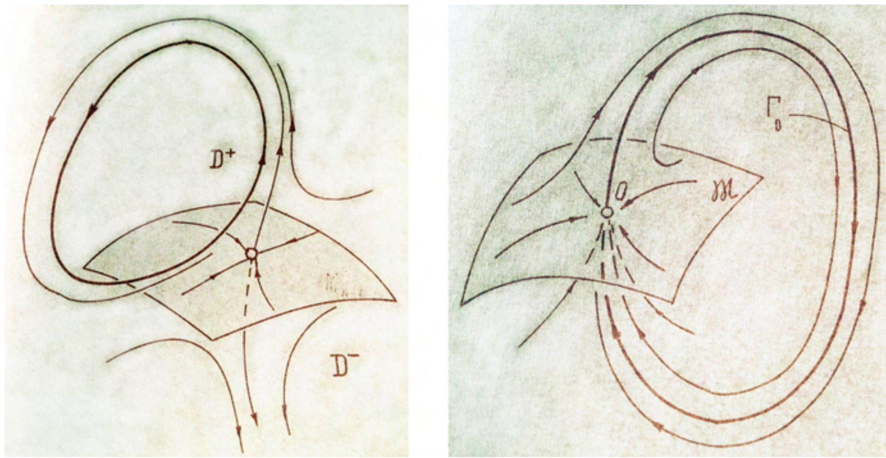

Fig. 2. (Left) A stable periodic orbit bifurcating from a homoclinic loop of saddle, and (right) a homoclinic loop of a saddle-node in $\mathbb{R}^3$. From L. P. Shilnikov's PhD thesis (1962).

of a "homoclinic Ω-explosion" that instantaneously causes chaotic dynamics upon crossing the boundary of Morse–Smale systems.

In a series of papers in 1965–67, Leonid published the complete solution to the famous Poincaré–Birkhoff problem on the structure of the set of trajectories in a neighborhood of a transverse homoclinic to a saddle periodic orbit. He proved that a system restricted to such an invariant set is conjugated to a suspension over the Bernoulli subshift on two symbols. Leonid considered this result as pivotal and kept reiterating that a transverse Poincaré homoclinic orbit would be a universal building block of Chaos Theory. Western authors often refer to this chaos criterion as S. Smale's result, which was published slightly earlier. It is fair to say that Shilnikov's theorem gave a complete description of the nonwandering set, whereas Smale described only a certain subset limited to the fulfillment of extra conditions on linearization properties, which may not always hold. To overcome problems with a linearization in resonant cases, Shilnikov had introduced an original technique for handling boundary-value problems near nonlinear saddles, known as the Shilnikov coordinates today. Leonid applied the new technique to study the structure of the set of trajectories near a homoclinic tube to an invariant torus. Later, with L. M. Lerman, they formulated and studied an analog of such homoclinic situations for general nonautonomous systems. His students further perfected this approach to solve a number of other homoclinic bifurcation problems.

Next came a series of works by Leonid himself, as well as papers done jointly with his students, which laid down the foundations for a new direction in dynamical systems, namely, the theory of global bifurcations. These can be subdivided into three groups: bifurcations within the Morse–Smale class; bifurcations occurring on boundaries between Morse–Smale systems and those with complex dynamics; and bifurcations in a class of systems with complex dynamics. Basic (codimension-1) nonlocal bifurcations of the first group were studied in Shilnikov's PhD thesis and his subsequent works (1968). A homoclinic loop of a Shilnikov saddle-focus (1965) was the first example of bifurcations from the third group. The first example from the second group is a Shilnikov saddle-node (1969). A bit later, he and his first PhD student N. K. Gavrilov studied homoclinic tangencies in a classic two publication series (1972–73). In the 1980s, Shilnikov continued the trend with his new coauthors S. V. Gonchenko and D. V. Turaev. Their findings, along with S. Newhouse's theory on dense structural instability, became the foundations for a new homoclinic chaos theory.

