Georgia <u>State</u> University

(*This paper consists of pages.*)

Test I **September 22, 2003** **Points: A** $\geq 91+$

Last name:	____________________	**POINTS**
First name:	____________________	

Show all of your work. Calculators are not needed or permitted. Write neatly. Place answers in the space provided.

1 (15 points). Let $L : V \to W$ be a linear operator. Prove that $\ker(L)$ is a subspace in W.

2 (15 points). Find the kernel and the range of the following linear transformation $L : P_3 \to P_3$
L(p(x))=p(0)x-p'(x)(x-1)

3 (15 points). *** Let L be a linear transformation of the vector space into itself. Show by induction that $L^{n+1}(\mathbf{x}) = L(L^n(\mathbf{x}))$ is a linear transformation as well.

4 (20 points). *** Prove the matrix representation theorem: L maps $\mathbf{v}$ into $\mathbf{w}$ iff $A\mathbf{x} = \mathbf{y}$, where $\mathbf{x} = [\mathbf{v}]_E$ and $\mathbf{y} = [\mathbf{w}]_F$

5 (10 points). Find the standard matrix for the following linear operator: (a) rotation though $\pi/3$ clockwise followed by reflection around the vertical axis, followed by projection on the x_1-axis.

6 (15 points). Let $L(p(x)) = p'(x) + p(0)x$. Find the matrix representation of L with respect to the ordered bases $E = \{1, x, x^2\}$ and $F = \{-1, x+1\}$. Find the image of $-x^2 + 3x - 2$ in the given bases.

7 (15 points). Let $L : R^3 \to R^3$ be multiplication given by

$$\begin{pmatrix} 1 & 3 & 4 \\ 3 & 4 & 7 \\ -2 & 2 & 0 \end{pmatrix}$$

Show that the kernel of L is a line through the origin. Find its equation (parametric or symmetric).

8 (15 points). Let $L : R^2 \to R^3$ be defined by $L((x_1, x_2)^T) = (x_1 + 2x_2, -x_1, 0)^T$. Find the matrix representation of L with respect to the bases $E = \{\mathbf{u}_1, \mathbf{u}_2\}$ and $F = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, where

$$\mathbf{u}_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}, \quad \mathbf{u}_2 = \begin{pmatrix} -2 \\ 4 \end{pmatrix}, \quad \mathbf{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$$

9 (30 points). Let $L : R^3 \to R^3$ be defined by

$$L\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + 2x_2 - x_3 \\ -x_1 \\ x_1 + 7x_3 \end{pmatrix}.$$

(a) Find the matrix representation of L with respect to the standards. (b) Find the matrix representation of L with respect to $F = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, where

$$\mathbf{u}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{u}_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{u}_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

(c) Find the transition matrix S and its inverse.

10 (10 points). *** Suppose $A = \Lambda B \Lambda^{-1}$, where Λ is a diagonal matrix with elements $(\lambda_1, \cdots, \lambda_n)$. Show that $A\mathbf{s}_i = \lambda \mathbf{s}_i$, where $i = 1, \ldots, n$.

11 (15 points). Show that if A and B are similar, then their multiplicative inverses are similar too.

12 (20 points). *** Prove the change of basis theorem.