

CHAPTER 1

MAKING SENSE OF DATA AND FUNCTIONS

OVERVIEW

How can you describe patterns in data? We explore in this chapter how to use graphs to visualize the shape of single-variable data and to show changes in two-variable data. Functions, a fundamental concept in mathematics, are introduced and used to model change.

After reading this chapter, you should be able to

- describe patterns in single- and two-variable data
- construct a “60-second summary”
- define a function and represent it in multiple ways
- identify properties of functions
- use the language of functions to describe and create graphs

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1.1 Describing Single-Variable Data

This course starts with you. How would you describe yourself to others? Are you a 5-foot 6-inch, black, 26-year-old female studying biology? Or perhaps you are a 5-foot 10-inch, Chinese, 18-year-old male English major. In statistical terms, characteristics such as height, race, age, and major that vary from person to person are called *variables*. Information collected about a variable is called *data*.¹

Some variables, such as age, height, or number of people in your household, can be represented by a number and a unit of measure (such as 18 years, 6 feet, or 3 people). These are called *quantitative variables*. For other variables, such as gender or college major, we use categories (such as male and female or biology and English) to classify information. These are called *categorical* (or *qualitative*) data. The dividing line between classifying a variable as categorical or quantitative is not always clear-cut. For example, you could ask individuals to list their years of education (making education a quantitative variable) or ask for their highest educational category, such as college or graduate school (making education a categorical variable).



Exploration 1.1
provides an opportunity to collect your own data and to think about issues related to classifying and interpreting data.

Many of the controversies in the social sciences have centered on how particular variables are defined and measured. For nearly two centuries, the categories used by the U.S. Census Bureau to classify race and ethnicity have been the subject of debate. For example, Hispanic used to be considered a racial classification. It is now considered an ethnic classification, since Hispanics can be black, or white, or any other race.

In this section we focus on analyzing single-variable data using visual displays (bar charts, histograms, and pie charts) and numeric descriptors (mean and median). We'll also discuss the elements of producing a good chart, including citing a reliable source.

Visualizing Single-Variable Data

Humans are visual creatures. Converting data to an image can make it much easier to recognize patterns.

Bar charts: How well educated are Americans?

Categorical data are usually displayed with a bar chart. Typically the categories are listed on the horizontal axis. The height of the bar above a single category tells you either the *frequency count* (the number of observations that fall into that category) or the *relative frequency* (the percentage of total observations). Since the relative size of the bars is the same using either frequency or relative frequency counts, we often put the two scales on different vertical axes of the same chart. For example, look at the vertical scales on the left- and right-hand side of Figure 1.1, a bar chart of the educational attainment of Americans age 25 or older in 2002. Note that the data source is the U.S. Bureau of the Census, so we would expect the numbers to be trustworthy.

The vertical scale on the left tells us the number (the frequency count) of Americans who fell into each educational category. For example, in 2002 over 50 million Americans age 25 or older had a high school degree but never went on to college.

It's often more useful to know the percentage (the relative frequency count) of all Americans who have at most a high school degree. Given that in 2002 the number of

¹*Data* is the plural of the Latin word *datum* (meaning “something given”)—hence one datum, two data.

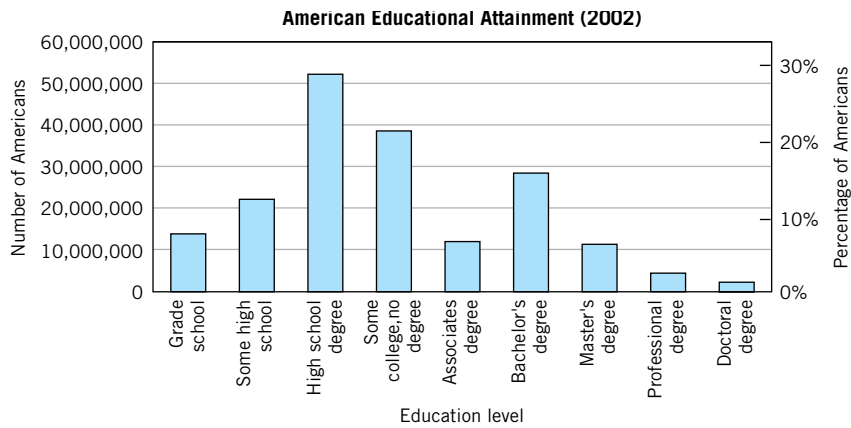


Figure 1.1 Bar chart showing the education levels for Americans age 25 or older. Source: U.S. Bureau of the Census, www.census.gov

people 25 years or older was 182,211,639 and the number who had at most a high school degree was 52,168,981, then the percentage of those with at most a high school degree was

$$\frac{\text{Number with at most a high school degree}}{\text{Total number of people age 25 or older}} = \frac{52,168,981}{182,211,639} = 0.286 \text{ (in decimal form) or } 28.6\%$$

The vertical scale on the right tells us the percentage (relative frequency). Using this scale, the percentage of Americans with at most a high school degree was slightly under 30%, which is consistent with our calculation.

EXAMPLE 1 What does the bar chart tell us?

- Using Figure 1.1, estimate the number and percentage of people age 25 or older who have bachelor’s degrees, but no further advanced education.
- Estimate the total number of people who have at least a high school education.
- What doesn’t the bar chart tell us?
- Write a brief summary of educational attainment in the United States.

- Solution**
- Those with bachelor’s degrees but no further education number about 28 million, or 16%.
 - Those who have completed a high school education include everyone with a high school degree up to a Ph.D. We could add up all the numbers (or percentages) for each of those seven categories. But it’s easier to subtract from the whole those who do not meet the conditions, that is, subtract those with either a grade school or only some high school education from the total population (people age 25 or older) of about 182 million.

| Grade School + Some High School = Total without High School Degree | | |
|--|-------------------------|--------------|
| Number | 14 million + 22 million | = 36 million |
| Percentage | 8% + 12% | = 20% |

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The number of Americans (age 25 or over) with a high school degree is about $182 \text{ million} - 36 \text{ million} = 146 \text{ million}$. The corresponding percentage is about $100\% - 20\% = 80\%$. So about four out of five Americans 25 years or older have completed high school.

- c. The bar chart does not tell us the total size of the population or the total number (or percentage) of Americans who have a high school degree. For example, if we include younger Americans age 18 or older, the percentage with a high school degree is almost 90%.
- d. About 80% of adult Americans (age 25 or older) have at least a high school education. About 30% completed high school but did not go on. An additional 40% have some college (up to a bachelor's degree) and about 10% have graduate degrees. This is not surprising, since the United States ranks among the mostly highly educated countries in the world.

An important aside: What a good graph should contain

When you encounter a graph in an article or you produce one for a class, there are three elements that should always be present:

1. An informative title that succinctly describes the graph
2. Clearly labeled axes (or a legend) including the units of measurement (e.g., indicating whether age is measured in months or years)
3. The source of the data cited in the data table, in the text, or on the graph



See the program "F1: Histograms."

Histograms: How well do Americans age?

Histograms are a specialized form of bar charts that are used to visualize single-variable quantitative data. Typically, the horizontal line on a histogram is a subset of the real numbers with the unit attached (representing, for example, number of years). Tick marks are placed on the real-number line to specify the size of each interval. The intervals are usually evenly spaced to facilitate comparisons (e.g., placed every 10 years). The size of the interval can help or obscure patterns in the data. As with a bar chart, the vertical axis can be labeled with a frequency or a relative frequency count. For example, Figure 1.2 shows the distribution of ages in the United States in 2002 (data again from the U.S. Bureau of the Census).

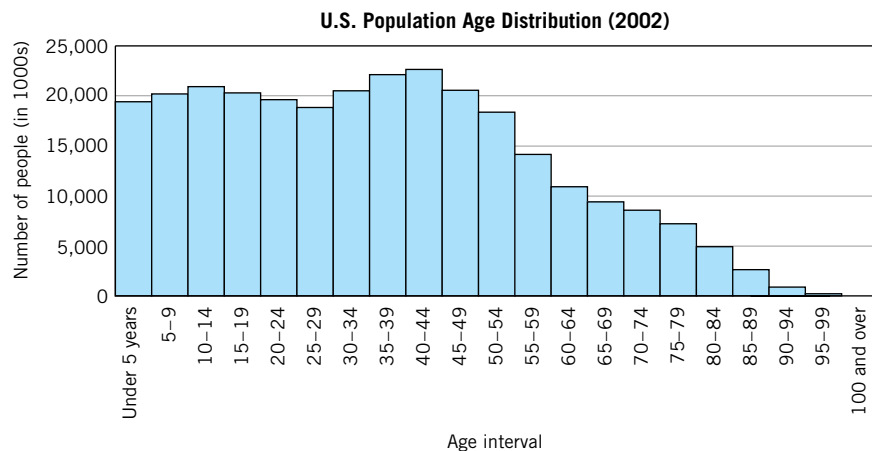


Figure 1.2 Age distribution of the U.S. population in 5-year intervals.

Source: U.S. Bureau of the Census, www.census.gov

EXAMPLE 2

- What 5-year age interval contains the most Americans? Roughly how many are in that interval? (Refer to Figure 1.2)
- Estimate the number of people under age 20.
- What might the bump between ages 30 and 50 mean?
- Construct a topic sentence for a report about the U.S. population.

Solution

- The interval between 40 and 44 years contains the largest number of Americans, about 23 million.
- The sum of the frequency counts for the four intervals below age 20 is about 80 million.
- This bump is most likely the “baby boom” resulting from the surge in births right after World War II.
- According to the U.S. Census Bureau 2002 data, the number of Americans in each 5-year age interval remained fairly flat up to age 30, peaked between ages 30 to 50, then fell in a gradual decline.

EXAMPLE 3

Describe the age distribution for Tanzania, one of the poorest countries in the world (Figure 1.3).

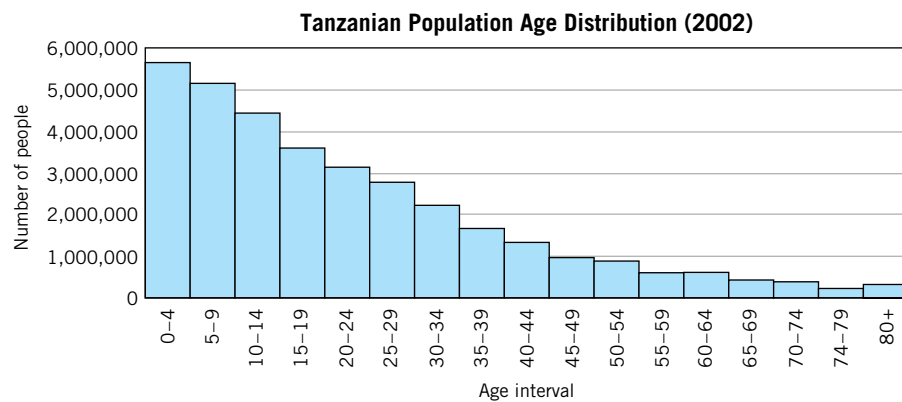


Figure 1.3 The age distribution in 2002 of the Tanzanian population in 5-year intervals.
Source: U.S. Bureau of the Census, International Data Base, www.census.gov

Solution

The age distributions in Tanzania and the United States are quite different. Tanzania is a much smaller country and has a profile typical of a developing country; that is, each subsequent 5-year interval has fewer people. For example, there are about 5.7 million children between 0 to 4 years old, but only about 5.1 million children age 5–9 years, a drop of over 10%. For ages 35 to 40 years, there are only about 1.7 million people, less than a third of the number of children between 0 and 4 years. Although the histogram gives a static picture of the Tanzanian population, the shape suggests that mortality rates are much higher than in the United States.

**SOMETHING TO THINK ABOUT**

What are some trade-offs in using pie charts versus histograms?

Pie charts: Who gets the biggest piece?

Both histograms and bar charts can be transformed into pie charts. For example, Figure 1.4 shows two pie charts of the U.S. and Tanzanian age distributions (both now divided into 20-year intervals). One advantage of using a pie chart is that it clearly shows the size of

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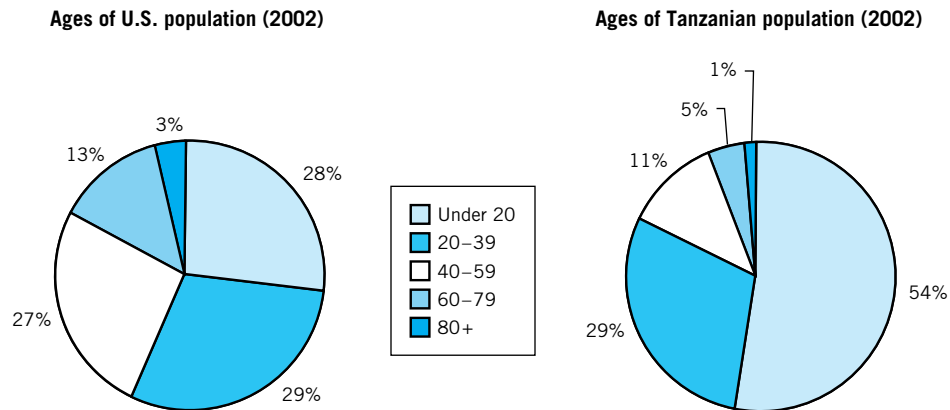


Figure 1.4 Two pie charts displaying information about the U.S. and Tanzanian age distributions. Source: U.S. Bureau of the Census, www.census.gov.

each piece relative to the whole. Hence, they are usually labeled with percentages rather than frequency counts.

In the United States the first three 20-year age intervals (under 20, 20–39, and 40–59 years) are all approximately equal in size and together make up 84% of the population. Those 60 and older represent the remaining 16% of the population.

In Tanzania, the proportions are entirely different. Over half of the population are under 20 years and more than 80% are under 40 years old. Those 60 and older make up only 6% of Tanzania’s population.

EXAMPLE 4

Federal spending is divided into two broad categories, mandatory and discretionary. Mandatory spending includes most entitlement programs such as Social Security, Medicare, and Medicaid, as well as interest on the national debt. Discretionary spending covers everything from road building to police protection, from medical research to national defense. Figure 1.5 contains two pie charts displaying the proportions of mandatory and discretionary components of the budget in 1966 and 2006 (projected).

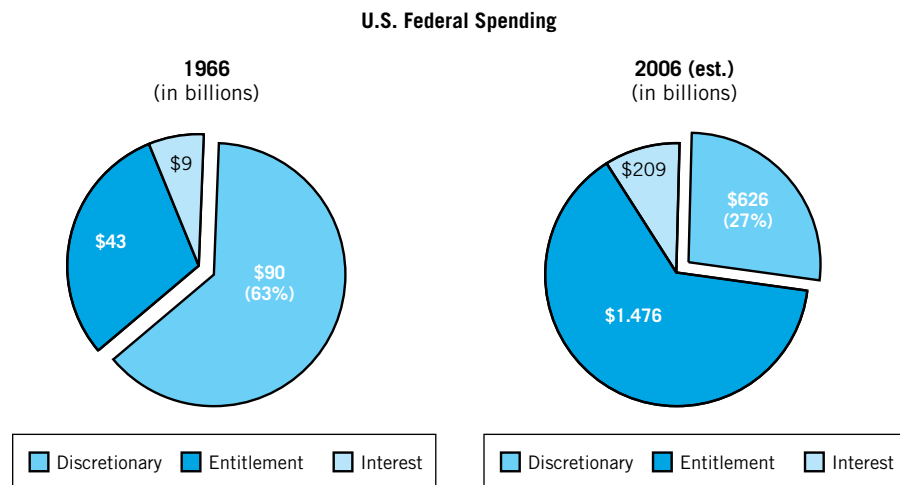


Figure 1.5 Breakdown of federal spending in 1966 and 2006 (projected). Source: U.S. Senate Committee on Appropriations, appropriations.senate.gov/budgetprocess

- a. What were the total federal expenditures in 1966? What are they projected to be in 2006?
- b. What striking changes take place between the 1966 and 2006?

Solution

- a. Total federal expenditures were \$142 billion in 1966 and are projected to be \$2,311 billion (or \$2.311 trillion) in 2006.
- b. Federal spending is projected to increase dramatically, along with a substantial shift in budget proportions. Mandatory spending (on entitlements and the debt interest) will shift from 37% to 73% of the budget. However, because of the increased total budget size, the expenditures in each category will increase considerably.

Mean and Median: What Is “Average” Anyway?

In 2002 the U.S. Bureau of the Census reported that the mean age for Americans was 36.1 and the median age was 35.7.

The Mean and Median

The *mean* is the sum of a list of numbers divided by the number of terms in the list.

The *median* is the middle value of an ordered numerical list; half the numbers lie at or below the median and half at or above it.



Read the excerpt from *CHANCE News*, an electronic magazine, to see how the mean is sometimes incorrectly used in the popular press.

The mean age of 36.1 represents the sum of the ages of every American divided by the total number of Americans. The median age of 35.7 means that if you placed all the ages in order, 35.7 would lie right in the middle; that is, half of Americans are younger or equal to 35.7 and half are 35.7 or older.

The significance of the mean and median

The median divides the number of entries in a data set into two equal halves. If the median age in a large urban housing project is 17, then half the population is 17 or under. Hence, issues such as day care, recreation, and education should be high priorities with the management. If the median age were 55, then issues such as health care and wheelchair accessibility might dominate the management’s concerns.

The median is unchanged by changes in values above and below it. For example, as long as the median income is larger than the poverty level, it will remain the same even if all poor people suddenly increase their incomes up to that level and everyone else’s income remains the same.

The mean is the most commonly cited statistic in the news media. One advantage of the mean is that it can be used for calculations relating to the whole data set. Suppose a corporation wants to open a new factory similar to its other factories. If the managers know the mean cost of wages and benefits for employees, they can make an estimate of what it will cost to employ the number of workers needed to run the new factory:

$$\text{total employee cost} = (\text{mean cost for employees}) \cdot (\text{number of employees})$$

The mean, unlike the median, can be affected by a few extreme values called *outliers*. For example, suppose Bill Gates, founder of Microsoft and the richest man in the world, were to move into a town of 10,000 people, all of whom earned nothing.



See the reading “The Median Isn’t the Message” to find out how an understanding of the median gave renewed hope to the renowned scientist Stephen J. Gould when he was diagnosed with cancer.

The median income would be \$0, but the mean income would be in the millions. That's why income studies usually use the median.

EXAMPLE 5 “Million-dollar Manhattan apartment? Just about average”²

According to *The New York Times*, in 2003 the median price of purchasing an apartment in Manhattan was \$575,000 and the mean price was \$916,959. How could there be such a difference in price? Which value do you think better represents apartment prices in Manhattan?

Solution A few apartments that sold for exorbitant prices (in the millions) could act as outliers, raising the mean above the median. If you want to buy an apartment in Manhattan, the median price is probably more important because it tells you that half the apartments cost \$575,000 or less.



SOMETHING TO THINK ABOUT

If someone tells you that in his town “all of the children are above average,” you might be skeptical. (This is called the “Lake Wobegon effect.”) But could most (more than half) of the children be above average? Explain.

In the press you will most likely encounter the word “average” rather than the term “mean” or “median.”³ The term “average” is used very loosely. It usually represents the mean, but it could also represent the median or something much more vague, such as the “average” American household. For example, the media reported that:

- The *average* American household has a mean of 2.58 people and a median household income of about \$42,655.⁴
- In the *average* household, the TV is turned on an average of 7 hours a day.⁵
- The *average* household has more than seven credit cards with a credit card balance of \$8,387 on which they are paying an “average” annual interest rate of 18.9%.⁶

An Introduction to Algebra Aerobics

In each section of the text there are “Algebra Aerobics” workouts with answers in the back of the book. They are intended to give you practice in the algebraic skills introduced in the section and to review skills we assume you have learned in other courses. These skills should provide a good foundation for doing the exercises at the end of each chapter. The exercises include more complex and challenging problems and have answers for only the odd-numbered ones. We recommend you work out these Algebra Aerobics practice problems and then check your solutions in the back of this book. The Algebra Aerobics are numbered according to the section of the book in which they occur.

²Source: *The New York Times*, Oct. 15, 2003.

³The word “average” has an interesting derivation according to Klein’s etymological dictionary. It comes from the Arabic word *awariyan*, which means “merchandise damaged by seawater.” The idea being debated was that if your ships arrived with water-damaged merchandise, should you have to bear all the losses yourself or should they be spread around, or “averaged,” among all the other merchants? The words *averia* in Spanish, *avaria* in Italian, and *avarie* in French still mean “damage.”

⁴Source: U.S. Bureau of the Census, www.census.gov.

⁵Source: 2002, *Trans Union LLC, TrenData Report*.

⁶Source: 2002, *Trans Union LLC, TrenData Report*.

