Georgia <u>State</u> University

(*This paper consists of pages.*)

Test II **due October 15, 2003** **Points: A** $\geq 91+$

Last name: ______________________________

First name: ______________________________

POINTS

Calculators are not needed or permitted. Write neatly. Place answers in the space provided. **The full credit is given only if all intermediate calculations and the entire work are shown.**

♦♦♦♦ **Problems for 6000 level are marked with asterisks**

1 (15 points). $* * * * **$

(a) Let $\mathbf{v} \neq \mathbf{0}$ in $\mathbb{R}^n$. Show that $\mathbf{v}^T\mathbf{y} = \mathbf{v}^T\mathbf{x}$ does not necessary imply that $\mathbf{y} = \mathbf{x}$. Draw the figure.

(b) Show that if $\mathbf{u}^T\mathbf{x} = \mathbf{u}^T\mathbf{y}$ for all **unit** vectors then $\mathbf{y} = \mathbf{x}$.

2 (10 points). $* * * * * * *$
Show that $\mathbf{u} \perp \mathbf{v}$ in $\mathbb{R}^n$ iff $||\mathbf{v} + \mathbf{u}|| = ||\mathbf{v} - \mathbf{u}||$

3 (10 points). Verify the *parallelogram* law for vectors v and u in $\mathbb{R}^n$

$$||\mathbf{v}+\mathbf{u}||^2 + ||\mathbf{v}-\mathbf{u}||^2 = 2||\mathbf{u}||^2 + 2||\mathbf{v}||^2$$

Sketch the figure.

4 (5 points). Find an equation in x, y and z for the plane that passes through the given points $P(3,2,-1)$, $Q(3,-2,4)$ and $Q(1,-1,3)$

5 (5 points). Find the distance between the planes $2x - 4y + 2z - 6 = 0$ and $-x + 2y - 1z - 3 = 0$

6 (5 points). Let $W = \text{Span}\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$. Show that if $\mathbf{x}$ is orthogonal to each $\mathbf{v}_i$, $1 \le i \le n$, then $\mathbf{x}$ is orthogonal to every vector in W.

7 (10 points). $******$
Given $\mathbf{u} \neq 0$ in $\mathbb{R}^n$, let $L = \text{Span}\{\mathbf{u}\}$. Show that the mapping

$$\mathbf{v} \to 2\frac{\mathbf{u}^T\mathbf{v}}{\mathbf{u}^T\mathbf{u}}\mathbf{v} - \mathbf{v}$$

is a linear transformation. Draw the figure. Define this transformation.

8 (10 points). Let B be a square matrix with *orthonormal* (i.g. orthogonal and normalized) columns. Show that B is invertible. Mention all theorems you use.

9 (10 points). Show that the mapping $\mathbf{x} \to B\mathbf{x}$ with such matrix B above preserves the norm of the vector $\mathbf{x}$.

10 (5 points). Find the point on the line $x + 2y - 1 = 0$ closest to the point $P(2, 1)$.

11 (10 points). Find R^{A^T}, $N(A)$, R^A and $N(A^T)$ for

$$A = \begin{pmatrix} 2 & -2 \\ 3 & 4 \\ 1 & 0 \end{pmatrix}$$

12 (5 points). Find the basis for $S^{\perp} \in \mathbb{R}^4$ where S is spanned by $(1,1,0,0)^T$ and $(0,0,1,1)$

13 (10 points). ****** Let A be an $m \times n$ matrix with linearly independent columns. Use the normal equation $A^T A\hat{\mathbf{x}} = A^T\mathbf{b}$ to produce a formula for $\hat{\mathbf{b}}$ — a vector projection of $\mathbf{b}$ onto $R(A)$. Hint: find $\hat{\mathbf{x}}$ first.

14 (10 points). ******* Verify that the error vector $\mathbf{b} - A\hat{\mathbf{x}}$, where $\hat{\mathbf{x}}$, is orthogonal to the column space of the matrix A; here

$$A = \begin{pmatrix} 1 & 1 \\ -1 & 1 \\ -1 & 2 \end{pmatrix} \qquad b = \begin{pmatrix} 7 \\ 0 \\ -7 \end{pmatrix}$$

15 (15 points). $* * * * **$

Find the equation $y = c_1 x + c_2 x^2$ of the least square quadratic curve that best fits the given data:

x	-1	0	1	2
y	1	0	2	5

Put the points and draw the found curve in (x, y)-plane.

★★★ **Must!!!** Double-check yourself — solve this problem using the **calculus approach** based on minimization of a function of two variables.