Another bifurcation of multidimensional systems, studied jointly with V. S. Afraimovich in 1974, described the disappearance of a saddle-node periodic orbit with the unstable

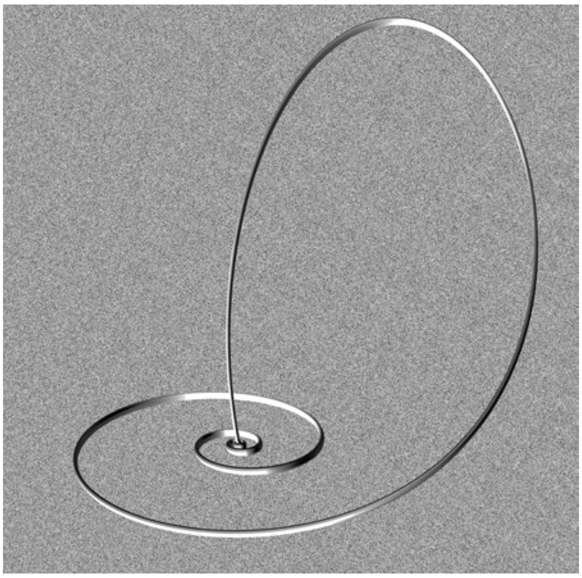

Fig. 3. Shilnikov saddle-focus loop.

manifold returning to the orbit throughout its stable region. This bifurcation only appears similar to a resonant torus bifurcation. However, they found a new exciting dynamical phenomenon at the breakdown of the torus, namely, the onset of chaotic dynamics and intermittency. This bifurcation happened to be quite generic and has been reported in various dynamical systems, and applied models which experience this "torus-chaos" transition. Later in 1978, Leonid and his student V. I. Lukyanov examined a similar bifurcation of a saddle-node periodic orbit with an unstable manifold returning transversally to a strongly stable one. This result provided a rigorous theoretical explanation to the phenomenon known as "transition to chaos through intermittency," which had been observed frequently in various applications featuring the coexistence of chaos with stable periodic orbits.

The key principle of the Andronov School of nonlinear oscillations in Gorky, and now in Nizhny Novgorod, was always its permanent focus on applications from physics, biology, etc. This principle was carried over by Professor Shilnikov as well: mathematically this meant solving a theoretical problem subjected to only a minimal number of constrains and conjectures. This would allow other researchers to use these transparent and sound mathematical methods for solving other applied problems as well. An exemplary problem, which became highly popular in the mid-1970s and remains so still, dealt with the structure of the strange attractor in the famous Lorenz model. The Lorenz attractor was a de-facto proof of dynamical chaos as a new fundamental phenomenon in nature. Unlike hyperbolic attractors, which were well understood, the strange attractor in the Lorenz model is structurally unstable and undergoes homoclinic and heteroclinic bifurcations as parameters vary. Leonid recognized this as soon as he learned about the Lorenz attractor, and he knew right away that his school had already developed and possessed the mathematical toolkit needed for its study. He was convinced that examination of this model would inevitably lead to very important breakthroughs for in-depth understanding of the nature and origin of deterministic chaos. Leonid proposed a phenomenological construction, called the geometric Lorenz model. This geometric model explained bifurcations, which occurred in the Lorenz model and led to the emergence of the strange attractor. These theoretical and numerical findings on the Lorenz attractor were reported in two remarkable papers done jointly with V. S. Afraimovich and V. V. Bykov in 1977 and 1982. These papers elaborated on the bifurcations and structure of the Lorenz attractor as a genuinely chaotic attractor (without stable orbits), which is structurally unstable in contrast to hyperbolic attractors. The team's research considerably advanced the state of the dynamical system theory beyond its time. Nearly simultaneously there came out a great array of independent papers on the Lorenz model and strange attractors in the West. The Afraimovich–Bykov–Shilnikov theory has remained the most complete and practical theory for other similar models and applications. It allows one to make feasible predictions on the evolutions of the Lorenz-like attractors, including the existence of lacunae, the detection of the existence of the Lorenz-like attractors in various models, as well as effective computations of various metric and ergodic properties of strange attractors. Later, Leonid proposed the new concept of a "quasi-attractor" — an attracting set containing coexisting hyperbolic subset and stable periodic orbits of long periods, as the most adequate mathematical image of dynamical chaos, which has since been reported frequently in a variety of applications.