Algebra Aerobics 1.1

1. Fill in Table 1.1. Round decimals to the nearest thousandth.

| Fraction | Decimal | Percent |
|-----------------|---------|---------|
| $\frac{7}{12}$ | | |
| | 0.025 | 2% |
| $\frac{1}{200}$ | | |
| | 0.35 | 0.75% |

Table 1.1

2. Calculate the following:
- A survey reported that 80 people, or 16% of the group, were smokers. How many people were surveyed?
 - Of the 236 students who took a test, 16.5% received a B grade. How many students received a B grade?
 - Six of the 16 people present were from foreign countries. What percent were foreigners?
3. When looking through the classified ads, you found that sixteen jobs had a starting salary of \$20,000, eight had a starting salary of \$32,000, and one had a starting salary of \$50,000. Find the mean and median starting salary for these jobs.⁷
4. Find the mean and median grade point average (GPA) from the data given in Table 1.2.

| GPA | Frequency Count |
|-----|-----------------|
| 1.0 | 56 |
| 2.0 | 102 |
| 3.0 | 46 |
| 4.0 | 12 |

Table 1.2

5. Americans tend to work more than other workers in the world. See Figure 1.6.
- Find the median of the average hours worked per year for the workers listed in Figure 1.6.
 - Assuming a 40-hour work week, how many more *weeks* per year do Americans work than Norwegians?

| | |
|-------------------|-------|
| Americans | 1,966 |
| Japanese | 1,889 |
| Australians | 1,866 |
| Canadians | 1,732 |
| British | 1,731 |
| French | 1,656 |
| Germans | 1,560 |
| Norwegians | 1,399 |

Figure 1.6 Average hours worked per year per worker in 1996–1997.

Source: International Labor Organization.

6. a. Fill in Table 1.3. Round your answers to the nearest whole number.

| Age | Frequency Count | Relative Frequency (%) |
|-------|-----------------|------------------------|
| 1–20 | | 38 |
| 21–40 | 35 | |
| 41–60 | 28 | |
| 61–80 | | |
| Total | 137 | |

Table 1.3

- Calculate the percentage of the population that is over 40 years old.
7. Use Table 1.3 to create a histogram and pie chart.
8. From the histogram in Figure 1.7, create a frequency distribution table. Assume that the total number of people represented by the histogram is 1352. (*Hint:* Estimate the relative frequencies from the graph and then calculate the frequency count in each interval.)

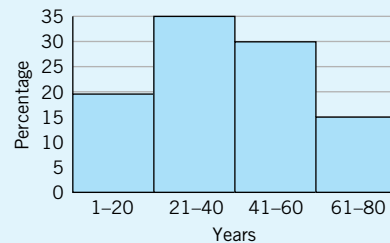


Figure 1.7 Distribution of ages (in years).

⁷Recall that given the list of numbers 9, 2, -2, 6, 5, the *mean* = the sum $9 + 2 + (-2) + 6 + 5$ divided by 5 (the number of items in the list) = $20/5 = 4$; the *median* = the middle number of the list in ascending order -2, 2, 5, 6, 9, which is 5. If the list had an even number of elements—for example -2, 2, 5, 6—the median would be the mean of the middle two numbers on the ordered list, in this case $(2 + 5)/2 = 7/2 = 3.5$.

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9. Calculate the mean and median for the following data:
 - a. \$475, \$250, \$300, \$450, \$275, \$300, \$6000, \$400, \$300
 - b. 0.4, 0.3, 0.3, 0.7, 1.2, 0.5, 0.9, 0.4
10. Explain why the mean may be a misleading numerical summary of the data in Problem 9(a).
11. A \$200 TV set goes on sale for 10% off. You have a card that reduces the sale price by another 10%.
 - a. How much do you pay for the TV set?
 - b. How much did you save by buying it during the sale and using your discount card?
 - c. By what total percentage has the TV set been reduced from the original price?
12. Figure 1.8 presents information about the Hispanic population in the United States from 1980 to 2000.
 - a. What does the bar chart tell you that the pie chart does not?
 - b. Using the bar and pie charts, what was the U.S. population in 1980, 1990, and 2000?
 - c. What does the state map tell you that the other charts do not?
 - d. If you were an investigative reporter interested in Hispanic issues, and you wanted to visit states with at least a 10% Hispanic population, how many states would you visit?
13. Describe the changes in families over the 50-year period 1947 to 1997 as represented in Figure 1.9.

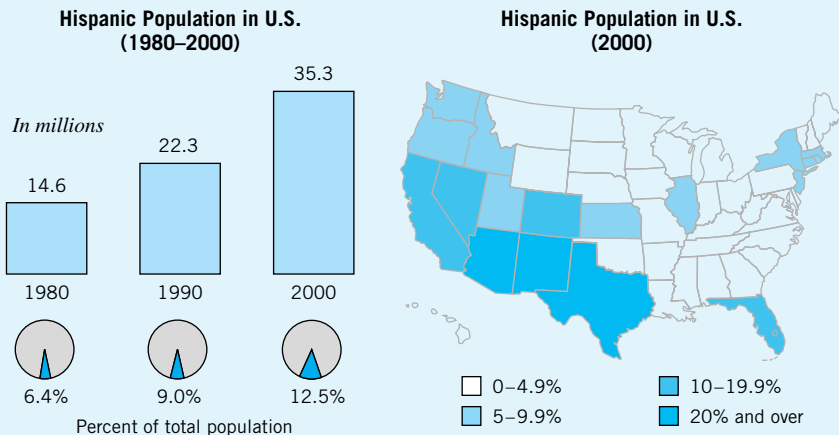


Figure 1.8 Change in the Hispanic population in the United States.
 Source: U.S. Bureau of the Census, www.census.gov

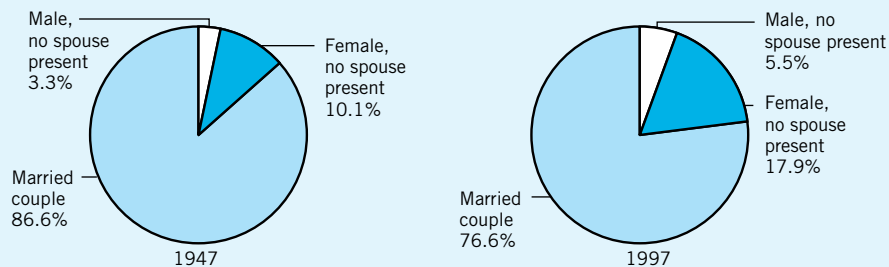


Figure 1.9 Changes in families from 1947 to 1997.
 Source: U.S. Bureau of the Census, March Current Population Survey.

1.2 Describing Relationships between Two Variables

By looking at two-variable data, we can learn how change in one variable affects change in another. How does the weight of a child determine the amount of medication prescribed by a pediatrician? How does median age or income change over time? In this section we examine how to describe these changes with graphs, data tables, written descriptions, and equations.

Visualizing Two-Variable Data

EXAMPLE 1 Scatter Plots

Table 1.4 shows data for two variables, year and median age of the U.S. population. Plot the data in Table 1.4 and then use your graph to describe the changes in the U.S. median age over time.

Solution We can think of the two numbers in each of the rows in Table 1.4 as an ordered pair of the form (year, median age). For example, the first row corresponds to the ordered pair (1850, 18.9) and the second row corresponds to (1860, 19.4). Figure 1.10 shows a scatter plot of the data. The graph is called a *time series* because it shows changes over time. In newspapers and magazines, the time series is the most frequently used form of data graphic.⁸



Median Age of U.S. Population, 1850–2050*

| Year | Median Age |
|------|------------|
| 1850 | 18.9 |
| 1860 | 19.4 |
| 1870 | 20.2 |
| 1880 | 20.9 |
| 1890 | 22.0 |
| 1900 | 22.9 |
| 1910 | 24.1 |
| 1920 | 25.3 |
| 1930 | 26.4 |
| 1940 | 29.0 |
| 1950 | 30.2 |
| 1960 | 29.5 |
| 1970 | 28.0 |
| 1980 | 30.0 |
| 1990 | 32.8 |
| 2000 | 35.3 |
| 2002 | 35.8 |
| 2005 | 36.7 |
| 2025 | 38.5 |
| 2050 | 38.8 |

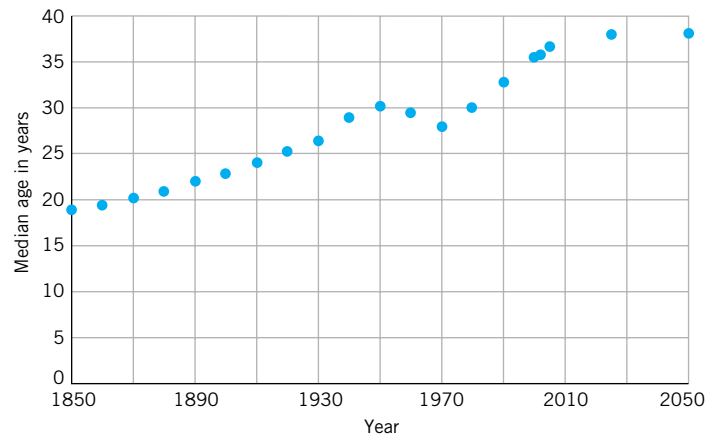


Figure 1.10 Median age of U.S. population over time.

Table 1.4

*Data for 2000–2050 are projected.

Source: U.S. Bureau of the Census, *Statistical Abstract of the United States*, 2003.

⁸Edward Tufte in *The Visual Display of Information* (Cheshire, Conn.: Graphics Press, 2001, p. 28) reported on a study that found that more than 75% of all graphics published were time series.

**SOMETHING TO THINK ABOUT**

What are some of the trade-offs in using the median instead of the mean age to describe changes over time?

Our graph shows that the median age of the U.S. population grew quite steadily for one hundred years, from 1850 to 1950. Although the median age decreased between 1950 and 1970, since 1970 it has continued to increase. From 1850 to the present, the median age nearly doubled, and projections for 2025 and 2050 indicate continued increases, though at a slower pace.

Constructing a “60-Second Summary”

To communicate effectively, you need to describe your ideas succinctly and clearly. One tool for doing this is a “60-second summary”—a brief synthesis of your thoughts that could be presented in one minute. Quantitative analyses strive to be straightforward and concise. They often start with a topic sentence that summarizes the key idea followed by supporting quantitative evidence.

After you have identified a topic you wish to write about or present orally, some recommended steps for constructing a 60-second summary are

- Collect relevant information (possibly from multiple sources, including the Internet).
- Search for patterns, taking notes.
- Identify a key idea (out of possibly many) that could provide a topic sentence.
- Select evidence and arguments that support your key idea.
- Examine counterevidence and arguments and decide if they should be included.
- Construct a 60-second summary, starting with your topic sentence.

You will probably weave back and forth among the steps in order to refine or modify your ideas. You can help your ideas take shape by putting them down on paper. Quantitative reports should not be written in the first person. For example, you might say something like “The data suggest that . . .” rather than “I found that the data . . .”

EXAMPLE 2 A 60-second summary

The annual federal surplus (+) or deficit (–) since World War II is shown in Table 1.5 and Figure 1.11 (a scatter plot where the points have been connected). Construct a 60-second summary describing the changes over time.

Solution Between 1945 and 2003 the U.S. federal deficit moved from a 30-year stable period, with as little as \$0 deficit, to a period of oscillations. From 1971 to 1992, the federal budget ran an annual deficit, which generally was getting larger until it reached almost \$300 billion in 1992. From 1992 to 1997, the deficit steadily decreased, and from 1998 to 2001 there were relatively large surpluses. The maximum surplus occurred in 2000, when it reached \$236 billion. But by 2002 the federal government was again running large deficits. In 2003 the deficit reached \$374 billion, the largest recorded up to that time.

Federal Budget: Surplus (+) or Deficit (-)

| Year | Billions of Dollars | Year | Billions of Dollars |
|------|---------------------|------|---------------------|
| 1945 | -\$48 | 1985 | -\$212 |
| 1950 | -\$3 | 1986 | -\$221 |
| 1955 | -\$3 | 1987 | -\$150 |
| 1960 | \$0 | 1988 | -\$155 |
| 1965 | -\$1 | 1989 | -\$152 |
| 1970 | -\$3 | 1990 | -\$221 |
| 1971 | -\$23 | 1991 | -\$269 |
| 1972 | -\$23 | 1992 | -\$290 |
| 1973 | -\$15 | 1993 | -\$255 |
| 1974 | -\$6 | 1994 | -\$203 |
| 1975 | -\$53 | 1995 | -\$164 |
| 1976 | -\$74 | 1996 | -\$108 |
| 1977 | -\$54 | 1997 | -\$22 |
| 1978 | -\$59 | 1998 | \$69 |
| 1979 | -\$41 | 1999 | \$126 |
| 1980 | -\$74 | 2000 | \$236 |
| 1981 | -\$79 | 2001 | \$127 |
| 1982 | -\$128 | 2002 | -\$159 |
| 1983 | -\$208 | 2003 | -\$374 |
| 1984 | -\$185 | | |

Table 1.5

Source: U.S. Bureau of the Census, *Statistical Abstract of the United States*, 2003.

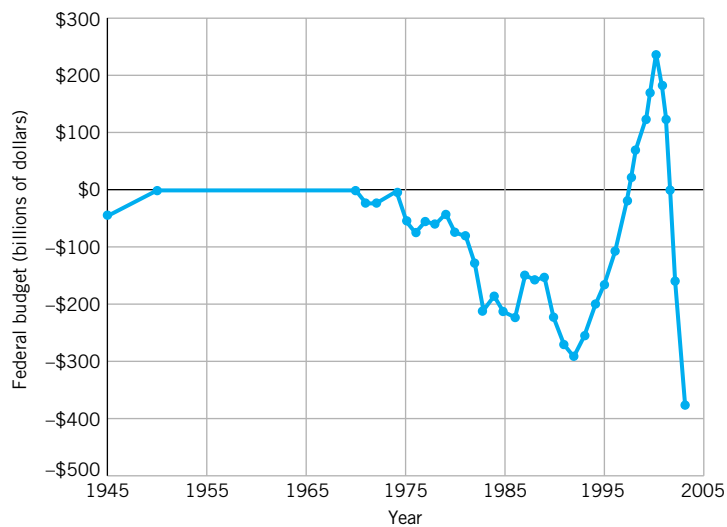


Figure 1.11 Annual federal budget surplus or deficit in billions of dollars.

Algebra Aerobics 1.2a

- The U.S. Census Bureau keeps data on the educational attainment of Americans over time. See Figure 1.12.
 - Estimate the percentage of Americans 25 years and older who were high school graduates in 1940, 1980, and 1999.
 - Estimate the percentage of Americans 25 years and older who were college graduates in 1940, 1980, and 1999.
 - Write a topic sentence describing the educational attainment of Americans 25 years and older over the time period 1940 to 1999.
- From Figure 1.13 approximately determine:
 - The year when the world population reached 4 billion
 - The year that it is projected to reach 8 billion
 - The number of years it will take to grow from 4 to 8 billion
- Use Figure 1.13 to estimate the following projections for the year 2150.
 - The total world population.
 - The total populations of all the more developed countries.
 - The total populations of all the less developed countries.
 - Write a topic sentence about the estimated world population in 2150.
- From Figure 1.13:
 - The world population in 2000 was how many times greater than the world population in 1900? What is the difference in population size?
 - The world population in 2100 is projected to be how many times greater than the world population in 2000? What is the difference in population size?
 - Describe the difference in the growth in world population in the twentieth century (1900–2000) versus the projected growth in the twenty-first century (2000–2100).

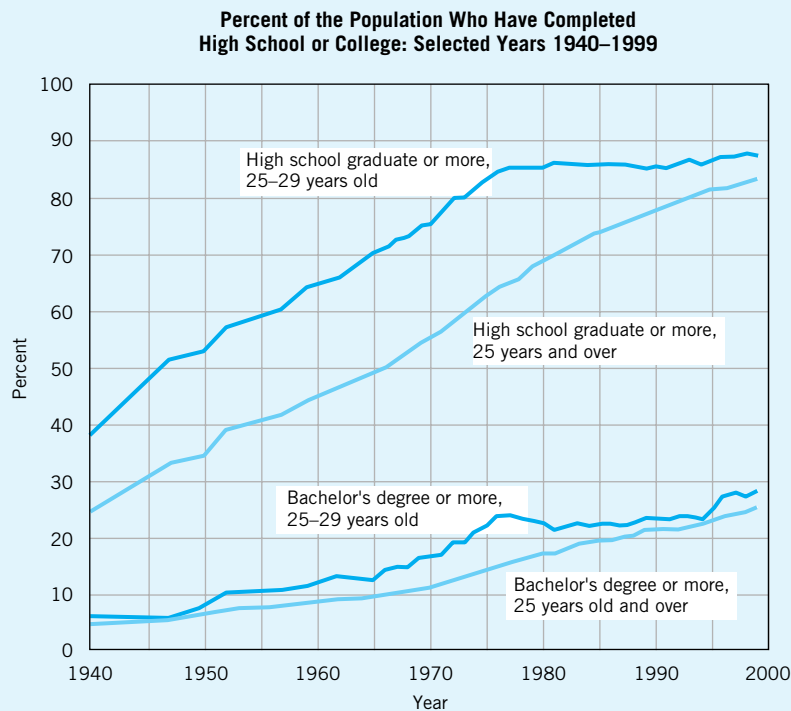


Figure 1.12 Educational attainment.

Source: U.S. Bureau of the Census, www.census.gov

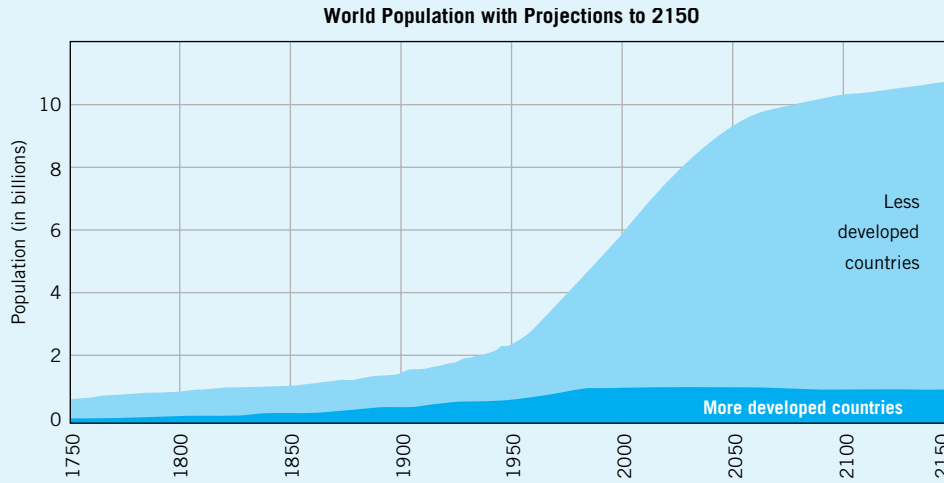


Figure 1.13 World population growth, 1750–2150 (est.).

Source: Population Reference Bureau, www.prb.org

5. The net worth of a household at any given time is the difference between assets (what you *own*) and liabilities (what you *owe*). Table 1.6 and Figure 1.14 show the median net worth of U.S. households, adjusted for inflation.⁹
- a. Write a few sentences about the trend in U.S. median household net worth.
 - b. What additional information might be useful in describing the trend in median net worth?

Median Net Worth of Households (adjusted for inflation using year 2000 dollars)

| Year | Median Net Worth (\$) |
|------|-----------------------|
| 1984 | 50,018 |
| 1988 | 49,855 |
| 1991 | 44,615 |
| 1993 | 43,567 |
| 1995 | 44,578 |
| 1998 | 49,932 |
| 2000 | 55,000 |

Table 1.6

Source: U.S. Bureau of the Census, www.census.gov

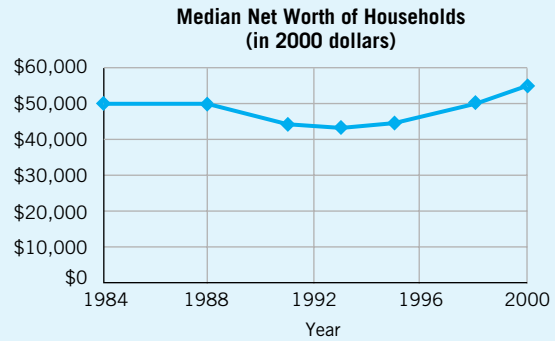


Figure 1.14

Using Equations to Describe Change

The relationship between two variables can also be described with an equation. An equation gives a rule on how change in the value of one variable affects change in the value of the other. If the variable n represents the number of years of education beyond grammar school and e represents yearly median earnings (in dollars) for people living in the United States, then the following equation models the relationship between e and n :

$$e = 3780 + 4320n$$

⁹“Constant dollars” is a measure used by economists to compare incomes and other variables in terms of purchasing power, eliminating the effects of inflation. To say the median income in 1986 was \$37,546 in “constant 2000 dollars” means that the median income in 1986 could buy an amount of goods and services that would cost \$37,546 to buy in 2000. The actual median income in 1986 (measured in what economists call “current dollars”) was much lower. Income corrected for inflation is sometimes called “real” income.

This equation provides a powerful tool for describing how earnings and education are linked and for making predictions. For example, to predict the median earnings, e , for those with a high school education, we replace n with 4 (representing 4 years beyond grammar school, or a high school education) in our equation to get

$$\begin{aligned} e &= 3780 + 4320 \cdot 4 \\ &= \$21,060 \end{aligned}$$

Thus our equation predicts that for those with a high school education, median earnings will be about \$21,060.

In “Extended Exploration: Looking for Links between Education and Income,” which follows Chapter 2, we show how such equations are derived and how they are used to analyze the relationship between education and earnings. An equation that is used to describe a real-world situation is called a *mathematical model*. Such models offer compact, often simplified descriptions of what may be a complex situation. Thus, the accuracy of the predictions made with such models can be questioned and disciplines outside of mathematics may be needed to help answer such questions. Yet these models are valuable guides in our quest to understand social and physical phenomena in our world.

