Georgia State University
(*This paper consists of pages.*)

Final **December 15, 2003** **Points: A** $\geq 91+$

Last name: ____________________
First name: ____________________

POINTS

Calculators are not needed or permitted. Write neatly. Place answers in the space provided. Full credit is given only if an entire work including intermediate calculations is shown.

♦♦♦♦ **6000 level "Must" problems are marked with asterisks**

1 (15 points). $******$
Prove that a matrix $A \in R^{n\times n}$ with n distinct eigenvalues has n linearly independent vectors.

2 (15 points). $* * * * * * *$
Prove that all eigenvalues of an Hermitian matrix are real, and that eigenvectors corresponding to distinct eigenvalues are orthogonal.

3 (20 points). $* * * * * * *$
Compute A^6, where

$$A = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix}$$

Show that $AX = DX$ too.

4 (10 points). Show that e^A is nonsingular for any diagonalizable matrix A

5 (10 points). Show that if A is stochastic, then $\lambda = 1$ is its eigenvalue.

6 (15 points). $*******$
Find a unitary diagonalizing matrix for each of the following:

$$A = \begin{pmatrix} 1 & 3+i \\ 3-i & 4 \end{pmatrix}$$

Is A Hermitian?

7 (15 points). * * * * * * *
Let U be a unitary matrix. Prove:

(a) U is normal;

(b) $||U\mathbf{x}|| = ||\mathbf{x}||$ for any $\mathbf{x} \in C^n$;

(c) if λ is an eigenvalue of U, then $|\lambda| = 1$

8 (15 points). Prove that a eigenvectors of a normal matrix $A \in C^{n \times n}$ form an orthonormal basis for C^n