Leonid Shilnikov and his students were responsible for instituting an array of research trends. Leonid published several influential papers proposing mathematical foundations and new plausible bifurcation scenarios for the onset of turbulence and spatial chaos. His collaboration with V. M. Eleonsky's physics group from Moscow focused on studies of bifurcations in conservative and Hamiltonian systems, where they discovered and described a new class of solutions — homoclinic trajectories to homoclinic loops. Notwithstanding, his main research theme focused on global bifurcations and strange attractors. With D. V. Turaev they developed the theory of wild and pseudo-hyperbolic strange attractors, which despite the presence of homoclinic tangencies contain no stable orbits. This passion of the father's continued in his son Andrey in examinations of local and global bifurcations leading to the emergence of strange attractors.

Leonid's joint work with D. V. Turaev (1995) brought in the generalization of both global bifurcation constructions described above. They discovered several novel bifurcations, called by the stunning name — "blue sky catastrophe." They showed that such codimension-1 homoclinic saddle-node bifurcations of periodic orbits could lead to the formation of strange hyperbolic attractors. In addition, they proposed a new bifurcation underlying the stability loss of periodic orbits, which remains the 7th, and last in the complete list of bifurcations of stable periodic orbits known to date. Leonid's works (jointly with D. V. Turaev, N. K. Gavrilov and A. L. Shilnikov) on the blue sky catastrophe have become very popular among specialists in neuronal dynamics with multiple time scales.

Leonid had his own, "Shilnikov's nonaxiomatic" style: the conditions of his theorems were meant to be understood with ease. Perhaps, because of that Leonid became a global attractor for many colleagues and research fellows from mathematics to physics, biology, neuroscience, chemistry and engineering, and he stayed that way always. Many academicians

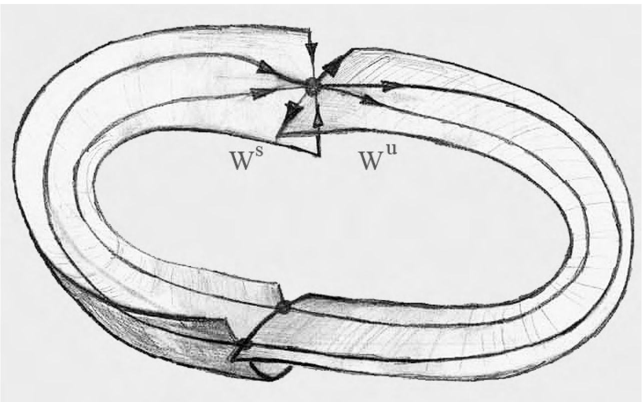

Fig. 4. Drawing of a Shilnikov saddle-node, or a saddle-saddle, with two homoclinic connections.

acknowledge that Shilnikov's ideas and charisma vastly influenced their own development, both professional and personal.

For several years, Professor Shilnikov ran the laboratory at the Department of Differential Equations at Research Institute for Applied Mathematics and Cybernetics, which was headed by E. A. Leontovich-Andronova. In 1984, Professor Shilnikov became the Head of the Department, which soon transformed into an informal science club for former students and like-minded colleagues. A weekly, two-hour long seminar led by Leonid was always a center of vivid and passionate discussions. To give a presentation at that seminar was considered both an honor and an invaluable experience. In addition to dynamical systems, the scope of the seminars was of a broad range, from algebra to theoretical physics. Shilnikov disseminated new ideas and findings from dynamical system theory, educating researchers in other fields about advances that would enable and inform their own research. He co-organized and delivered lectures at a large number of meetings and workshops on nonlinear dynamics.

For nearly four decades, Leonid Shilnikov lectured at his *alma mater* Gorky, and then later at Nizhny Novgorod State University. His course "Bifurcations of high-dimensional dynamical systems" was very popular with students of mathematics and physics. Leonid was able to demonstrate the beauty of mathematics to students, and is a part of why students with a keen interest in the sciences, highly valued his lectures and seminars. He was a magnet for strongly motivated students. Anyone who stepped into his attraction basin, sensed the extraordinary atmosphere of a true scientific environment.