Describing the relationship between abstract variables

Variables can represent quantities that are not associated with real objects or events. The following equation or mathematical sentence defines a relationship between two quantities, which are named by the abstract variables x and y :

$$y = x^2 + 2x - 3$$

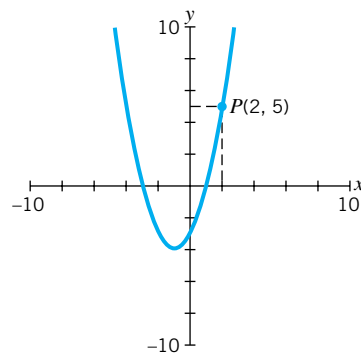
By substituting various values for x and finding the associated values for y , we can generate pairs of values for x and y , called *solutions to the equation*, that make the sentence true. By convention, we express these solutions as ordered pairs of the form (x, y) . Thus, $(1, 0)$ would be a solution to $y = x^2 + 2x - 3$, since $0 = 1^2 + 2(1) - 3$, whereas $(0, 1)$ would not be a solution, since $1 \neq 0^2 + 2(0) - 3$.

There are infinitely many possible solutions to the equation $y = x^2 + 2x - 3$, since we could substitute any real number for x and find a corresponding y . Table 1.7 lists a few solutions.

| x | y |
|-----|-----|
| -4 | 5 |
| -3 | 0 |
| -2 | -3 |
| -1 | -4 |
| 0 | -3 |
| 1 | 0 |
| 2 | 5 |
| 3 | 12 |

Table 1.7

We can use technology to graph the equation (see Figure 1.15). All the points on the graph represent solutions to the equation, and every solution is a point on the graph of the equation.



Coordinates of point P are:
 (horizontal coordinate, vertical coordinate)
 (x, y)
 $(2, 5)$

Figure 1.15 Graph of $y = x^2 + 2x - 3$ with one solution (of infinitely many solutions) labeled.

Solutions of an Equation

The *solutions* of an equation in two variables x and y are the ordered pairs (x, y) that make the equation a true statement.

Graph of an Equation

The *graph* of an equation in two variables displays the set of points that are solutions to the equation.

EXAMPLE 3 Solutions for equations in one or two variables

Describe how the solutions for the following equations are similar and how they differ.

$$3x + 5 = 11$$

$$x + 2 = x + 2$$

$$3 + x = y + 5$$

Solution The solutions are similar in the sense that each solution for each particular statement makes the statement true. They are different because:

There is only one solution ($x = 2$) of the single-variable equation $3x + 5 = 11$.

There are an infinite number of solutions for x of the single-variable equation $x + 2 = x + 2$, since any real number will make the statement a true statement.

There are infinitely many solutions, in the form of ordered pairs (x, y) , of the two-variable equation $3 + x = y + 5$.

EXAMPLE 4 Estimating solutions from a graph

The graph of the equation $x^2 + 4y^2 = 4$ is shown in Figure 1.16.

- From the graph, estimate three solutions of the equation.
- Check your solutions using the equation.

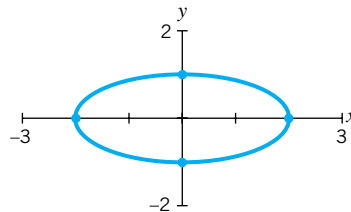


Figure 1.16 Graph of $x^2 + 4y^2 = 4$, an ellipse.

- Solution**
- The coordinates $(0, 1)$, $(-2, 0)$, and $(1, 0.8)$ appear to lie on the ellipse, which is the graph of the equation $x^2 + 4y^2 = 4$.
 - If substituting the ordered pair $(0, 1)$ into the equation makes it a true statement, then $(0, 1)$ is a solution.

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| | |
|--------------------------------|----------------------|
| Given | $x^2 + 4y^2 = 4$ |
| substitute $x = 0$ and $y = 1$ | $(0)^2 + 4(1)^2 = 4$ |
| evaluate | $4 = 4$ |

We get a true statement, so $(0, 1)$ is a solution to the equation.

For the ordered pair $(-2, 0)$:

| | |
|---------------------------------|-----------------------|
| Given | $x^2 + 4y^2 = 4$ |
| substitute $x = -2$ and $y = 0$ | $(-2)^2 + 4(0)^2 = 4$ |
| evaluate | $4 + 0 = 4$ |
| | $4 = 4$ |

Again we get a true statement, so $(-2, 0)$ is a solution to the equation.

For the ordered pair $(1, 0.8)$:

| | |
|----------------------------------|------------------------|
| Given | $x^2 + 4y^2 = 4$ |
| substitute $x = 1$ and $y = 0.8$ | $(1)^2 + 4(0.8)^2 = 4$ |
| evaluate | $1 + 4(0.64) = 4$ |
| | $3.56 \neq 4$ |

We get a false statement, so $(1, 0.8)$ is not a solution, although it is close to a solution.

Algebra Aerobics 1.2b

Note: Problem 4 requires technology.

1. a. Describe in your own words how to compute the value for y given a value for x using the following equation:

$$y = 3x^2 - x + 1$$

- b. Which of the following ordered pairs represent solutions to the equation?

$(0, 0)$, $(0, 1)$, $(1, 0)$, $(-1, 2)$, $(-2, 3)$, $(-1, 0)$

- c. Use $x = 0, \pm 1, \pm 2, \pm 3$ to generate a small table of values that represent solutions to the equation.
2. Repeat the directions in Problem 1(a), (b), and (c) using the equation $y = (x - 1)^2$.
3. Given the equations $y_1 = 4 - 3x$ and $y_2 = -2x^2 - 3x + 5$, fill in Table 1.8.
- a. Use the table to create two scatter plots, one for the ordered pairs (x, y_1) and the other for (x, y_2) .
- b. Draw a smooth curve through the points on each graph.
- c. Is $(1, 1)$ a solution for equation y_1 ? For y_2 ?
- d. Is $(-1, 6)$ a solution for equation y_1 ? For y_2 ?
- e. Look at the graphs. Is the ordered pair $(-3, 2)$ a solution for either equation? Verify your answer by substituting the values into each equation.

4. Use technology to graph the equation

$$y = x^2 - 3x + 2.$$

- a. What is the ordered pair that represents the y -intercept?
- b. Estimate the ordered pairs that represent the x -intercepts.
- c. If $x = 3/2$, find y .
- d. Find two points that are *not* solutions to this equation.

| x | y_1 | y_2 |
|-----|-------|-------|
| -6 | | |
| -4 | | |
| -2 | | |
| -1 | | |
| 0 | | |
| 1 | | |
| 2 | | |
| 4 | | |
| 6 | | |

Table 1.8

1.3 An Introduction to Functions

What Is a Function?

When we speak informally of one quantity being a function of some other quantity, we mean that one depends on the other. For example, someone may say that what they wear is a function of where they are going, or what they weigh is a function of what they eat, or how well a car runs is a function of how well it is maintained.

In mathematics, the word “function” has a precise meaning. A function is a special relationship between two quantities. If the value of one quantity uniquely determines the value of a second quantity, then the second quantity is a *function* of the first.

Median age and the federal deficit are functions of time since each year determines a unique (one and only one) value of median age or the federal deficit. The equation $y = x^2 + 2x - 3$ defines y as a function of x since each value of x we substitute in the equation determines a unique value of y .

Definition of a Function

A variable y is a *function* of a variable x if each value of x determines a unique value of y .

Representing Functions in Multiple Ways

We can think of a function as a “rule” that takes certain inputs and assigns to each input value exactly one output value. The rule can be described using words, data tables, graphs, or equations.

EXAMPLE 1 Sales tax

Eleven states have a sales tax of 6%; that is, for each dollar spent in a store in these states, the law says that you must pay a tax of 6 cents, or \$0.06.¹⁰ Represent the sales tax as a function of purchase price using an equation, table, and graph.

Solution Using an equation

We can write this as an equation where T represents the amount of sales tax and P represents the price of the purchase (both measured in dollars):

$$\begin{aligned}\text{Amount of sales tax} &= 0.06 \cdot \text{price of purchase} \\ T &= 0.06P\end{aligned}$$

Our function rule says: “Take the given value of P and multiply it by 0.06; the result is the corresponding value of T .” The equation represents T as a function of P , since for each value of P the equation determines a unique (one and only one) value of T . The purchase price, P , is restricted to dollar amounts greater than or equal to zero.

¹⁰In 2004, a sales tax of 6% was the most common rate for a sales tax in the United States. See www.taxadmin.org for a listing of the sales tax rates for all of the states.

Using a Table

We can use this formula to make a table of values for T determined by the different values of P (see Table 1.9). Such tables were once posted next to many cash registers.

| P (purchase price in \$) | T (sales tax in \$) |
|----------------------------|-----------------------|
| 0 | 0.00 |
| 1 | 0.06 |
| 2 | 0.12 |
| 3 | 0.18 |
| 4 | 0.24 |
| 5 | 0.30 |
| 6 | 0.36 |
| 7 | 0.42 |
| 8 | 0.48 |
| 9 | 0.54 |
| 10 | 0.60 |

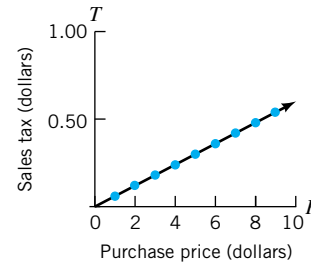


Table 1.9

Figure 1.17 Graph of \$0.06 sales tax.

Using a Graph

We can use the points in Table 1.9 to create a graph of the function (Figure 1.17). The table shows the sales tax only for selected purchase prices, but we could have used any positive dollar amount for P . We connected the points on the scatter plot to suggest the many possible intermediate values for price. For example, if $P = \$2.50$, then $T = \$0.15$.

Independent and Dependent Variables

Since a function is a rule that assigns to each input a unique output, we think of the output as being dependent on the input. We call the input of a function the *independent variable* and the output the *dependent variable*. When a set of ordered pairs represents a function, then each ordered pair is written in the form

$$(\text{independent variable}, \text{dependent variable})$$

or equivalently,

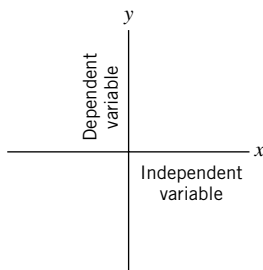
$$(\text{input}, \text{output})$$

If x is the independent and y the dependent variable, then the ordered pairs would be of the form

$$(x, y)$$

The mathematical convention is for the first variable, or input of a function, to be represented on the horizontal axis and the second variable, or output, on the vertical axis.

Sometimes the choice of the independent variable is arbitrary or not obvious. For example, economists argue as to whether wealth is a function of education or education is a function of wealth. As seen in the next example, there may be more than one correct choice.



EXAMPLE 2 Independent and dependent variables

In the sales tax example, the equation $T = 0.06P$ gives the sales tax, T , as a function of purchase price, P . In this case T is the dependent variable, or output, and P is the independent variable, or input. But this equation also gives us P as a function of T ; that is, each value of T corresponds to one and only one value of P . It is easier to see the relationship if we solve for P in terms of T , to get

$$P = T/0.06$$

Now we are thinking of the purchase price, P , as the dependent variable, or output, and the sales tax, T , as the independent variable, or input. So, if you tell me how much tax you paid, I can find the purchase price. In practical terms, however, we would usually think of the purchase price as the input.

When Is a Relationship Not a Function?

Not all relationships between two quantities describe functions. In the following set of ordered pairs in the form (x, y) ,

$$(1, 2), (1, 3), (2, 4), (3, 5)$$

y is not a function of x . When $x = 1$, y can be either 2 or 3. Thus, there is not one unique value of y for each value of x .

EXAMPLE 3 How can we tell if a table represents a function?

Consider the set of data in Table 1.10. The first column shows the year of the Olympics, T . The second column shows the record distance, R (in feet), for the men's Olympic 16-pound shot put.

- a. Is R a function of T ?
- b. Is T a function of R ?
- c. What should be your choice for the dependent and independent variables?

Solution a. R is a function of T .

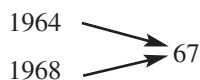
To determine if R is a function of T , we need to find out if each value of T (the input) determines one and only one value for R (the output). So, the ordered pair representing the relationship would be of the form (T, R) . Using Table 1.10, we can verify that for each T , there is one and only one R . So R , the record shot put distance, is a function of the year of the Olympics, T . Note that different inputs (such as 1964 and 1968) can have the same output (67 feet), and the relationship can still be a function. There are even 5 different years that have 70 feet as their output.

Olympic Shot Put

| Year, T | Record Distance in Feet Thrown, R |
|--------------|--|
| 1960 | 65 |
| 1964 | 67 |
| 1968 | 67 |
| 1972 | 70 |
| 1976 | 70 |
| 1980 | 70 |
| 1984 | 70 |
| 1988 | 74 |
| 1992 | 71 |
| 1996 | 71 |
| 2000 | 70 |

Table 1.10

Source: *The World Almanac and Book of Facts*, 2003.



b. T is *not* a function of R .

To determine if T is a function of R , we need to find out if each value of R (now the input) determines one and only one value for T (the output). Now the ordered pairs representing the relationship would be of the form (R, T) . Table 1.11 shows this new pairing, where R is thought of as the input and T as the output.

The year, T , is *not* a function of R , the record distance. Some values of R give more than one value for T . For example, when $R = 67$ there are two corresponding values for T : 1964 and 1968, and this violates the condition of a unique (one and only one) output for each input.

c. R the record distance, is a function of the year, T , but T is *not* a function of R . The record, R , depends on the year, T , but T does not depend on R . To construct a function relating T and R , we must choose T as the independent variable and R as the dependent variable. The ordered pairs that represent the function would be written as (T, R) .

Olympic Shot Put

| Record Distance in Feet Thrown, R | Year, T |
|-------------------------------------|-----------|
| 65 | 1960 |
| 67 | 1964 |
| 67 | 1968 |
| 70 | 1972 |
| 70 | 1976 |
| 70 | 1980 |
| 70 | 1984 |
| 74 | 1988 |
| 71 | 1992 |
| 71 | 1996 |
| 70 | 2000 |

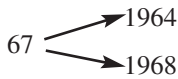


Table 1.11

EXAMPLE 4 How would the axes be labeled for each graph of the following functions:

- a. Density of water is a function of temperature.
- b. Radiation intensity is a function of wavelength.
- c. A quantity Q is a function of time t .

Solution

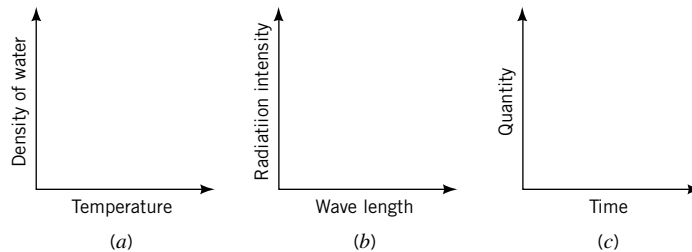


Figure 1.18 Various labels for axes, depending on the context.

How to tell if a graph represents a function: The vertical line test

For a graph to represent a function, each value of the input on the horizontal axis must be associated with one and only one value of the output on the vertical axis. If you can draw a vertical line that intersects a graph in more than one point, then at least one input is associated with two or more outputs, and the graph does not represent a function.

The graph in Figure 1.19 represents y as a function of x . For each value of x , there is only one corresponding value of y . No vertical line intersects the curve in more than one point. The graph in Figure 1.20 does *not* represent a function. One can draw a vertical line (an infinite number, in fact) that intersects the graph in more than one point. Figure 1.20 shows a vertical line that intersects the graph at both $(4, 2)$ and $(4, -2)$. That means that the value $x = 4$ does not determine one and only one value of y . It corresponds to y values of both 2 and -2 .

 **Exploration 1.2**
will help you develop an intuitive sense of functions.

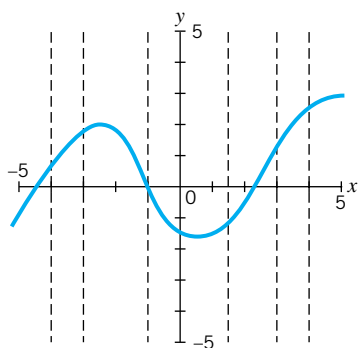


Figure 1.19 The graph represents y as a function of x since there is no vertical line that intersects the curve at more than one point.

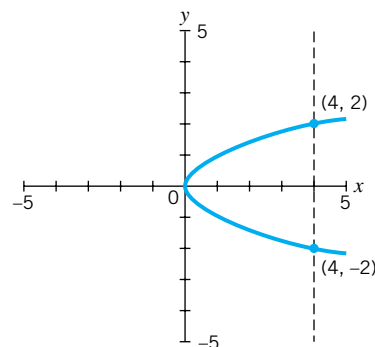


Figure 1.20 The graph does not represent y as a function of x since there is at least one vertical line that intersects this curve at more than one point.

Algebra Aerobics 1.3a

1. Refer to the graph in Figure 1.21. Is y a function of x ?

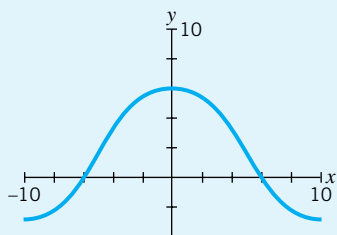


Figure 1.21 Graph of an abstract relationship between x and y .

2. Which of the graphs in Figure 1.22 represent functions and which do not? Why?

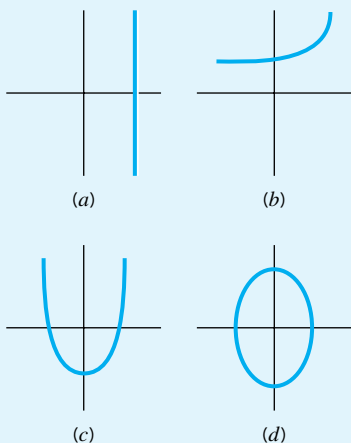


Figure 1.22

3. Consider the chart in Figure 1.23. Is weight a function of height? Is height a function of weight? Explain your answer.

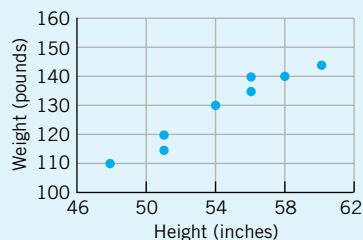


Figure 1.23 Graph of weight versus height.

4. Consider Table 1.12.
 a. Is D a function of Y ?
 b. Is Y a function of D ?

| Y | D |
|------|---------|
| 1992 | \$2.50 |
| 1993 | \$2.70 |
| 1994 | \$2.40 |
| 1995 | -\$0.50 |
| 1996 | \$0.70 |
| 1997 | \$2.70 |

Table 1.12

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5. Plot the following points with x on the horizontal and y on the vertical axis. Draw a line through the points and determine if the line represents a function.

| | | | | | |
|-----------|-----|-----|-----------|-----|-----|
| a. | x | y | b. | x | y |
| | -1 | 3 | | -2 | -1 |
| | 0 | 3 | | -2 | 0 |
| | 2 | 3 | | -2 | 1 |
| | 3 | 3 | | -2 | 4 |

6. **a.** Write an equation for computing a 15% tip in a restaurant. Does your equation represent a function? If so, what are your choices for the independent and dependent variables?
b. How much would the equation suggest you tip for an \$8 meal?
c. Compute a 15% tip on a total check of \$26.42.
7. If we let D stand for ampicillin dosage expressed in milligrams and W stand for a child's weight in kilograms, then the equation

$$D = 50W$$

gives a rule for finding the safe maximum daily drug dosage of ampicillin (used to treat respiratory infections)

for children who weigh less than 10 kilograms (about 22 pounds).¹¹

- a.** What are logical choices for the independent and dependent variables?
b. Does the equation represent a function? Why?
c. Generate a small table and graph of the function.
d. Think of the function $D = 50W$ for ampicillin dosage as an abstract mathematical equation. How will the table and graph change?
8. Table 1.13 lists the pulse rate, P , and the respiration rate, R , taken at different times for one individual.

- a.** Based on this table, is R a function of P ?
b. Based on this table, is P a function of R ?

| P (beats/min) | R (breaths/min) |
|-----------------|-------------------|
| 75 | 12 |
| 95 | 17 |
| 75 | 14 |
| 130 | 20 |
| 110 | 19 |

Table 1.13

Function Notation

As we have seen in Section 1.2, not all equations represent functions. But functions have important qualities, so it is useful to have a way to indicate when a relationship is a function. To indicate that a quantity y is a function of a quantity x , we can write

y is a function of x

or in abbreviated form,

y equals “ f of x ”

or using function notation,

$$y = f(x)$$

The expression $y = f(x)$ means that the rule f is applied to the input value x to give the output value, $f(x)$:

$$\text{output} = f(\text{input})$$

¹¹Information extracted from Anna M., Curren and Laurie D Muntlay, *Math for Meds; Dosages and Solutions*, 6th ed. (San Diego: W. I. Publications, 1990), p. 198.

or

$$\text{dependent variable} = f(\text{independent variable})$$

For example, if f is defined by the equation $T = 0.06P$, then assuming P is the input and T the output,

$$f(P) = T \quad \text{where } T = 0.06P$$

or

$$f(P) = 0.06P$$

We use the letter f to represent the relationship between P (the input) and T (the output), but we could use any other letter to represent this relationship.

