Georgia State University
(*This paper consists of pages.*)

Test III **November 24, 2003** **Points: 91+ is A**

Last name:	______________________
First name:	______________________

POINTS

Show all of your work. Calculators are not needed or permitted. Write neatly. Place answers in the space provided.

1 (5 points). Find the point(s) at which the twisted cubic

$$\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$$

intersects the plane $4x + 2y + z = 24$. Find the angle of intersection.

2. A surface is represented by the equation $F(x, y, z) = xy + 2xz^2 + 3yz = 56$. Find

a. (10 points) the equation of the plane tangent to this surface at$(2, 1, 3)$;

b (10 points) Find the directional derivative of $F(x, y, z)$ at the point $(2, 1, 3)$ in the direction of $\mathbf{v} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$;

c. (5 points) Find $\frac{\partial z}{\partial y}$ on this surface at $(2, 1, 3)$ by forming a **difference quotient**.

3. Find and classify the stationary points of the function $f(x, y) = x^3 - xy^3 + 3xy$.

4 (10 points). Find the most general function, if any!, with the given gradient

$$\left(2\ln(3y) + \frac{1}{x}\right)\mathbf{i} + \left(\frac{2x}{y} + y^2\right)\mathbf{j}$$

5 (10 points). Let $z = x^2 - y^2 - xy$ and let C be the curve of intersection of the surface with the plane $y = 3$. Find the equation for the tangent line to the graph of C at the point $(-3, 3, 9)$.

6 (10 points). Let both x and y be changing with time t and at a certain instant you know that $x = 1, \quad y = 2, \quad x'(t) = 2, \quad y'(t) = -2$. Use the chain rule to find $Q'(t)$ at this instant.

7 (10 points). Find $\lim_{(x,y)\to(0,0)} \dfrac{x^2y^2}{x^4+y^4}$ or show that it does not exist.

8 (15 points). Let $f(x,y) = 2x\sqrt{x+2y^2}$.

Find the gradient vector of $f(x,y)$ $\quad \triangledown f(x,y) =$

Find the directional derivative of $f(x,y)$ at $(x,y) = (2,-1)$ in the direction of the vector $\mathbf{a} = -\mathbf{i} + 4\mathbf{j}$

Find a unit vector in the direction in which $f(x,y)$ does not change at $(x,y) = (2,-1)$

9 (10 points). Determine whether a function $z = f(x, y)$ with the following gradient $\bigtriangledown f(z, y) = (x^2 + y)\mathbf{i} + (y^3 + x)\mathbf{j}$ may exist. If so, find such a function.

10 (10 points). Let $f(x, y) = 2x^2 - y^2 - 1$ be C^1 everywhere. Let $\mathbf{a}(0, 0)$ and $\mathbf{b}(1, 1)$. Find the point $\mathbf{c}$ on the line segment connecting $\mathbf{a}$ and $\mathbf{b}$ where $f(\mathbf{b}) - f(\mathbf{a}) = \bigtriangledown f(\mathbf{c}) \cdot (\mathbf{b} - \mathbf{a})$.

11 (10 points). Let $f(x, y) = 2x^2/y + \sin(ln(x)e^{xy})$. Find dy/dx.

12 (10 points). Show that the sphere $x^2 + y^2 + z^2 - 8x - 8y - 6z + 24$ is tangent to the ellipsoid $x^2 + 3y^2 + 2z^2 = 9$ at the point $(2, 1, 1)$. Find the equations of the tangents planes at this point. What is the equation of the normal line at the point of tangency?