L. P. Shilnikov was one of the celebrated and influential figures in the theory of dynamical systems. He possessed the very pivotal qualities of an exceptional researcher and human being — ethics, integrity and scientific bravery. He did science his way, by creating new directions and original methods. Leonid was a wise and principled mentor for his PhD students, who became active and independent researchers: N. K. Gavrilov, V. S. Afraimovich, A. D. Morozov, L. M. Lerman, L. A. Belyakov, V. V. Bykov, V. I. Lukyanov, A. N. Bautin, S. V. Gonchenko, M. I. Malkin, A. L. Shilnikov, D. V. Turaev, M. V. Shashkov, O. V. Stenkin, I. V. Belykh, V. S. Gonchenko, and scholars at his department: I. M. Ovsyannikov and V. S. Biragov, as well as G. M. Polotovskii, N. V. Roschin, Ya. L. Umanskii, V. Z. Grines and E. V. Zhuzhoma.

Leonid was the leader, spiritual and fraternal, of the Nizhny Novgorod mathematical community. He helped found the Nizhny Novgorod Mathematical Society, and was its first president. He did not miss a single instance of its meetings, unless he was traveling.

Leonid published nearly two hundred research papers and coauthored a few books. Among those are, *Methods of Qualitative Theory in Nonlinear Dynamics* (*Parts 1 and 2*) with A. L. Shilnikov, D. V. Turaev, and L. Chua (1998, 2001), and its recent translations in Russian (2003, 2009) and Chinese (2010), as well as *Bifurcation Theory* with V. I. Arnold, V. S. Afraimovich, Yu. S. Ilyashenko, in Russian (1986) and English (1994). In the last papers he returned to his favorite subject — the Lorenz attractor and its multidimensional generalizations.

Professor Shilnikov's scientific achievements were acknowledged by several awards, including A. M. Lyapunov Award of Russian Academy of Science (1998), M. A. Lavrentiev Award of National Academy of Science of Ukraine (2005), and Professorship of Alexander von Humboldt Foundation of Germany in 2002. He served on editorial boards for many peer-reviewed journals. He was a keynote speaker at a large number of conferences and workshops held in Russia and all over the world. He was invited to visit and speak at leading research universities in USA, Belgium, France, Israel, Germany, Italy and China. For several years, Leonid's group collaborated with Nobel Prize awardee I. R. Prigogine and his colleagues at Universite Libre de Bruxelles beginning with the acclaimed conference

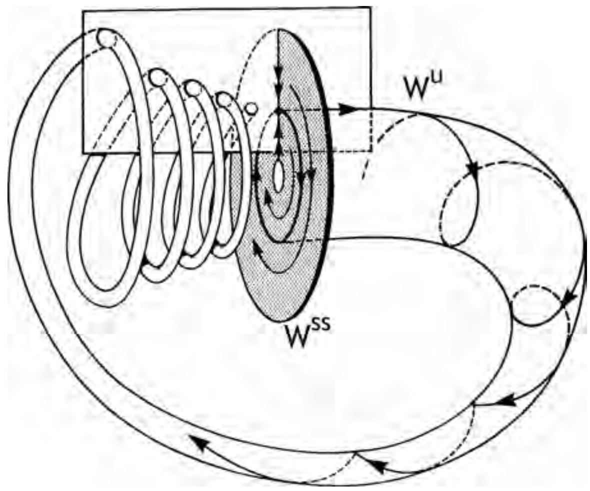

Fig. 5. Shilnikov–Turaev construction of the blue sky catastrophe giving rise to a stable periodic orbit of infinite length and period.

"Homoclinic Chaos" (Brussels, 1991), honoring Shilnikov's contributions to mathematics and nonlinear dynamics.

During his university years, Leonid Pavlovich met his future wife Lyudmila Ivanovna, and they lived together for 55 years. He was her faithful companion and friend, and for their children and grandchildren, he remained their considerate and fair family head. Leonid Pavlovich was an unquestionable role model worthy of the utmost respect. He was a book collector and an avid history pundit, particularly the history of science. He was an aficionado of football since his teenage years, and summertime fishing on the Volga River was his passion and source of tranquility.

For each and every one of us, Leonid Pavlovich Shilnikov will always remain a Teacher, an extraordinary Expert and a Visionary in science.

On behalf of the Organizers of the L. P. Shilnikov memorial conference

Guest Editors
A. L. Shilnikov and D. V. Turaev
along with L. M. Lerman, V. S. Afraimovich, S. V. Gonchenko
and L. A. Belyakov
(*Institute for Applied Mathematics and Cybernetics*
Lobachevsky State University of Nizhny Novgorod)