Function notation is particularly useful when a function is being evaluated at a specific point. For example, instead of saying “the value for T when $P = 10$,” we simply write “ $f(10)$.” Then we have

$$\begin{aligned} f(10) &= (0.06)(10) \\ &= 0.6 \end{aligned}$$

Similarly,

$$\begin{aligned} f(0.5) &= (0.06)(0.5) & f(6.5) &= (0.06)(6.5) \\ &= 0.03 & &= 0.39 \end{aligned}$$

The function notation $f(x)$ emphasizes the choice of one variable as the input (or independent variable) and the other as the output (or dependent variable). We encounter this notation regularly in mathematics texts, but it is used less frequently in social science and science texts. Scientists may be reluctant to classify one variable as depending upon another or may want to retain the flexibility of an equation that can be rewritten in many different forms.

Common Error

The expression $f(x)$ does not mean “ f times x .” It means the function f evaluated at x .

EXAMPLE 5 Using function notation with equations

a. Write the following relationship using function notation:

The intensity of light, I , is a function of the distance, d , from its source.

b. Evaluate $g(0)$, $g(2)$, and $g(-2)$ for the function

$$g(x) = \frac{1}{x - 1}$$

Solution a. $I = f(d)$

b.

$$g(0) = \frac{1}{0 - 1} = \frac{1}{-1} = -1$$

$$g(2) = \frac{1}{2 - 1} = \frac{1}{1} = 1$$

$$g(-2) = \frac{1}{-2 - 1} = \frac{1}{-3} = -\frac{1}{3}$$

EXAMPLE 6 Using function notation with data tables

Use Table 1.14 to fill in the missing values:

- a. $s(0) = ?$ b. $s(-1) = ?$ c. $s(?) = 4$

| | | | | | |
|--------|----|----|---|---|---|
| x | -1 | -2 | 0 | 1 | 2 |
| $s(x)$ | 1 | 4 | 0 | 1 | 4 |

Table 1.14

Solution a. $s(0) = 0$ b. $s(-1) = 1$ c. $s(-2) = 4$ and $s(2) = 4$

EXAMPLE 7 Using function notation with graphs

Use the graph in Figure 1.24 to estimate the missing values:

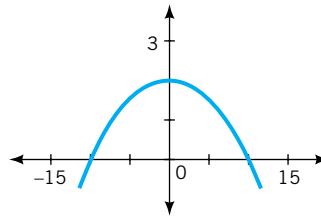


Figure 1.24 Graph of a function.

- a. $f(0) = ?$ b. $f(-5) = ?$ c. $f(?) = 0$

Solution a. $f(0) = 2$ b. $f(-5) = 1.5$ c. $f(10) = 0$ and $f(-10) = 0$

Rewriting equations using function notation

In order to use function notation, an equation needs to be in the form

$$\text{output} = \text{some rule applied to input}$$

or equivalently

$$\text{dependent variable} = \text{some rule applied to independent variable}$$

Translating an equation into this format is called putting the equation in *function form*. Many graphing calculators and computer graphing programs accept only equations in function form as input.

To put an equation into function form, we first need to identify the independent and the dependent variables. Sometimes the choice may be obvious, at other times arbitrary. If we use the mathematical convention that x represents the input or independent variable and y the output or dependent variable, when we put equations into function form, we want

$$y = \text{some rule applied to } x$$

EXAMPLE 8 Analyze the equation $4x - 3y = 6$. Decide whether or not the equation represents a function. If it does, write the relationship using function notation.

Solution First, put the equation into function form. Assume y is the output.

$$\begin{array}{ll} \text{Given the equation} & 4x - 3y = 6 \\ \text{subtract } 4x \text{ from both sides} & -3y = 6 - 4x \\ \text{divide both sides by } -3 & \frac{-3y}{-3} = \frac{6 - 4x}{-3} \\ \text{simplify} & y = \frac{6}{-3} + \frac{-4x}{-3} \\ \text{simplify and rearrange terms} & y = \frac{4}{3}x - 2 \end{array}$$

We now have an expression for y in terms of x .

Using technology or by hand, we can generate a graph of the equation (see Figure 1.25). Since the graph passes the vertical line test, y is a function of x .

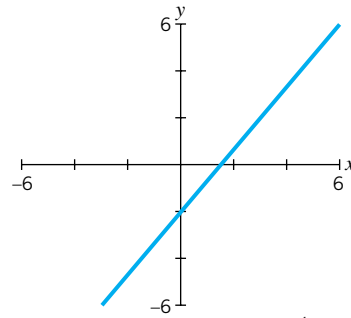


Figure 1.25 Graph of $y = \frac{4}{3}x - 2$.

If we name our function f , then using function notation, we have

$$y = f(x) \quad \text{where } f(x) = (4/3)x - 2$$

EXAMPLE 9 Analyze the equation $y^2 - x = 0$. Generate a graph of the equation. Decide whether or not the equation represents a function. If the equation represents a function, write the relationship using function notation. Assume y is the output.

Solution Put the equation in function form:

$$\begin{array}{ll} \text{Given the equation} & y^2 - x = 0 \\ \text{add } x \text{ to both sides} & y^2 = x \end{array}$$

To solve this equation, we take the *square root* of both sides of the equation and we get

$$y = \pm \sqrt{x}$$

This gives us two solutions for any value of $x > 0$ as shown in Table 1.15. For example, if $x = 4$, then y can either be 2 or -2 since both $2^2 = 4$ and $(-2)^2 = 4$.

| x | y |
|-----|---------------------------|
| 0 | 0 |
| 1 | 1 or -1 |
| 2 | $\sqrt{2}$ or $-\sqrt{2}$ |
| 4 | 2 or -2 |
| 9 | 3 or -3 |

Table 1.15

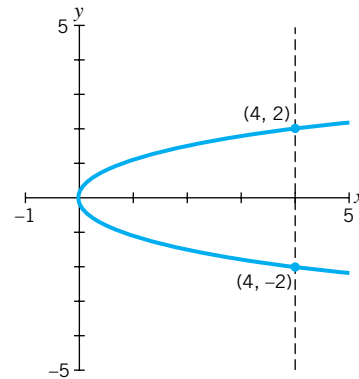


Figure 1.26 Graph of the equation $y^2 = x$.

The graph of the equation in Figure 1.26 does not pass the vertical line test. In particular, the solutions $(4, -2)$ and $(4, 2)$ lie on the same vertical line. So y is not a function of x and we cannot use function notation to represent this relationship.

Domain and Range

A function is often defined only for certain values of the input (or independent variable). The set of all possible values for the input is called the *domain* of the function. The set of corresponding values of the output (or dependent variable) is called the *range* of the function.

Domain and Range of a Function

The *domain* of a function is the set of possible values of the input. The *range* is the set of corresponding values of the output.

EXAMPLE 10 Finding the domain and range

In the sales tax example at the beginning of this section, we used the equation

$$T = 0.06P$$

to represent the sales tax, T , as a function of the purchase price, P (where all units are in dollars). What are the domain and range of this function?

Solution Since negative values for P are meaningless, P is restricted to dollar amounts greater than or equal to zero. In theory there is no upper limit on prices, so we assume P has no maximum amount. In this example,

the domain is all dollar values of P greater than or equal to 0

We can express this more compactly with the symbol \geq , which reads left to right, “greater than or equal to.” So,

the domain is all values of P such that $P \geq 0$

or abbreviated to the domain is $P \geq 0$

What are the corresponding values for the tax T ? The values for T in our model will also always be nonnegative. As long as there is no maximum value for P , there will be no maximum value for T . So,

the range is all dollar values of T greater than or equal to 0

or we can shorten this to the range is $T \geq 0$

Representing the Domain and Range with Interval Notation

For continuous variables, domains and ranges are often written using interval notation. In this notation:

The set of numbers x such that $a \leq x \leq b$ is written $[a, b]$.

We read the statement $a \leq x \leq b$ as: “ x is greater than or equal to a and less than or equal to b .”

The set of numbers x such that $a < x < b$ is written (a, b)

We read the statement $a < x < b$ as: “ x is greater than a and less than b .”

Note: Since the notation (a, b) can also mean the coordinates of a point, we will say the *interval* (a, b) when we want to refer to an interval.

For example, if the domain is values of n greater than or equal to 50 and less than or equal to 100, then

$$\begin{aligned} \text{domain} &= \text{all } n \text{ values with } 50 \leq n \leq 100 \\ &= [50, 100] \end{aligned}$$

If the domain is values of n greater than 50 and less than 100, then

$$\begin{aligned} \text{domain} &= \text{all } n \text{ values with } 50 < n < 100 \\ &= \text{interval } (50, 100) \end{aligned}$$

The symbols $[]$ are called *closed brackets* and the symbols $()$ are called *open brackets*. We can also use one closed and one open bracket. For example, if we want to exclude 50 but include 100 as part of the domain, we would represent the interval as $(50, 100]$. The interval can be displayed on the real number line as:



In general, a hollow dot indicates exclusion and a solid dot inclusion.

EXAMPLE 11a Are the Pensacola tides a function of time of day?

The graph in Figure 1.27 shows the water level of the tides in Pensacola, Florida, over a 24-hour period. Are the Pensacola tides a function of the time of day? If so, identify the independent and dependent variables. What are the domain and range of this function?

Solution The Pensacola tides are a function of the time of day since the graph passes the vertical line test. The independent variable is time, and the dependent variable is water level. The domain is from 0 to 24 hours, and the range is from about -10 to $+10$ centimeters. Using interval notation:

$$\begin{aligned} \text{domain} &= [0, 24] \\ \text{range} &= [-10, 10] \end{aligned}$$

EXAMPLE 11b *Is the time of day a function of the Pensacola tides?*

Use Figure 1.28 to determine if the time of day is a function of the Pensacola tides. Justify your answer.

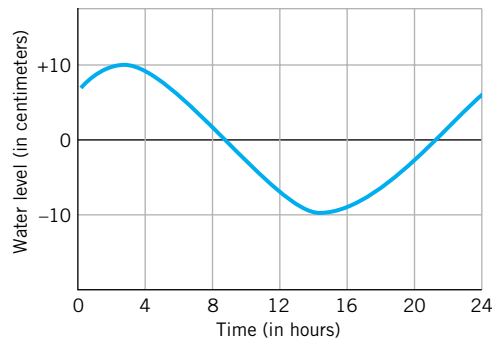


Figure 1.27 Diurnal tides in a 24-hour period in Pensacola, Florida.

Source: Adapted from Fig. 8.2 in *Oceanography: An Introduction to the Planet Oceanus*, by Paul R. Pinet. Copyright © 1992 by West Publishing Company, St. Paul, MN. All rights reserved.

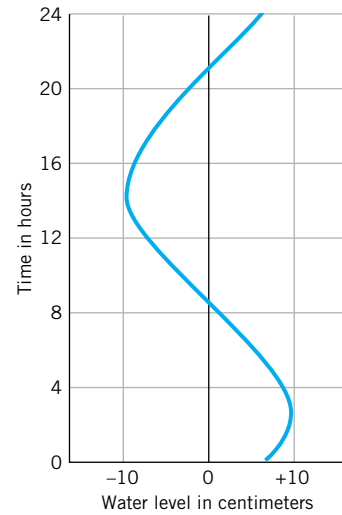


Figure 1.28 The time of day is not a function of the tide.

Solution Time of day is *not* a function of the Pensacola tides.

Figure 1.28 also shows the water level of the Pensacola tides, now with water level on the horizontal axis and time on the vertical axis. The graph fails the vertical line test. For instance, the dark vertical line corresponding to a water level of 0 centimeters crosses the graph in two different places (at about 9 and 21 hours). There is not one unique hour associated with a water level of 0 centimeters. Therefore, the time of day is not a function of water level.

Is the function undefined for certain values?

When specifying the domain and range of a function, we need to consider whether the function is undefined for any values. For example, for the function

$$y = \frac{1}{x}$$

the expression $1/x$ is undefined when $x = 0$. For any other value for x , the function is defined. Thus,

the domain is all real numbers except 0

To find the range, we need to determine the possible output values for y . Sometimes it is easier to find the y values that are not possible. In this case, y can't equal zero. Why? Our rule says to take 1 and divide by x , but it is impossible to divide 1 by a real number in our domain and get zero as a result. Thus,

the range is all real numbers except 0

The interval $(-\infty, +\infty)$ or $(-\infty, \infty)$ represents all real numbers. To represent all real numbers except 0 using interval notation, we use the union, represented by \cup , of two intervals:

$$(-\infty, 0) \cup (0, \infty)$$

SOMETHING TO THINK ABOUT
 What happens to the value of y as x approaches 0?

Using the graph of the function, we can check to see if our domain and range are reasonable.

Figure 1.29 suggests that x comes very close to 0 but does not equal 0. The same is true for y .

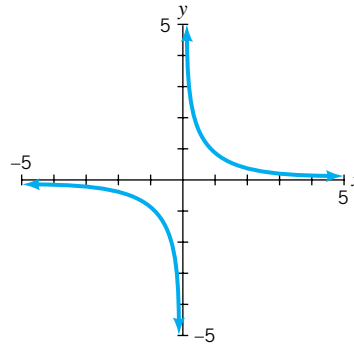


Figure 1.29 Graph of $y = \frac{1}{x}$.

EXAMPLE 12 When is a function undefined?

Match each function in Figure 1.30 with the appropriate domain and range listed in parts (a) to (e). [Note: The dotted line in the graph of $f(x)$ is not part of the function.]

- a. Domain: $[0, \infty)$ Range: $[0, \infty)$
- b. Domain: $(0, \infty)$ Range: $(0, \infty)$
- c. Domain: $(-\infty, 1) \cup (1, \infty)$ Range: $(-\infty, 1) \cup (1, \infty)$
- d. Domain: $(-\infty, 1) \cup (1, \infty)$ Range: $(-\infty, 0) \cup (0, \infty)$
- e. Domain: $(-\infty, 0) \cup (0, \infty)$ Range: $(0, \infty)$

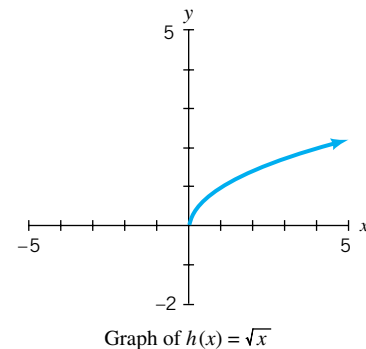
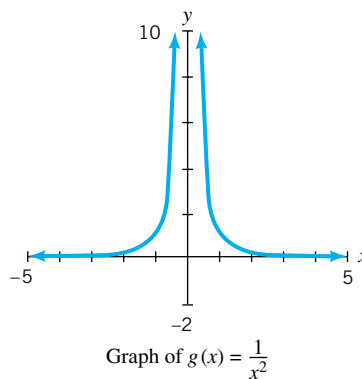
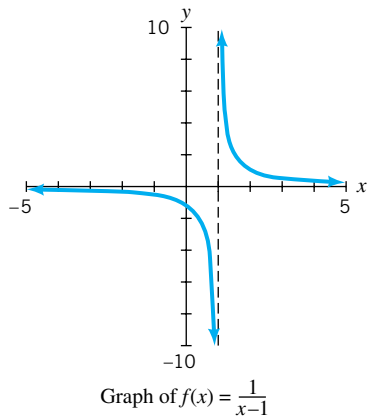


Figure 1.30 Graphs for three functions.

Solution $f(x) = 1/(x - 1)$ matches with (d)
 $g(x) = 1/x^2$ matches with (e)
 $h(x) = \sqrt{x}$ matches with (a)

Algebra Aerobics 1.3b

- Express each of the following using interval notation.
 - $x > 2$
 - $4 \leq x < 20$
 - $t \leq 0$ or $t > 500$
- Express the given interval as an inequality.
 - $[-3, 10)$
 - $(-2.5, 6.8]$
 - $(-\infty, 5] \cup [12, \infty)$
- Express each of the following statements in interval notation.
 - Harry's GPA is at least 2.5 but at most 3.6.
 - A good hitter has a batting average of at least 0.333.
 - The estimated time of death was between 10 and 11 in the morning.
 - Starting annual salary at a position is anything from \$35,000 to \$50,000 depending upon experience.
- Consider the function $f(x) = x^2 - 5x + 6$. Find $f(0)$, $f(1)$, and $f(-3)$.
- Given the function $f(x) = \frac{2}{x-1}$, evaluate $f(0)$, $f(-1)$, $f(1)$, and $f(-3)$.
- Determine the value of t for which each of the functions has a value of 3.

$$r(t) = 5 - 2t \quad p(t) = 3t - 9 \quad m(t) = 5t - 12$$

In Problems 7–11 solve for y in terms of x . Determine if y is a function of x . If it is, rewrite using $f(x)$ notation and determine the domain.

 - $2(x - 1) - 3(y + 5) = 10$
 - $x^2 + 2x - 3y + 4 = 0$
 - $7x - 2y = 5$
 - $2xy = 6$
 - $x/2 + y/3 = 1$
- From the graph in Figure 1.31, estimate $f(-4)$, $f(-1)$, $f(0)$, and $f(3)$. Find two approximate values for x such that $f(x) = 0$.

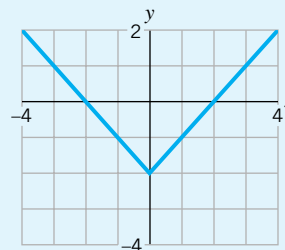


Figure 1.31 Graph of $f(x)$.

- From Table 1.16 find $f(0)$ and $f(20)$. Find two values of x for which $f(x) = 10$. Explain why $f(x)$ is a function.

| x | $f(x)$ |
|-----|--------|
| 0 | 20 |
| 10 | 10 |
| 20 | 0 |
| 30 | 10 |
| 40 | 20 |

Table 1.16

- Find values of x for which the function is undefined and determine the domain.

$$f(x) = \frac{x+1}{x+5} \quad g(x) = \frac{1}{x+1} \quad h(x) = 4 - x^2$$

1.4 Visualizing Functions

In this section we return to the question: How does change in one variable affect change in another variable? Graphs are one of the easiest ways to recognize change. We start with three basic questions:

Is There a Maximum or Minimum Value?

If a function has a *maximum* (or *minimum*) value, then it appears as the highest point (or lowest point) on its graph.

EXAMPLE 1 Determine if each function in Figure 1.32 has a maximum or minimum, then estimate its value.

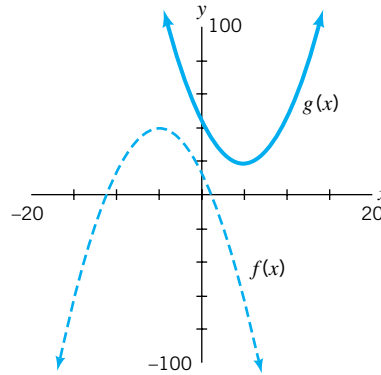


Figure 1.32

Solution The function $f(x)$ appears to have a maximum value of 40 when $x = -5$ but has no minimum value since both arms of the function extend indefinitely downward.

The function $g(x)$ appears to have a minimum value of 20 when $x = 5$, but no maximum value since both arms of the function extend indefinitely upward.

In Figure 1.33 the function $h(x)$ appears to have a maximum value of 200, which occurs when $x = 25$, and a minimum value of -50 , when $x = 5$.

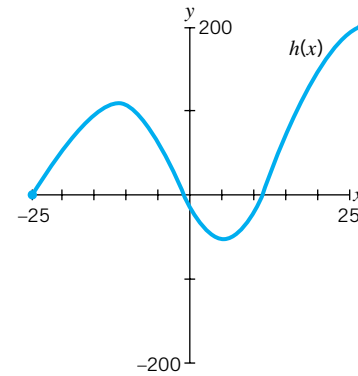


Figure 1.33

Is the Function Increasing or Decreasing?



A function f is *increasing* over a specified interval if the values of $f(x)$ increase as x increases in this interval. A function f is *decreasing* over a specified interval if the values of $f(x)$ decrease as x increases over the interval.

The graph of an increasing function climbs as we move from left to right. The graph of a decreasing function falls as we move from left to right.

EXAMPLE 2 Rising and falling stock prices

The graphs in Figure 1.34 show the price of a share of stock (in dollars) for two different companies on the New York Stock Exchange over a one-week period from January 9 to 16, 2004. Describe how the stock prices for each company changed during this time.

34 CHAPTER 1 MAKING SENSE OF DATA AND FUNCTIONS

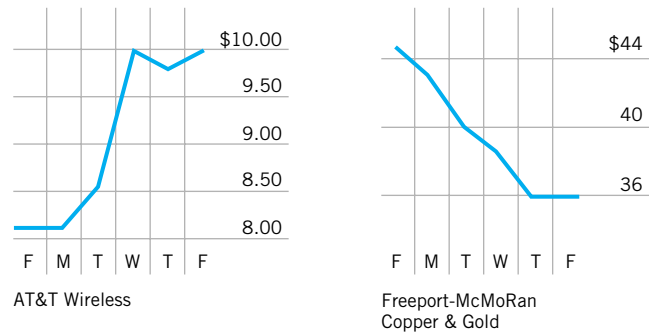


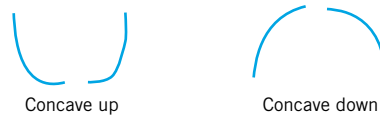
Figure 1.34 Stock prices for two companies.
 Source: *The New York Times*, Jan. 17, 2004.

Solution For the week from January 9 to 16, 2004, the stock price for AT&T Wireless started out at a low of a little over \$8 a share on Friday the 9th that continued through Monday the 12th. The stock price then started to rise steadily, reaching a maximum value of a slightly over \$10 a share on Wednesday. The price decreased slightly on Thursday and returned near the maximum on Friday the 16th.

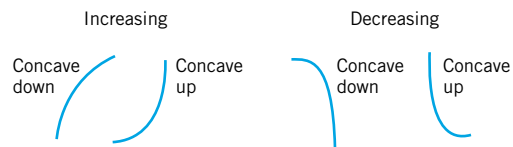
For the week from January 9 to 16, the stock price for FMMR Copper and Gold started out at a maximum value of about \$44 a share on Friday the 9th, but then steadily decreased in value until it reached a low of about \$36 a share on Thursday the 15th through Friday the 16th.

Is the Graph Concave Up or Concave Down?

What does the concavity of a graph mean? The graph of a function is *concave up* if it bends upward as we move left to right; it is *concave down* if it bends downward as we move left to right.



Concavity is independent of whether the function is increasing or decreasing.



EXAMPLE 3 **Graphs are not necessarily pictures of events¹²**

The graph in Figure 1.35 shows the speed of a roller coaster car as a function of time.

- Describe how the speed of the roller coaster car changes over time. Describe the changes in the graph as the speed changes over time.
- Draw a picture of a possible track for this roller coaster.

¹²Examples 3 is adapted from Shell Centre for Mathematical Education, *The Language of Functions and Graphs*. Manchester, England: University of Nottingham, 1985.

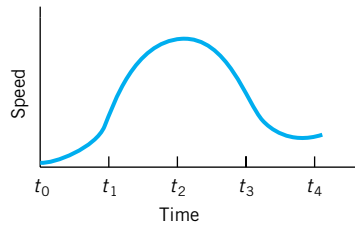


Figure 1.35

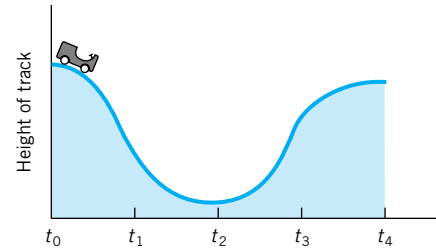


Figure 1.36

- Solution**
- a. The speed of the roller coaster car increases from t_0 to t_2 , reaching a maximum for this part of the ride at t_2 . The speed decreases from t_2 to t_4 .
The graph of speed versus time is concave up and increasing from t_0 to t_1 and then concave down and increasing from t_1 to t_2 . From t_2 to t_3 the graph is concave down and decreasing, and from t_3 to t_4 it is concave up and decreasing.
 - b. A picture for a possible track of the roller coaster is shown in Figure 1.36. Notice how the track is an upside-down picture of the graph of speed versus time. (Since when the roller coaster goes down the speed increases, when the roller coaster goes up, the speed decreases.)

EXAMPLE 4 Growth patterns

Figure 1.37 shows the growth patterns for three areas in Virginia. Compare the difference in growth for these areas between 1900 and 2000.

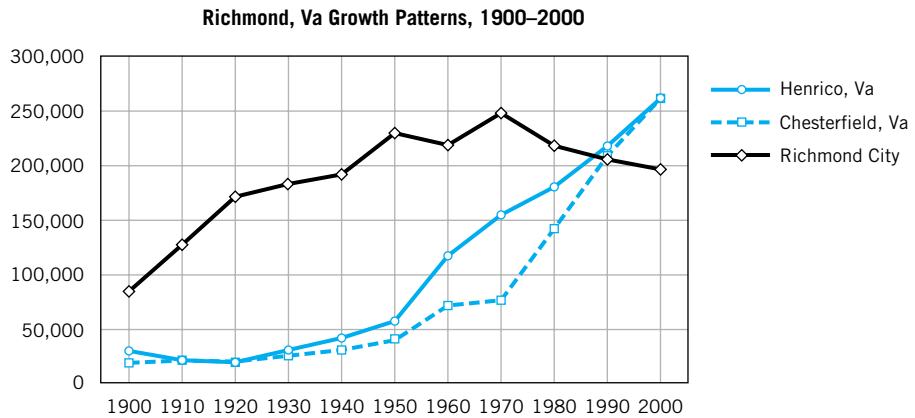


Figure 1.37 Richmond, Virginia, growth patterns, 1900–2000.

Source: www.savethebay.org/land/images

- Solution** From 1900 to 1990, Richmond City consistently had the largest population; however, Henrico and Chesterfield counties were growing faster than Richmond City. By 1990 all three areas had populations of about 200,000. After 1990, Henrico and Chesterfield’s populations continued to grow but Richmond’s continued to decline.

What Is Really Happening?

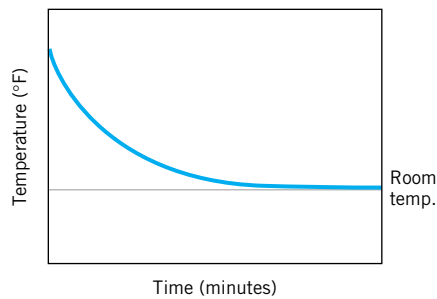
We now have the basic vocabulary for describing a function’s behavior. Think of a function as telling a story. We want to decipher not just the individual words, but the overall plot. In each situation we should ask, What is really happening here? What do the words tell us about the shape of the graph? What does the graph tell us about the underlying phenomenon?

EXAMPLE 5 Generate a rough sketch of each of the following situations.

- a. A cup of hot coffee cooling.
- b. U.S. venture capital (money provided by investment companies to business start-ups) increased modestly but steadily in the early 1990s, soared during the “dotcom bubble,” (in the late 1990’s), with a high in 2000, and then suffered a drastic decrease back to pre-dotcom levels.
- c. Using a simple predator-prey model: initially as the number of lions (the predators) increases, the numbers of gazelle (their prey) decrease. When there are not enough gazelles to feed all the lions, the number of lions decreases and the number of gazelles starts to increase.

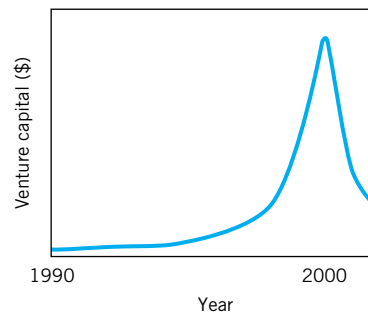
Solution a.

A Cup of Hot Coffee Cooling Over Time



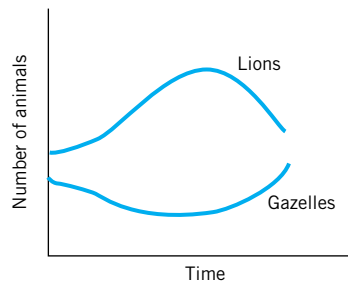
b.

U.S. Venture Capital from 1990–2002



c.

Predator-Prey Model of Lions vs. Gazelles



EXAMPLE 6 You are a TV journalist. Summarize for your viewers the essence of each of the following graphs in Figures 1.38, 1.39, and 1.40.

a.

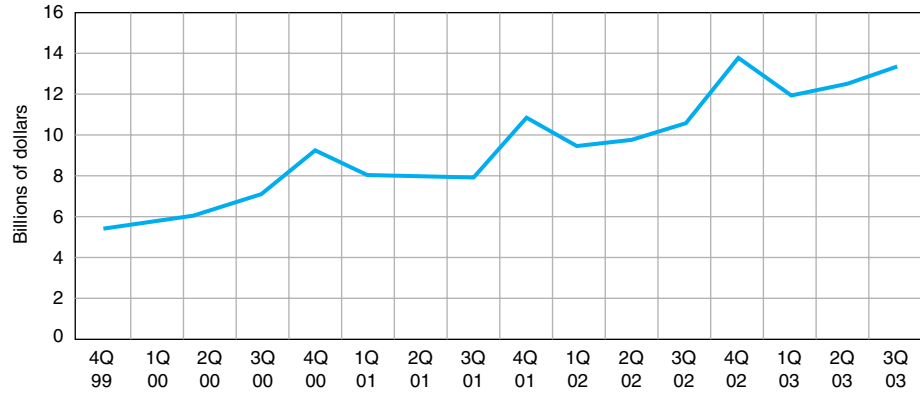
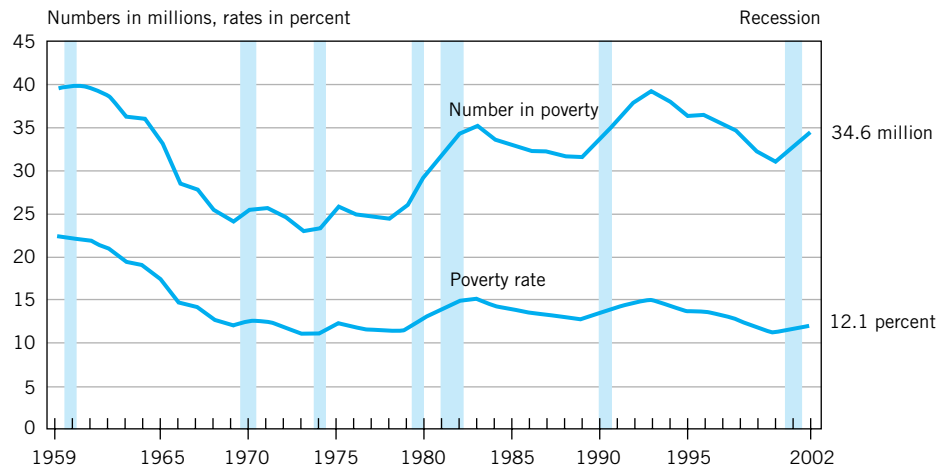


Figure 1.38 Estimated quarterly U.S. retail e-commerce sales (fourth quarter 1999 to third quarter 2003).

Source: U.S. Department of Commerce, www.census.gov

b. *Note:* The vertical scale is used in two different ways on this graph from the U.S. Bureau of the Census.



Note: The data points represent the midpoints of the respective years.

Figure 1.39 Number in poverty and poverty rate, 1959 to 2002.

Source: U.S. Census Bureau, Current Population Survey, 1960–2003 Annual Social and Economic Supplements.

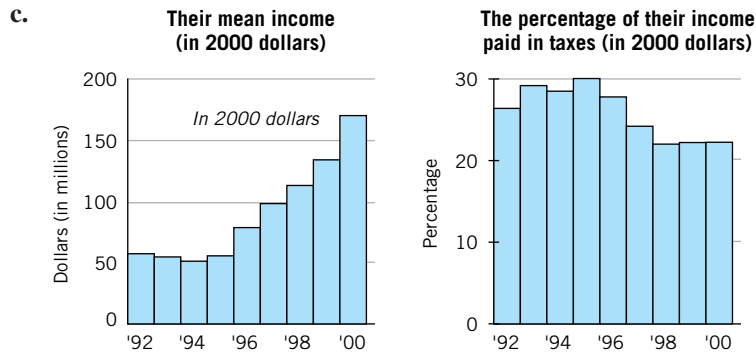


Figure 1.40 Data on the top 400 taxpayers in the United States.
Source: *The New York Times*, June 26, 2003.

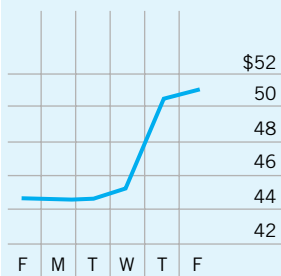
- Solution**
- Data released from the U.S. Department of Commerce show that e-commerce sales generally rose from the fourth quarter of 1999 to the third quarter of 2003. In each fourth quarter there is a distinct peak indicating an increase due to holiday sales.
 - In 1959, according to the U.S. Bureau of the Census, there were about 40 million Americans in poverty, representing almost 23% of the population. Between 1959 and 1974 both the number and the percentage of Americans in poverty decreased to a low of about 23 million and 12%, respectively. From 1974 to 2002, the percentage in poverty continued to hover between 12% and 15%. During the same time period the number in poverty vacillated, but the overall trend was an increase to almost 34.6 million in 2002.
 - From 1992 to 2000 the mean income of the top 400 taxpayers grew steadily, but the percentage of their income that went toward taxes decreased.

We will spend the rest of this text examining patterns in functions and their graphs. We'll study "families of functions"—linear, exponential, logarithmic, power, and polynomial—that will provide mathematical tools for describing the world around us.

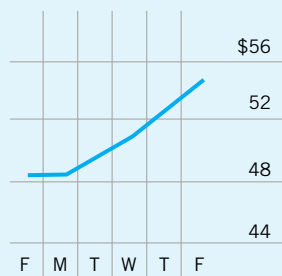
Algebra Aerobics 1.4

Note: Problem 4 requires technology.

- The graphs in Figure 1.41 show the price of shares of stock of two companies over a one-week period,
- Use Figure 1.42 to answer the following questions.



Bank One
NYSE: ONE



Genzyme
NNM: GENZ

Figure 1.41 Stock prices for two companies.
Source: *The New York Times*, Jan. 17, 2004.

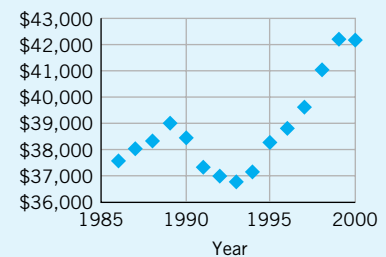


Figure 1.42 Median household income adjusted for inflation (in constant 2000 dollars).

- a. Estimate the maximum value for median household income during the time period represented on the graph? In what year does the maximum occur? What are the approximate coordinates at the maximum point?
 - b. What is the minimum value for median household income? In what year does this occur? What are the coordinates of this point?
 - c. Describe the changes in median household income between 1986 and 2000.
3. Examine the graphs of military reserve enlisted personnel in Figure 1.43.
 - a. In what year(s) did female and male enlisted personnel reach a maximum?
 - b. What was the maximum and minimum for both men and women over the time interval [1990, 2002]?
 - c. Describe the trend in number of female and male reserve enlisted personnel.
4. Consider the equation: $x^2 - 2y + 3 = 4x$. Solve for y , then use technology to graph the equation.
 - a. Find three points (x, y) that are solutions to the equation.
 - b. Find three points (x, y) that are not solutions to the equation.
 - c. Does the graph have a maximum or minimum point?
 - d. Over what interval is the function increasing? Decreasing?
 5. Figure 1.44 shows the graphs of the functions $f(x)$ and $g(x)$ on the interval $[-3, 3]$ for x .
 - a. Estimate three points that are solutions of each equation represented by $f(x)$ and $g(x)$.
 - b. Give three points that are not solutions to each equation.
 - c. Estimate intervals of x on which the function is increasing.

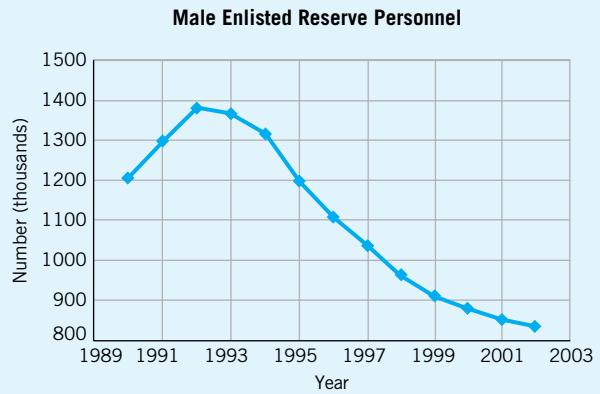
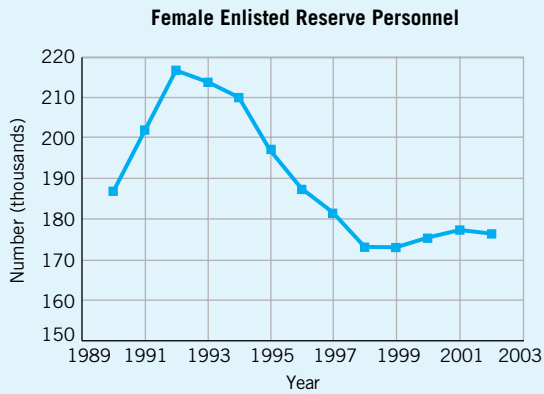


Figure 1.43 Female and male military reserve personnel.
Source: U.S. Bureau of the Census, *Statistical Abstract 2003*.

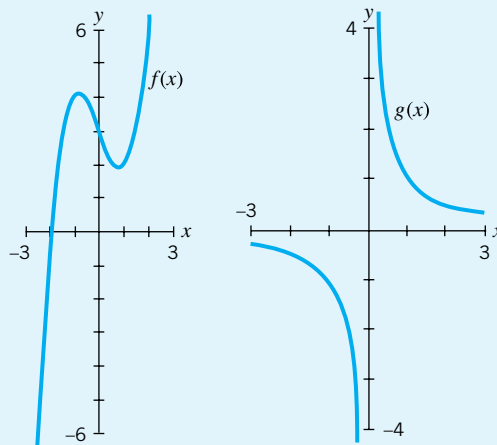


Figure 1.44 Graphs of two functions.

- d. Estimate intervals of x on which the function is decreasing.
 - e. Estimate intervals of x on which the function is concave down.
 - f. Estimate intervals of x on which the function is concave up.
6. Answer the following questions about poverty levels in the United States by examining the graph in Figure 1.45.
- a. What is the longest time interval over which the poverty rate for children (individuals under the age of 18 years) decreased?
 - b. Construct a headline to accompany Figure 1.45 for a newspaper article about poverty levels in the United States.
7. Choose the “best” graph in Figure 1.46 to describe the following situation and explain why the others are not good choices. Speed (S) is on the vertical axis and time (t) is on the horizontal axis.

A child in a playground tentatively climbs the steps of a large slide, first at a steady pace, then gradually slowing down until she reaches the top, where she stops to rest before sliding down.

8. Generate a rough sketch of each of the following situations.
- a. U.S. AIDS cases increased dramatically, reaching an all-time high for a relatively short period, and then consistently decreased, until a recent small increase.
 - b. When a soda is removed from the fridge, the internal pressure is slightly above the surrounding air pressure. With the can unopened, the internal pressure soon more than doubles, stabilizing at a level three times the surrounding air pressure.
 - c. Adding additional fertilizer to a crop initially increases productivity, but adding too much can be toxic, lowering productivity.

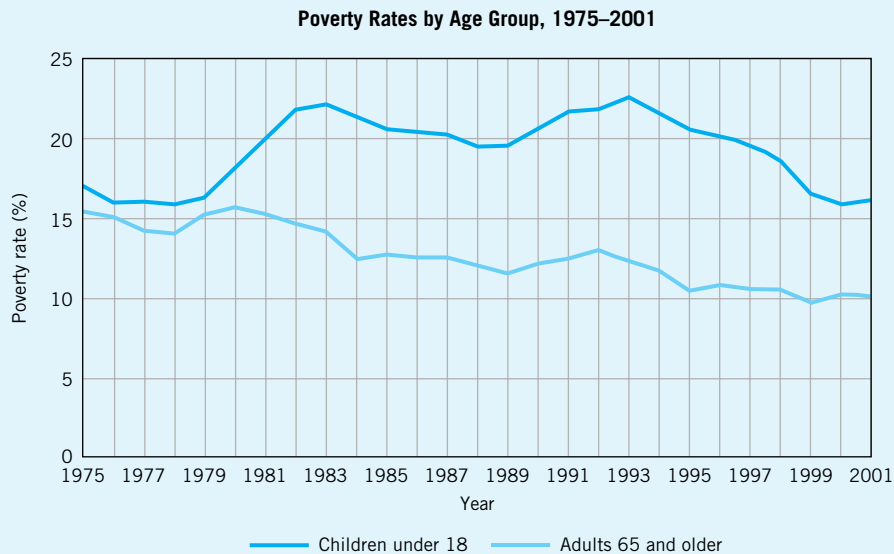


Figure 1.45 U.S. poverty rates.
 Source: National Center for Children in Poverty, Columbia University, www.nccp.org

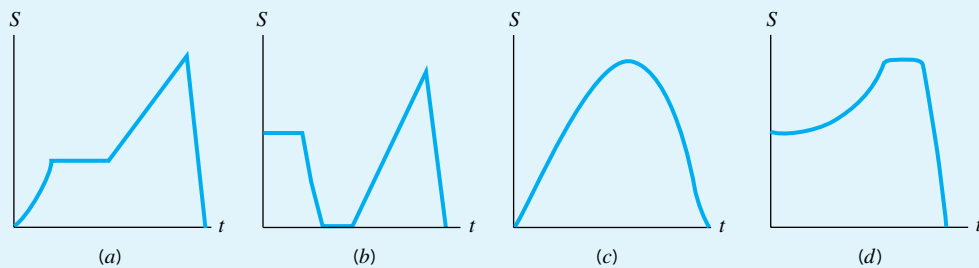


Figure 1.46

CHAPTER SUMMARY

Single-Variable Data Visualizing Single-Variable Data

Single-variable data are often represented with bar charts, pie charts, and histograms.

Numerical Descriptors

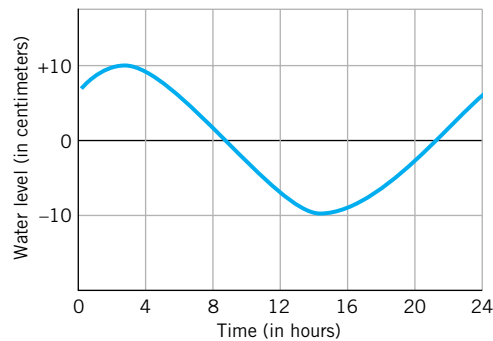
The *mean* of a list of numbers is their sum divided by the number of terms in the list. The mean is often referred to as the “average.”

The *median* separates a numerically ordered list of numbers into two parts, with half the numbers at or below the median and half at or above the median.

The *relative frequency* of any value is the fraction (often expressed as a percentage) of all the data in the sample having that value.

Relationships Between Two Variables Visualizing Two-Variable Data

Graphs of two-variable data can show how change in one variable affects change in the other. The graph shown below is called a *time series* because it shows changes over time.



Equations in Two Variables

The *solutions of an equation* in two variables x and y are the ordered pairs (x, y) that make the equation a true statement.

The *graph of an equation* in two variables displays the set of points that are solutions to the equation.

Functions A variable y is a *function* of a variable x if each value of x determines a unique (one and only one value) of y .

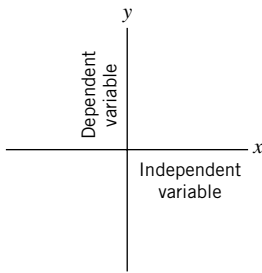
When a set of ordered pairs represents a function, then each ordered pair is written in the form

(independent variable, dependent variable)

(input, output)

(x, y)

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By convention, on the graph of a function, the independent variable is represented on the horizontal axis and the dependent variable on the vertical axis.

Function Notation

The expression $f(x)$ means the rule f is applied to the input value x to give the output value, $f(x)$:

$$f(\text{input}) = \text{output}$$

or $f(\text{independent variable}) = \text{dependent variable}$

For example, if f is defined by the equation $T = 0.06P$, then, assuming P is the input and T the output,

$$f(P) = T \quad \text{where} \quad T = 0.06P$$

or $f(P) = 0.06P$

Function notation is particularly useful when a function is being evaluated at a specific point. For example, $f(10) = (0.06)(10) = 0.6$

Domain and Range of a Function

The *domain* of a function is the set of all possible values of the independent variable.

The *range* is the set of corresponding values of the dependent variable.

Using Interval Notation to Represent Domain and Range

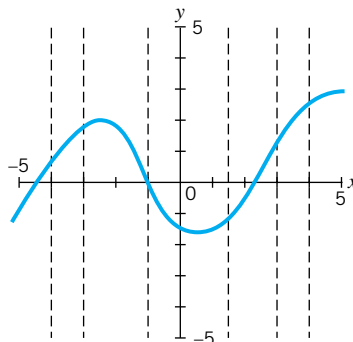
The set of numbers x such that $a \leq x \leq b$ is written $[a, b]$.

The set of numbers x such that $a < x < b$ is written (a, b) .

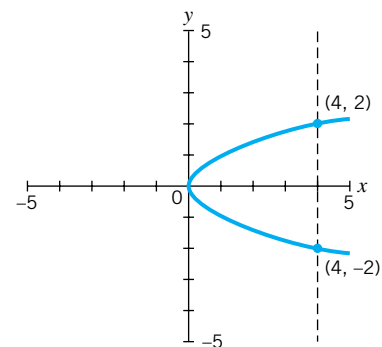
For example, if the domain is values of n greater than 50 and less than or equal to 100, then the domain = $(50, 100]$.

How Can We Tell if a Graph Represents a Function?

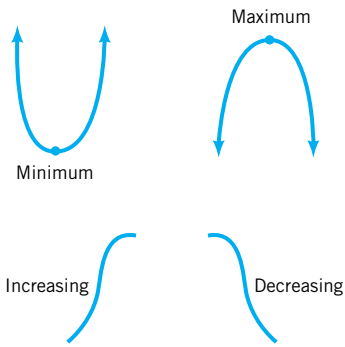
A graph does not represent a function if it fails the *vertical line test*. If you can draw a vertical line that crosses the graph two or more times, the graph does not represent a function.



This graph represents y as a function of x since there is no vertical line that intersects the curve at more than one point.



This graph does not represent y as a function of x since there is at least one vertical line that intersects this curve at more than one point.



Visualizing Functions

If a function has a *maximum* (or *minimum*) value, then it appears as the highest point (or lowest point) on its graph.

A function f is *increasing* over a specified interval if the values of $f(x)$ increase as x increases in this interval. A function f is *decreasing* over a specified interval if the values of $f(x)$ decrease as x increases over the interval.

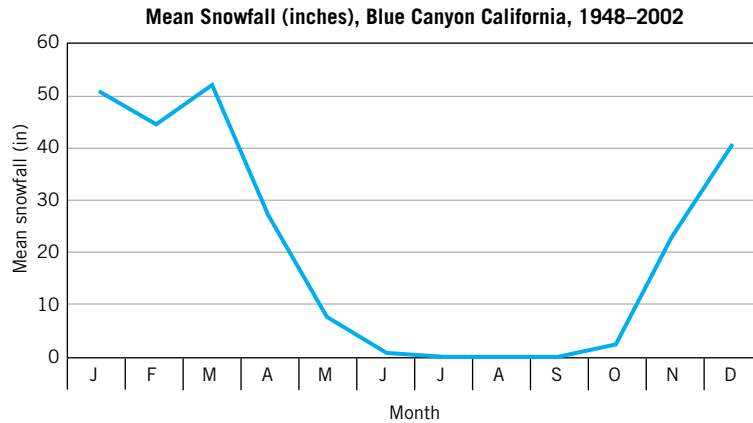
The graph of a function is *concave up* if it bends upward as we move left to right; it is *concave down* if it bends downward.



CHECK YOUR UNDERSTANDING

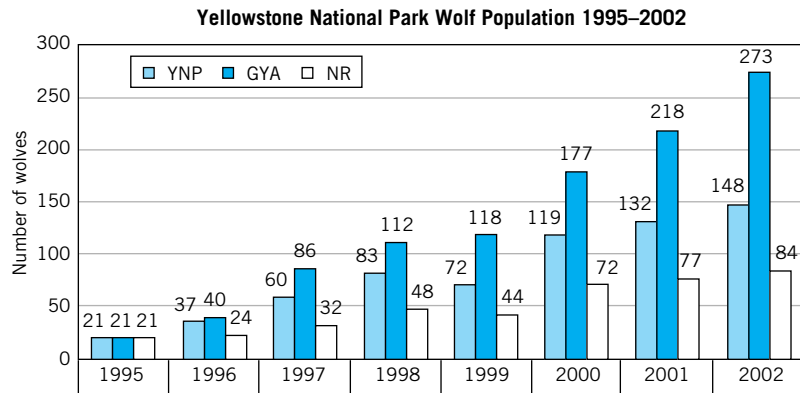
- I. Is each of the statements in Problems 1–27 true or false?
 1. Histograms, bar charts, and pie charts are used to graph single-variable data.
 2. Means and medians, both measures of central tendency, can be used interchangeably.
 3. A scatter plot is a plot of data points (x, y) for two-variable data.

Problems 4 and 5 refer to the accompanying figure.



Source: Western Regional Climate Center, www.wrcc.dri.edu.

4. The graph shows the changes in mean snowfall for each month of the year over the years 1948–2002.
5. The maximum snowfall occurs in March, with mean snowfall of about 52 inches.
6. The accompanying figure, of the wolf population in Yellowstone National Park (YNP), the Greater Yellowstone area (GYA), and the Northern Region (NR), shows that the wolf population in the Greater Yellowstone area is increasing.



Source: Yellowstone Wolf Project, Annual Report 2002, U.S. Department of the Interior.

7. The independent variable of a function is also called the output of the function.
8. The graph of the equation describing the sales revenue R (\$ million) as a function of the amount spent on advertising A (\$ thousand) is the set of ordered pairs (R, A) that satisfy the equation.
9. If sales revenue R is a function of advertising A , then R is the dependent variable and A is the independent variable.
10. If $M = F(q)$, then F is a function of q .

Problems 11 and 12 refer to the function $F(q) = \frac{2}{3q}$.

11. $F(-1) = \frac{-2}{3}$.

12. The domain of $F(q)$ is the set of all real numbers.

13. In the accompanying graph y is a function of x .

Problems 14–16 refer to the accompanying graph of $h(x)$.

14. The function $h(x)$ has a maximum value at $(1, 3)$.

15. The function $h(x)$ is concave down.

16. The function $h(x)$ increases to the right of $x = 1$ and increases to the left of $x = 1$.

Problems 17 and 18 refer to the following table.

| | | | | |
|-----|----|---|---|---|
| R | -1 | 2 | 0 | 3 |
| S | 10 | 8 | 6 | 4 |

17. The variable S is a function of R .

18. The variable S is decreasing over the domain for R .

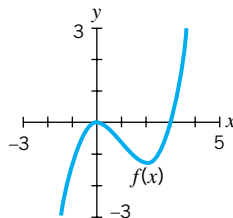
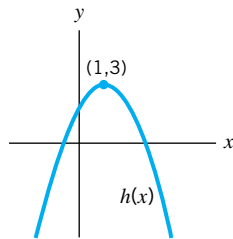
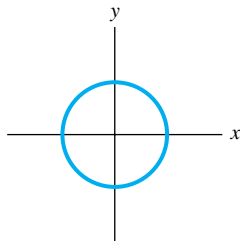
19. A set of ordered pairs of the form (M, C) implies that M is a function of C .

Problems 20–22 refer to the accompanying graph of $f(x)$.

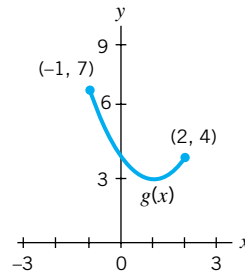
20. The function $f(x)$ has both a maximum and a minimum over the interval $[-1, 3]$.

21. The function $f(x)$ decreases over the interval $(0, 2)$.

22. The function $f(x)$ is concave up for $x > 1$.

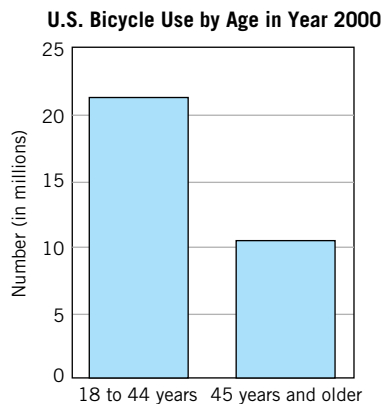


Problems 23 and 24 refer to the accompanying figure.



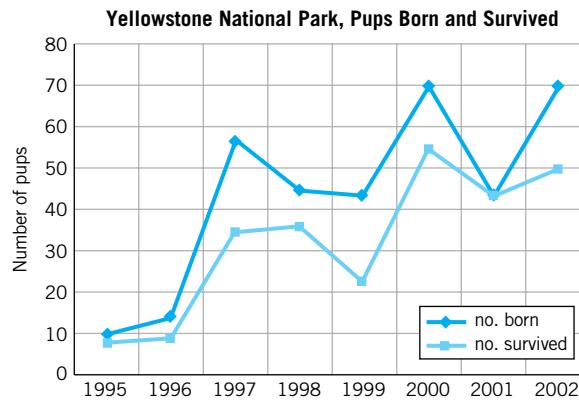
23. The domain of the function $g(x)$ is the interval $[-1, 2]$.
 24. The range of the function $g(x)$ is the interval $[4, 7]$.

Problems 25–27 refer to the accompanying figure.



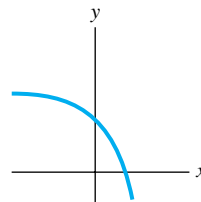
Source: National Survey on Recreation and the Environment, www.bts.gov

25. The single-variable histogram describes the *age* of bicycle riders.
 26. There are about twice as many bicycle riders who are 18 to 44 years old as there are 45 years or older.
 27. The total number of bicycle riders is over 30 million.
- II.** In Problems 28–34, give an example of a graph, relationship, function, or functions with the specified properties.
28. A relationship between two variables w and z described with an equation where z is not a function of w .
 29. A relationship between two variables w and z described as a table where w is a function of z but z is not a function of w .
 30. A graph of a function that is increasing and concave down.
 31. A graph of a function that is decreasing and concave up.
 32. A graph of a function that is concave up and has a minimum value at the point $(-2, 0)$.
 33. A graph of a function where the domain is the set of real numbers and that has no maximum or minimum values.
 34. A topic sentence describing the wolf pups that were born and survived in Yellowstone Park as shown in the accompanying scatterplot.

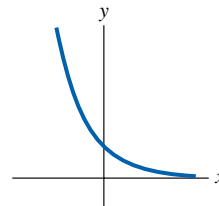


Source: Yellowstone Wolf Project, Annual Report 2002, U.S. Department of the Interior.

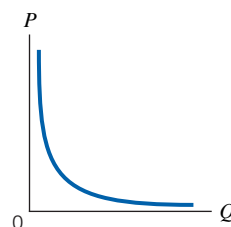
- III. Is each of the statements in Problems 35–42 true or false? If a statement is true, explain how you know. If a statement is false, give a counterexample or explain why it is false.
- 35. A function can have either a maximum or a minimum but not both.
 - 36. Neither horizontal nor vertical lines are functions.
 - 37. A function is any relationship between two quantities.
 - 38. The graph in the accompanying figure is decreasing and concave down.



39. The graph in the accompanying figure is increasing and concave up.



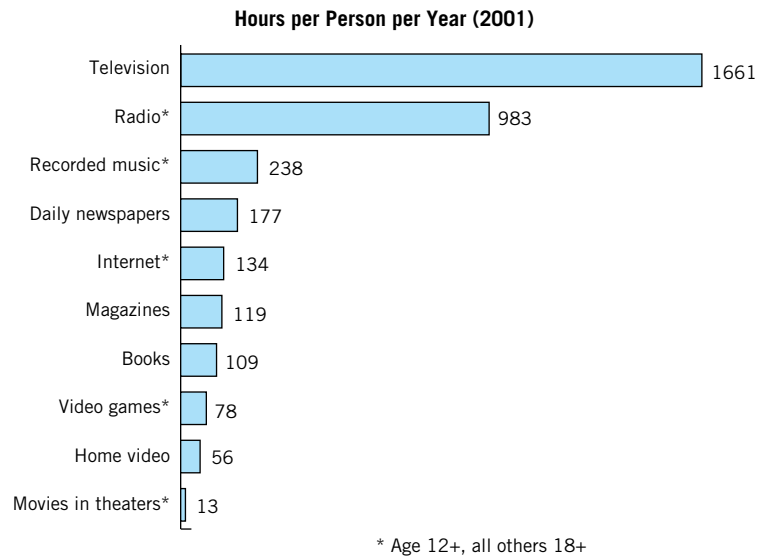
- 40. All functions have at least one minimum value.
- 41. Sometimes the choice as to which variable will be the independent variable for a function is arbitrary.
- 42. The graph in the accompanying figure represents P as a function of Q .



EXERCISES

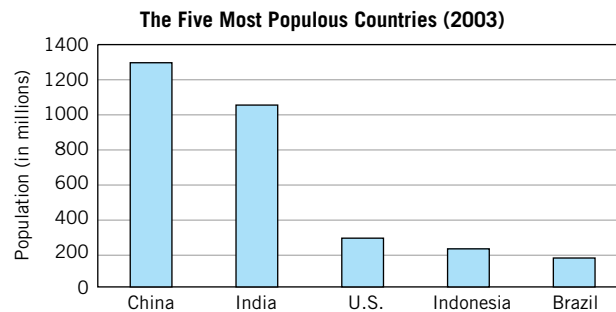
Exercises for Section 1.1

1. Many media sources compete for American's time (see accompanying figure). Assume the times spent on various activities do not overlap to answer the following questions.
 - a. How many total hours per person per year were spent in 2001 either listening to recorded music, using the Internet, or playing video games? Approximately how many hours per person per day?
 - b. How many total hours per person per year were spent reading either newspapers, magazines, or books? Approximately how many hours per person per day?
 - c. Compare the answers to parts (a) and (b) with the hours per day that a person watches television.



Source: Media Info Center, www.mediainfocenter.org

2. The accompanying bar chart shows the five countries with the largest populations in 2003.

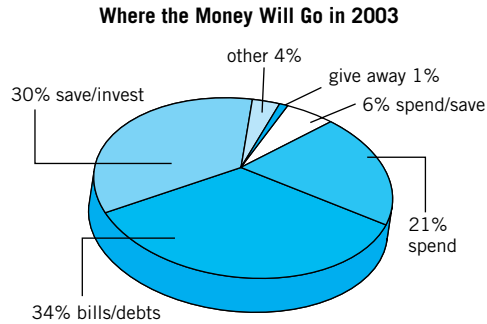


Source: "The CIA Factbook: 2003," www.odci.gov/cia

- a. What country has the largest population and approximately what is its population size?
- b. The population of India is projected in the near future to exceed the population of China. Given the current data, what is the minimum number of additional persons needed to make India's population larger than China's?
- c. The world population in 2003 is estimated to be about 6.3 billion. Approximately what percentage of the world's population live in China? In India? In the United States?

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3. In 2003 some taxpayers received \$300–600 tax rebates. Congress approved this spending as a means to stimulate the economy. According to a May 2003 ABC News/*Washington Post* poll, the accompanying pie chart shows how people would use the money.

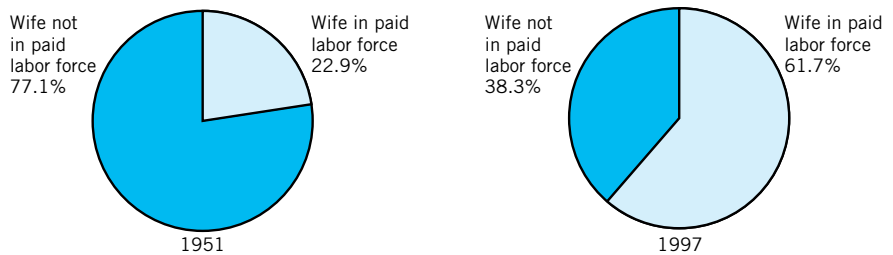


Source: ABCNEWS/*Washington Post* poll. May, 2003.

- a. What is the largest category on which people say they will spend their rebates? Why does the category look so much larger than its actual relative size?
 - b. What might make you suspicious about the numbers in this pie chart?
4. Compare the two sets of pie charts.

Wives Are an Increasingly Important Component of the Labor Force

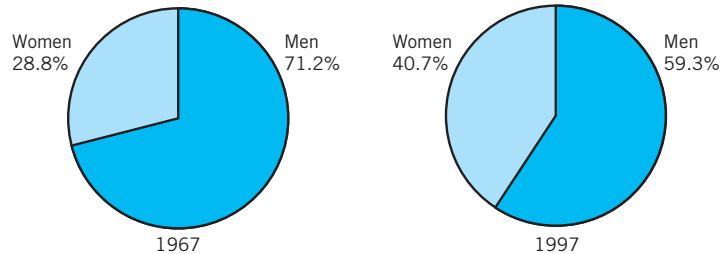
Percent of married-couple families with wives in the paid-labor force



Source: U.S. Bureau of the Census, March Current Population Survey.

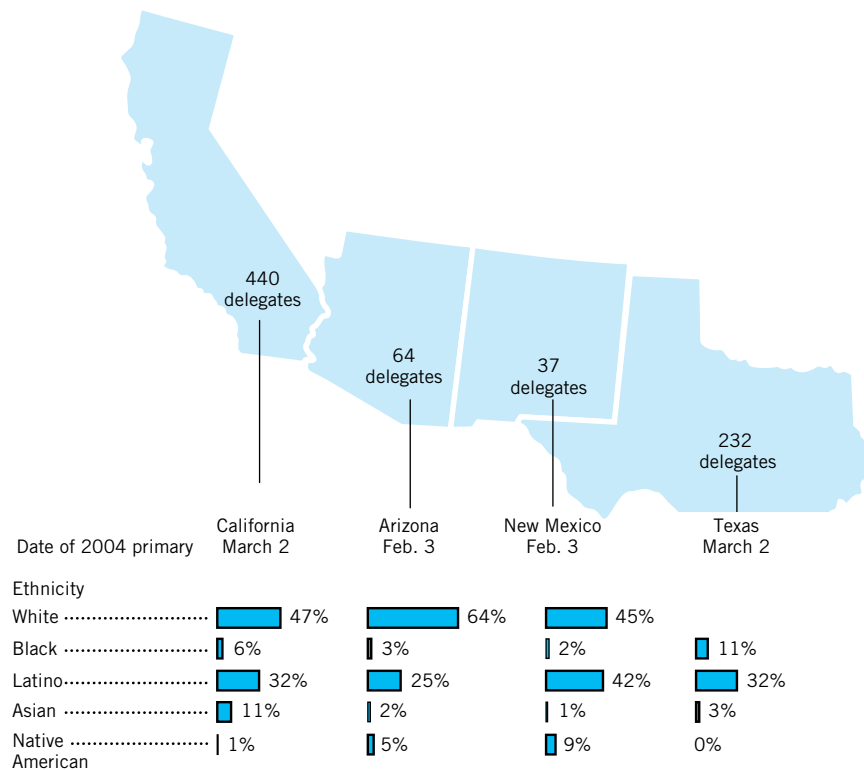
Women's Percent Share of the Labor Force Pie

(Percent of all full-time, year-round workers)



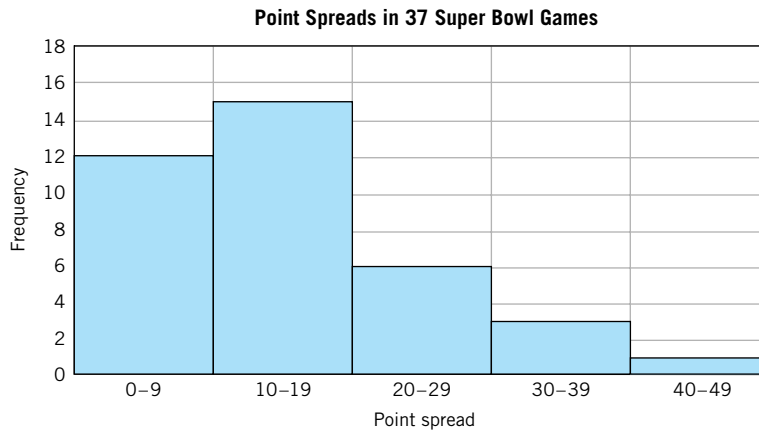
Source: U.S. Bureau of the Census, March Current Population Survey.

- a. What percentage of women were employed full-time in 1967? In 1997?
 - b. What percentage of wives worked in 1951? In 1997?
 - c. What do the pie charts not tell you about women or wives in the labor force?
5. During the 2004 presidential primaries, Democratic candidates were campaigning in four southwestern states. The accompanying charts have demographic information about the voters in these states. Use the percentages given to justify your answers to the following questions.



Sources: *The Green Papers*; *The Almanac of American Politics*.

- a. Why is California such a key state for any presidential candidate?
 - b. If the delegates proportionally reflect the diverse population of the state, how many delegates should be non-white from these four states?
6. The point spread in a football game is the difference between the winning team's score and the losing team's score. For example, in the 2004 Super Bowl game, the Patriots won with 32 points versus the Carolina Panthers' 29 points. So the point spread was 3 points.



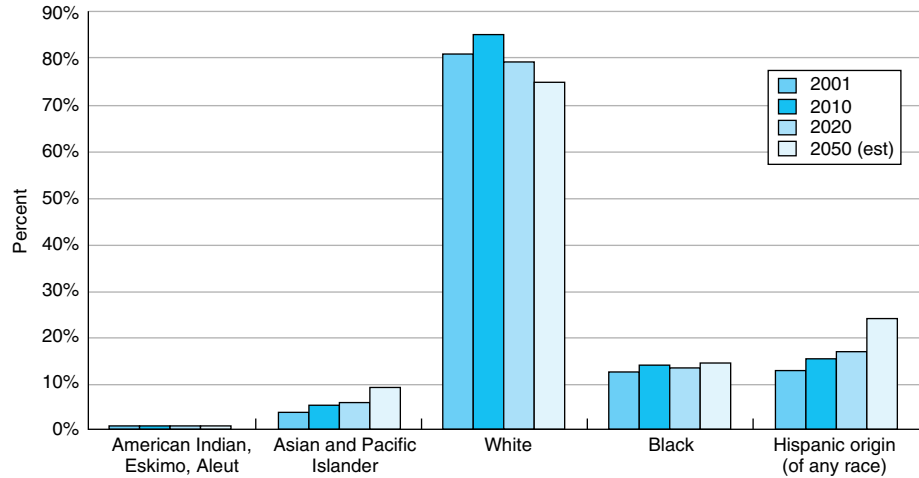
Source: www.sports-book.at

- a. What is the interval with the most likely point spread in a Super Bowl? The least likely?
 - b. What percentage of these Super Bowl games had a point spread of 9 or less? Of 19 or less? (Hint: First find the total number of 9 games.)
7. Given here is a table of salaries taken from a survey of recent graduates (with bachelor degrees) from a well-known university in Pittsburgh.

| Salary (in thousands) | Number of Graduates Receiving Salary |
|--------------------------|---|
| 11-15 | 2 |
| 16-20 | 3 |
| 21-25 | 10 |
| 26-30 | 20 |
| 31-35 | 9 |
| 36-40 | 1 |

- a. How many graduates were surveyed?
 - b. Is this quantitative or qualitative data? Explain.
 - c. What is the relative frequency of people having a salary between \$26,000 and \$30,000?
 - d. Create a histogram of the data.
8. The accompanying bar chart shows the predictions of the U.S. Census Bureau about the future racial composition of American society.
- a. Estimate the following percentages:
 - i. Asian and Pacific Islanders in the year 2050
 - ii. Combined white and black population in the year 2020
 - iii. Non-Hispanic population in the year 2001
 - b. The U.S. Bureau of the Census has projected that there will be approximately 392,031,000 people in the United States in the year 2050. Approximately how many people will be of Hispanic origin in the year 2050?
 - c. Write a topic sentence describing the overall trend.

Expected Change from 2001–2050 of U.S. Population by Race and Ethnicity

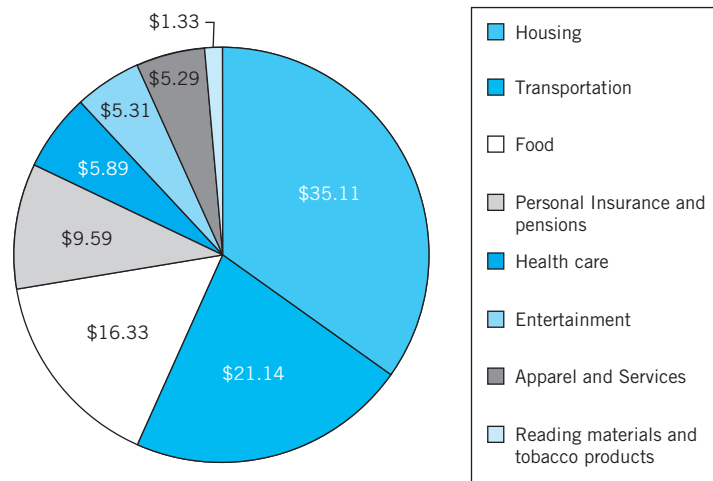


Sources: U.S. Bureau of the Census, *Statistical Abstract of the United States: 2002*.

9. Below is a pie chart of America’s spending patterns at the end of 2002.
- In what single category did Americans spend the largest percentage of their income? Estimate this percentage.
 - According to this chart, if an American family has an income of \$35,000, how much of it would be spent on food?
 - If you were to write a newspaper article to accompany this pie chart, what would your opening topic sentence be?

How Americans Spend Their Money

How the average American spends \$100, as measured in late 2002. The Labor Department uses this survey—along with price samples—to calculate the inflation rate.



Sources: U.S. Bureau of the Census, *Statistical Abstract of the United States: 2002*.

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10. Attendance at a stadium for the last 30 games of a college baseball team is listed as follows:


| | | | | | |
|------|------|------|------|------|------|
| 5072 | 3582 | 2504 | 4834 | 2456 | 3956 |
| 2341 | 2478 | 3602 | 5435 | 3903 | 4535 |
| 1980 | 1784 | 1493 | 3674 | 4593 | 5108 |
| 1376 | 978 | 2035 | 1239 | 2456 | 5189 |
| 3654 | 3845 | 673 | 2745 | 3768 | 5227 |

Create a histogram to display these data. Decide how large the intervals should be to illustrate the data well but not be overly detailed.

11. a. Compute the mean and median for the list: 5, 18, 22, 46, 80, 105, 110.
 b. Change one of the entries in the list in part (a) so that the median stays the same and the mean increases.
12. Suppose that a church congregation has 100 members, each of whom donates 10% of his or her income to the church. The church collected \$250,000 last year from its members.
- a. What was the mean contribution of its members?
 b. What was the mean income of its members?
 c. Can you predict the median income of its members? Explain your answer.
13. Suppose that annual salaries in a certain corporation are as follows:

| | |
|-------------------------|----------|
| Level I (30 employees) | \$18,000 |
| Level II (8 employees) | \$36,000 |
| Level III (2 employees) | \$80,000 |

Find the mean and median annual salary. Suppose that an advertisement is placed in the newspaper giving the average annual salary of employees in this corporation as a way to attract applicants. Why would this be a misleading indicator of salary expectations?

14. Suppose your grades on your first four exams were 78%, 92%, 60%, and 85%. What would be the lowest possible average that your last two exams could have so that your grade in the class, based on the average of the six exams, is at least 82%?
-  15. Read Stephen Jay Gould's article "The Median Isn't the Message" and explain how an understanding of statistics brought hope to a cancer victim.
16. a. On the first quiz (worth 25 points) given in a section of college algebra, one person received a score of 16, two people got 18, one got 21, three got 22, one got 23, and one got 25. What were the mean and median of the quiz scores for this group of students?
 b. On the second quiz (again worth 25 points), the scores for eight students were 16, 17, 18, 20, 22, 23, 25, and 25.
 i. If the mean of the scores for the nine students was 21, then what was the missing score?
 ii. If the median of the scores was 22, then what are possible scores for the missing ninth student?
17. In 2003 the prices (in dollars) of the top-selling digital cameras¹³ were: 150, 200, 230, 270, 280, 300, 330, 330, 350, 370, 400, 400, 430, 450, 500, 500, 550, 700, 700, 800.
- a. Calculate the mean and median of the camera prices.
 b. Use the data to fill in the following data table:

¹³Data gathered from www.circuitcity.com

| Dollar Interval | Frequency Count | Relative Frequency |
|-----------------|-----------------|--------------------|
| 100–199 | | |
| 200–299 | | |
| 300–399 | | |
| 400–499 | | |
| 500–599 | | |
| 600–699 | | |
| 700–799 | | |
| 800–899 | | |
| Total | | |

- c. Construct a frequency histogram using the table.
 - d. What price range has the most cameras from which to choose?
 - e. You decided that you are willing to pay \$300 or more, but less than \$600. How many camera choices would you have?
 - f. What percentage of all cameras cost below \$400?
18. Up to and including George W. Bush, the ages of the last fifteen presidents when they took office¹⁴ were 56, 55, 51, 54, 51, 60, 62, 43, 55, 56, 52, 69, 64, 46, 54.
- a. Find the mean and median ages of the past fifteen presidents when they took office.
 - b. If the mean age of the past sixteen presidents is 54.94, at what age did the missing president take office?
 - c. Beginning with age 40 and using 5-year intervals, find the frequency count for each age interval.
 - d. Create a frequency histogram using your results from part (c).
19. Herb Caen, a Pulitzer Prize–winning columnist for the *San Francisco Chronicle*, remarked that a person moving from state A to state B could raise the average IQ in both states. Is he right? Explain.
20. Why do you think most researchers use median rather than mean income when studying “typical” households?
21. According to the 2000 U.S. Census, the median net worth of American families was \$55,000 and the mean net worth was \$282,500. How could there be such a wide discrepancy?
22. Read the *CHANCE News* article and explain why the author was concerned.
23. The Greek letter Σ (called sigma) is used to represent the sum of all of the terms of a certain group. Thus, $a_1 + a_2 + a_3 + \cdots + a_n$ can be written as



$$\sum_{i=1}^n a_i = a_1 + a_2 + \cdots + a_n$$

which means to add together all of the values of a_i from a_1 to a_n .

- a. Using Σ notation, write an algebraic expression for the mean of the five numbers x_1, x_2, x_3, x_4, x_5 .
- b. Using Σ notation, write an algebraic expression for the mean of n numbers $t_1, t_2, t_3, \dots, t_n$.

¹⁴<http://www.campvishus.org/PresAgeDadLeft.htm#AgeOffice>.

c. Evaluate the following sum:

$$\sum_{k=1}^5 2k$$

24. The accompanying table gives the ages of students in a mathematics class.

Ages of Students

| Age Interval | Frequency Count |
|--------------|-----------------|
| 15–19 | 2 |
| 20–24 | 8 |
| 25–29 | 4 |
| 30–34 | 3 |
| 35–39 | 2 |
| 40–44 | 1 |
| 45–49 | 1 |
| Total | 21 |

- a. Use this information to estimate the mean age of the students in the class. Show your work. (*Hint:* Use the mean age of each interval).
- b. What is the largest value the actual mean could have? The smallest? Why?
25. (Use of calculator or other technology recommended) Use the following table to generate an estimate of the mean age of the U.S. population. Show your work.

Ages of U.S. Population in 2001 in 10-Year Intervals

| Age (years) | Population (thousands) |
|-------------|------------------------|
| Under 10 | 39,553 |
| 10–19 | 41,148 |
| 20–29 | 38,607 |
| 30–39 | 42,924 |
| 40–49 | 43,545 |
| 50–59 | 32,609 |
| 60–69 | 20,651 |
| 70–79 | 16,206 |
| 80 and over | 9,553 |
| Total | 284,796 |

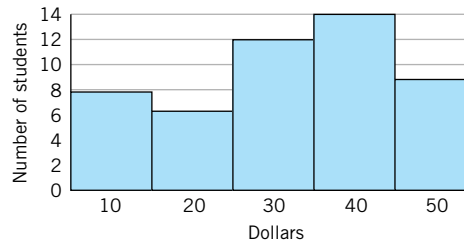
Source: U.S. Bureau of the Census, *Statistical Abstract of the United States 2002*.

26. An article titled “Venerable Elders” (*The Economist*, July 24, 1999) reported that “both Democratic and Republican images are selective snapshots of a reality in which the median net worth of households headed by Americans aged 65 or over is around double the national average—but in which a tenth of such households are also living in poverty.” What additional statistics would be useful in forming an opinion on whether elderly Americans are wealthy or poor compared with Americans as a whole?



The program "F4: Measures of Central Tendency" in *FAM 1000 Census Graphs* can help you understand mean and median and their relationship to histograms

27. Estimate the mean and median from the given histogram.



Monthly allowance of junior high school students.

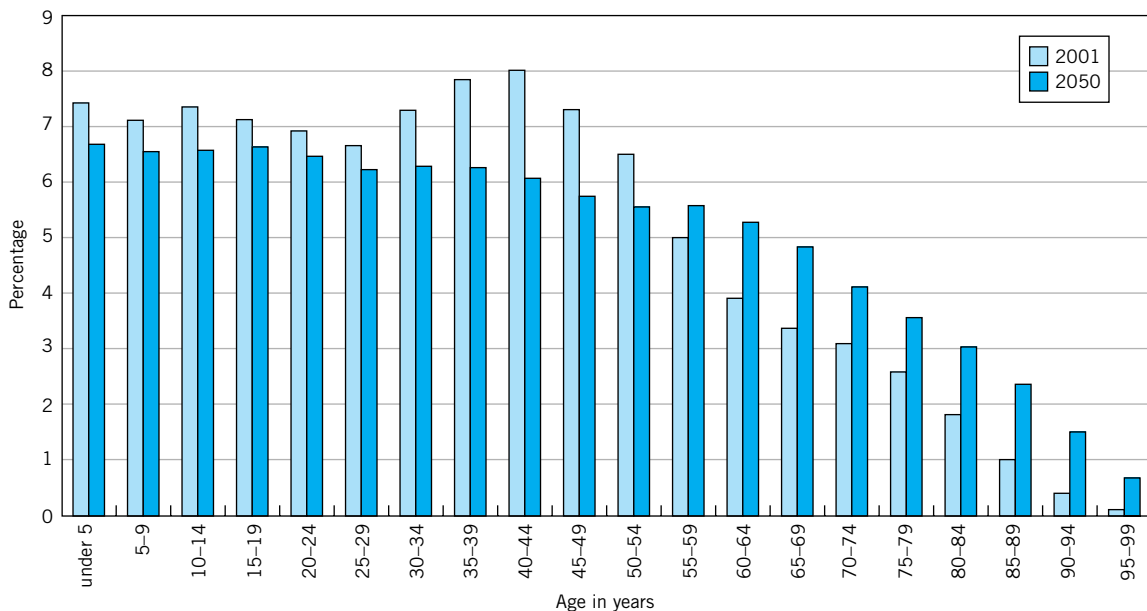
28. Choose a paragraph of text from any source (perhaps even this textbook) and construct a histogram of word lengths (the number of letters in the word). If the same word appears more than once, count it as many times as it appears. You will have to make some reasonable decisions about what to do with numbers, abbreviations, and contractions.

Compute the mean and median word lengths from your graph. Indicate how you would expect the graph to be different if you used:

- a. A children's book
- b. A work of literature
- c. A medical textbook



29. The following graph shows the distribution of ages in the U.S. population in 2001 and projections for 2050. (See Excel or graph link file PROJAGES.) Write at least two different headlines that capture some aspect of this graph.



Distribution of ages of U.S. population in 2001 and 2050 (projected).
 Source: U.S. Bureau of the Census, *Statistical Abstract of the United States 2002*.



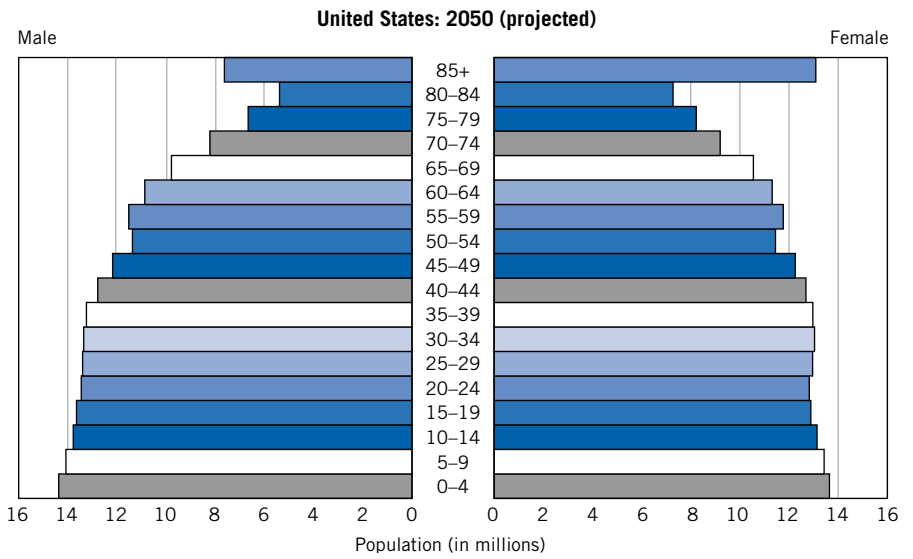
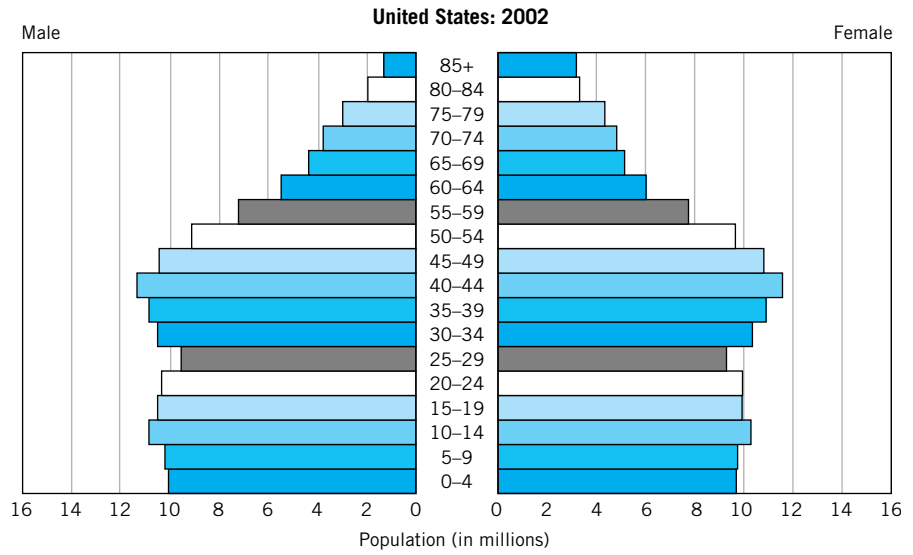
30. (Computer required) Open up the program “F1: Histograms” in *FAM1000 Census Graphs* in the course software. The 2003 U.S. Census data on 1000 randomly selected U.S. individuals and their families are imbedded in this program. You can use it to create histograms for education, age, and different measures of income. Try using different interval sizes to see what patterns emerge. Decide on one variable (say education) and compare the histograms of this variable for different groups of people. For example, you could compare education histograms for men and women or for people living in two different regions of the country. Pick a comparison that you think is interesting and, if possible, print out your histograms. Create a possible headline for these data. Describe three key features that support your headline.
31. Consider the data in the following table describing wolves captured and radio-collared in Montana and southern British Columbia, Canada, in 1993.
- Look at the variables sex, weight, and age. Identify each variable as quantitative or qualitative.
 - Calculate the mean and median weights of female wolves and of male wolves.
 - Discuss which measurement of central tendency (mean or median) might be more appropriate for describing the data in part (b).
 - Estimate the mean age of the females, of the males, and of the total population.

| Wolf Pack | Capture Date | Wolf Number | Ear Tag Number | Sex | Weight (lb) | Age (years) |
|--------------|--------------|-------------|----------------|-----|-------------|-------------|
| Spruce Creek | 6/14 | 9318 | 102103 | M | 100 | 2–3 |
| | 6/16 | 9381 | 104105 | F | 73 | 1–2 |
| North Camas | 6/2 | 9375 | None | F | 77 | 4–5 |
| | 6/4 | 9378 | 100101 | F | 72 | 3–4 |
| | 6/4 | 9376 | 9596 | F | 66 | 2–3 |
| | 6/4 | 9377 | 9798 | F | 63 | 1–2 |
| | 6/9 | 9379 | None | F | 59 | ? |
| South Camas | 6/11 | 9380 | None | F | 59 | 1 |
| | 5/27 | 9474 | 9091 | F | 61 | 1 |
| | 5/29 | 9317 | 9293 | M | 95 | 2–3 |
| Murphy Lake | 5/30 | 8756 | 2627 | F | 77 | 6 |
| | 6/22 | 1718 | 1718 | F | 73 | 3 |
| | 6/23 | 2627 | 2627 | F | 21 | Pup |
| | 6/23 | 3637 | 3637 | M | 96 | 4–5 |
| Sawtooth | 10/15 | 2223 | 2223 | F | 62 | Pup |
| | 2/26 | 8808 | 8808 | M | 122 | 5 |
| | 9/22 | 2829 | 2829 | F | 50 | Pup |
| Ninemile | 9/25 | 3031 | 3031 | F | 53 | Pup |
| | 8/27 | 4243 | 4243 | F | 73 | 1 |

Source: 1993 *Annual Report of the Montana Interagency Wolf Working Group*, prepared by the U.S. Fish and Wildlife Service.

32. Population pyramids are a type of chart used to depict the overall age structure of a society.
- Using the accompanying population pyramids of the United States, estimate the number of:
 - Males who were between the ages 35 and 39 years in 2002
 - Females who were between the ages 55 and 59 years in 2002
 - Males 85 years and older in the year 2050; females 85 years and older in the year 2050
 - All males and females between the ages of 0 and 9 years in the year 2050

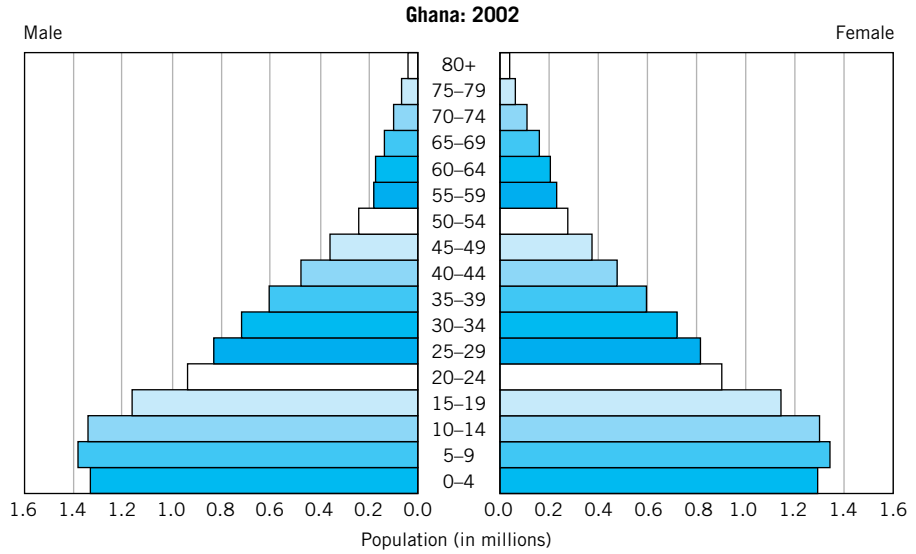
b. Describe two changes in the distribution of ages from the year 2002 to the predictions for 2050.



Population pyramid summary for the United States for 2002 and 2050.

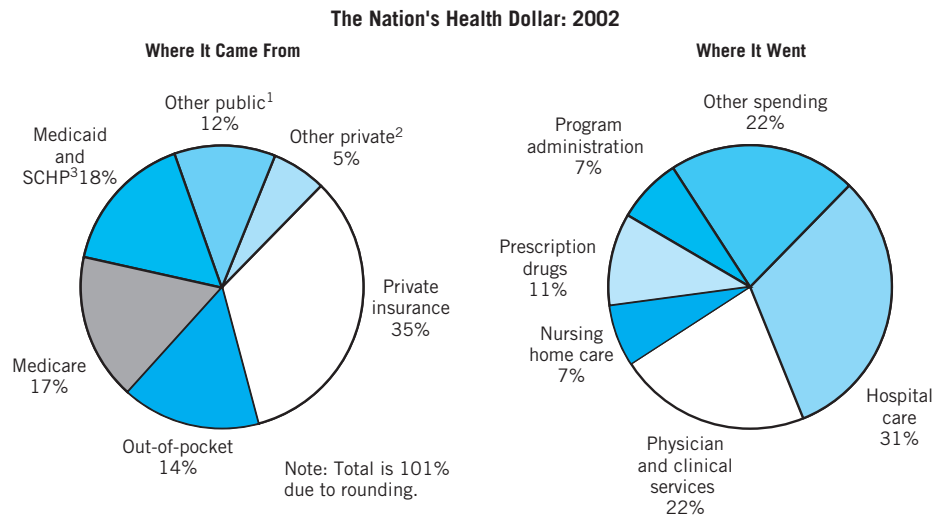
Source: U.S. Census Bureau, International Data Base, www.census.gov

33. The accompanying population pyramid shows the age structure in Ghana, a developing country in Africa, for 2002. The previous exercise contains a population pyramid for the United States, an industrialized nation, for 2002. Describe three major differences in the distribution of ages in these two countries in 2002.



Source: U.S. Census Bureau, International Data Base.

34. Why is the mean age larger than the median age in the United States? Do you think your prediction holds true for most communities within the United States? What predictions would you make for other countries? Check your predictions with data from the U.S. Census Bureau at www.census.gov.
35. While you are writing an article in the school newspaper about health care costs in the United States, you come across the following two pie charts.



¹“Other public” includes programs such as worker’s compensation, public health activity, Department of Defense, Department of Veterans Affairs, Indian Health Service, and state and local hospital subsidies and school health.

²“Other private” includes industrial in-plant, privately funded construction, and non-patient revenues, including philanthropy.

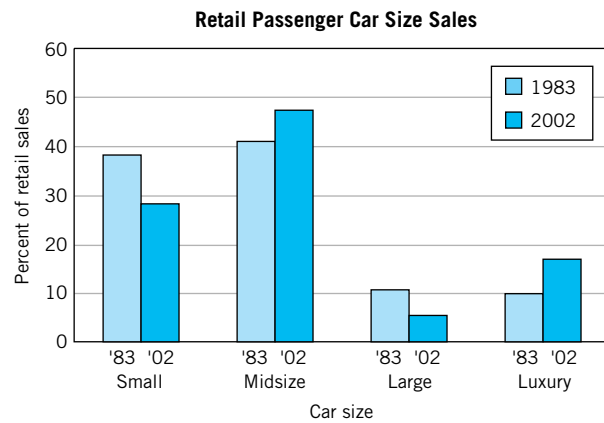
³SCHP refers to Social Security Health Programs.

Source: National Institutes of Health, www.nih.gov

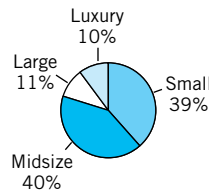
- a. What are the largest categories in each chart?
 - b. As our population ages, which spending category do you foresee increasing the most?
 - c. What percentage of health dollars come from either public or other government programs?
 - d. What don't these two charts tell you?
36. Shown are three forms of displaying data about retail passenger car size sales: a table, a bar chart, and pie charts.

Retail Passenger Car Size Sales

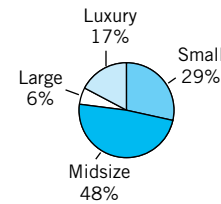
| Car Size | 1983 Percent of All Cars Sold | 2002 Percent of All Cars Sold |
|----------|-------------------------------|-------------------------------|
| Small | 39 | 29 |
| Midsize | 41 | 48 |
| Large | 11 | 6 |
| Luxury | 10 | 17 |



1983 Retail Passenger Car Size Sales



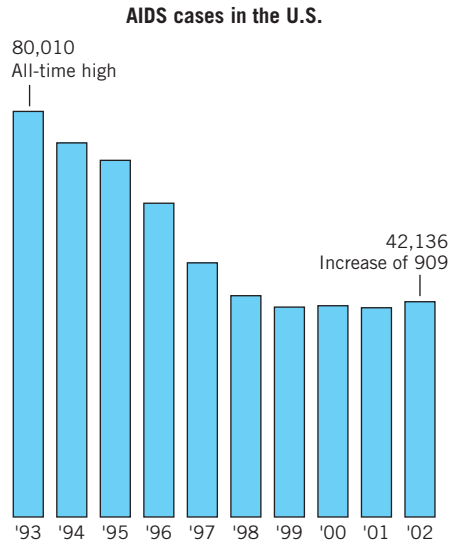
2002 Retail Passenger Car Size Sales



Write two of your own headlines and choose the chart or charts that would best make your point.

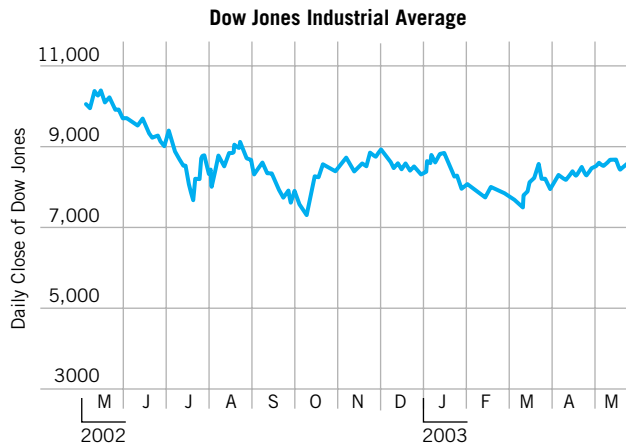
Exercises for Section 1.2

37. Assume you work for a newspaper and are asked to report on the following data.
- a. What are three important facts that emerge from this graph?
 - b. Create a title for the graph.
 - c. Construct a 60-second summary that will accompany the graph in the newspaper article.



Source: Centers for Disease Control and Prevention, www.cdc.gov

38. On May 23, 2003, *The New York Times* published the following graph of the Dow Jones Industrial Average. The Dow Jones is based on 30 stocks that trade on the New York Stock Exchange and is the best-known index of U.S. stocks.



Sources: Associated Press, Bloomberg Financial Markets.

- What time period does the graph cover?
- Estimate the lowest Dow Jones Industrial Average during that period. When did it occur?
- Estimate the highest value for the Dow Jones during that period. When did it occur?
- Write a topic sentence describing the change in the Dow Jones over the given time period.

39. The accompanying table shows the number of personal and property crimes in the U.S. from 1995 to 2000.

| Year | Personal Crimes (in thousands) | Property Crimes (in thousands) |
|------|-----------------------------------|--------------------------------------|
| 1995 | 10,436 | 29,490 |
| 1998 | 8,412 | 22,895 |
| 1999 | 7,565 | 21,215 |
| 2000 | 6,597 | 19,296 |

Source: U.S. Bureau of the Census, *Statistical Abstract*, 2001.

- Create a scatter plot of the personal crimes over time. Connect the points with line segments.
 - Create a scatter plot of the property crimes on the same graph. Connect the points with line segments.
 - Approximately how many *times* more property crimes than personal crimes were committed in 1995? Was this consistent throughout the interval of 1995 to 2000?
 - Write a topic sentence that compares property and personal crime over 1995 to 2000.
40. The National Cancer Institute now estimates that after 70 years of age, 1 woman in 8 will have gotten breast cancer. Fortunately, they also estimate that 95% of breast cancer can be cured, especially if caught early. The following data show how many women in different age groups are likely to get breast cancer.

Life Time Risk of Developing Breast Cancer

| Age Group | Chance of Developing Cancer | Chance in 1000 Women |
|--------------|--------------------------------|----------------------|
| 30–39 | 1 in 252 | 4 per 1000 |
| 40–49 | 1 in 68 | 15 per 1000 |
| 50–59 | 1 in 35 | 29 per 1000 |
| 60–69 | 1 in 27 | 37 per 1000 |
| 70+ | 1 in 8 | 125 per 1000 |

(Note: Men may get breast cancer too, but less than 1% of all breast cancer cases occur in men.)

- What is the overall relationship between age and breast cancer?
- Make a bar chart using the chance of breast cancer in 1000 women for the age groups given.
- Using the “Chance in 1000 women” data, estimate how much more likely that women in their 40s would have had breast cancer than women in their 30s. How much more likely for women in their 50s than women in their 40s?
- It is common for women to have yearly mammograms to detect breast cancer after they turn 50, and health insurance companies routinely pay for them. Looking at these data, would you recommend an earlier start for yearly mammograms? Explain your answer in terms of the interests of the patient and the insurance company.

(Note: Some research says that mammograms are not that good at detection.)

62 CHAPTER 1 MAKING SENSE OF DATA AND FUNCTIONS

41. The National Center for Chronic Disease Prevention and Health Promotion published the following data of the chance that a man has had prostate cancer at different ages.

Life Time Risk of Developing Prostate Cancer

| Age | 45 | 50 | 55 | 60 | 65 | 70 | 75 | 80 |
|--------------|-------------|----------|----------|---------|---------|---------|--------|--------|
| Risk | 1 in 25,000 | 1 in 476 | 1 in 120 | 1 in 43 | 1 in 21 | 1 in 13 | 1 in 9 | 1 in 6 |
| Percent Risk | 0.04% | 0.21% | 0.83% | 2.3% | 4.8% | 7.7% | 11.1% | 16.7% |

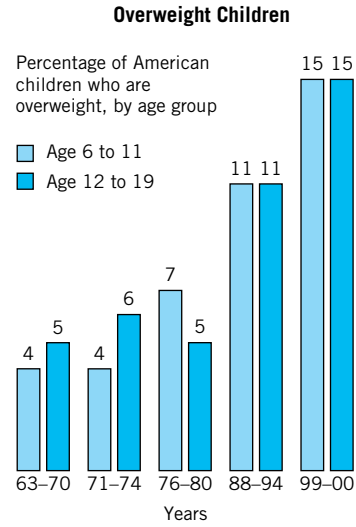
- What is the relationship between age and getting prostate cancer?
 - Make a scatter plot of the percent risk for men of the ages given.
 - Using the “Percent risk” data, how much more likely are men 50 years old to have had prostate cancer than men who are 45? How much more likely are men 55 years old to have had prostate cancer than men who are 50?
 - Looking at this data, when would you recommend annual prostate checkups for men? Explain your answer in terms of the interests of the patient and the insurance company.
42. Birth rate data in the United States is given as the number of live births per 1000 women in each age category.

| Age | 10–14 | 15–19 | 20–24 | 25–29 | 30–34 | 35–39 | 40–44 | 45–49 |
|------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1950 | 1.0 | 81.6 | 196.6 | 166.1 | 103.7 | 52.9 | 15.1 | 1.2 |
| 2000 | 0.9 | 48.5 | 112.3 | 121.4 | 94.1 | 40.4 | 7.9 | 0.5 |

Source: National Center for Health Statistics, U.S. Dept. of Health and Human Services.

- Construct a bar chart showing the birth rates for the year 1950. Which mother’s age category had the highest rate of live births? What percentage of women in that category delivered live babies? In which age category was the lowest rate of babies born? What percentage of women in that category delivered live babies?
 - Construct a bar chart showing the birth rates for the year 2000. Which mother’s age category had the highest rate of live births? What percentage of women in that category delivered live babies? In which age category was the lowest rate of babies born? What percentage of women in that category delivered live babies?
 - Write a paragraph comparing and contrasting the birth rates in 1950 and in 2000. Bear in mind that since 1950 there have been considerable medical advances in saving premature babies and in increasing the fertility of couples.
43. The National Center for Health Statistics published the accompanying chart on childhood obesity.
- How would you describe the overall trend in the weights of American children?
 - Over which years did the percentage of overweight children age 6 to 11 increase?
 - Over which time period was there no change in the percentage of overweight children age 6 to 11?
 - During which time period were there relatively more overweight 6- to 11-year-olds than 12- to 19-year-olds?

- e. One of the national health objectives for the year 2010 is to reduce the prevalence of obesity among children to less than 15%. Does this seem like a reasonable goal?



Source: National Center for Health Statistics, www.cdc.gov/nchs

44. Some years are more severe for influenza- and pneumonia-related deaths than others. The accompanying table shows data from Centers for Disease Control figures for selected years from 1950 to 2000.

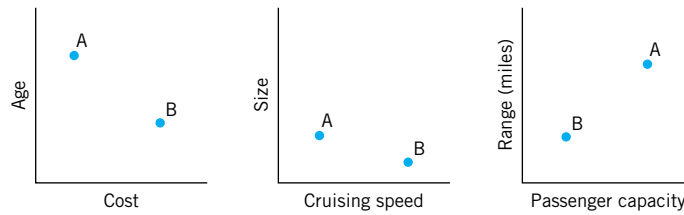
Age-Adjusted Death Rate for Influenza and Pneumonia

| Year | Death Rate per 100,000 | |
|------|------------------------|---------|
| | Males | Females |
| 1950 | 55 | 42 |
| 1960 | 66 | 44 |
| 1970 | 54 | 33 |
| 1980 | 42 | 25 |
| 1990 | 48 | 31 |
| 2000 | 29 | 21 |

Source: Centers for Disease Control and Prevention, www.cdc.gov

- Create a double bar chart showing the death rates both for men and for women who died of influenza and pneumonia between 1950 and 2000.
- In which year were death rates highest for both men and women?
- Were there any decades in which there was an increase in male deaths but a decrease for women?
- Write a 60-second summary about deaths due to influenza and pneumonia over the years 1950 to 2000.

45. The following three graphs describe two cars, A and B.



For parts (a)–(d), decide whether the statement is true or false. Explain your reasoning.

- a. The newer car is more expensive.
 - b. The slower car is larger.
 - c. The larger car is newer.
 - d. The less expensive car carries more passengers.
 - e. State two other facts you can derive from the graphs.
 - f. Which car would you buy? Why?
46. a. Which (if any) of the following ordered pairs (x, y) is a solution to the equation $y = x^2 - 2x + 1$? Show how you came to your conclusion.

$(-2, 7), (1, 0), (2, 1)$

- b. Find one additional ordered pair that is a solution to the equation above. Show how you found your solution.

47. Consider the equation $R = 2 - 5T$.

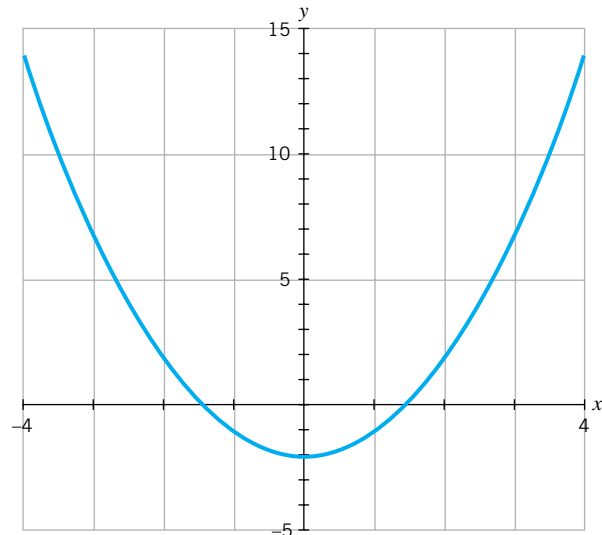
- a. Determine which, if any, of the following points (T, R) satisfy this equation.

$(0, 4), (1, -3), (2, 0)$.

- b. Find two additional ordered pairs that are solutions to the equation.
- c. Make a scatter plot of the solution points found.
- d. What does the scatter plot suggest about where more solutions could be found? Check your predictions.

48. Use the accompanying graph to estimate the missing values for x or y in the table.

| x | y |
|-----|-----|
| -3 | |
| | 2 |
| -1 | -2 |
| | -1 |
| 2 | |
| | 7 |



49. For parts (a)–(d) use the following equation: $y = \frac{x + 1}{x - 1}$.
- Describe in words how to find the value for y given a value for x .
 - Find the ordered pair that represents a solution to the equation when the value of x is 5.
 - Find the ordered pair that represents a solution to the equation when the value of y is 3.
 - Is there an ordered-pair solution to the equation when the value of x is 1? If so, find it; if not, explain why.
50. For parts (a)–(d) use the following equation: $y = \frac{1}{x + 1}$.
- Describe in words how to find the value for y given a value for x .
 - Find the ordered pair that represents a solution to the equation when the value of x is 0.
 - Find the ordered pair that represents a solution to the equation when the value of y is 4.
 - Is there an ordered-pair solution to the equation when the value of x is -1 ? If so, find it; if not, explain why.
51. For parts (a)–(d) use the following equation: $y = -2x^2$.
- If $x = 0$, find the value of y .
 - If x is greater than zero, what can you say about the value of y ?
 - If x is a negative number, what can you say about the value of y ?
 - Can you find an ordered pair that represents a solution to the equation when y is greater than zero? If so, find it; if not, explain why.
52. Find the ordered pairs that represent solutions to each of the following equations when $x = 0$, when $x = 3$, and when $x = -2$.
- | | |
|--------------------|--------------------------|
| a. $y = 2x^2 + 5x$ | c. $y = x^3 + x^2$ |
| b. $y = -x^2 + 1$ | d. $y = 3(x - 2)(x - 1)$ |
53. Given the four ordered pairs $(-1, 3)$, $(1, 0)$, $(2, 3)$, and $(1, 2)$, for each of the following equations, identify which points (if any) are solutions for that equation.
- | | |
|------------------|--------------------------|
| a. $y = 2x + 5$ | c. $y = x^2 - x + 1$ |
| b. $y = x^2 - 1$ | d. $y = \frac{4}{x + 1}$ |

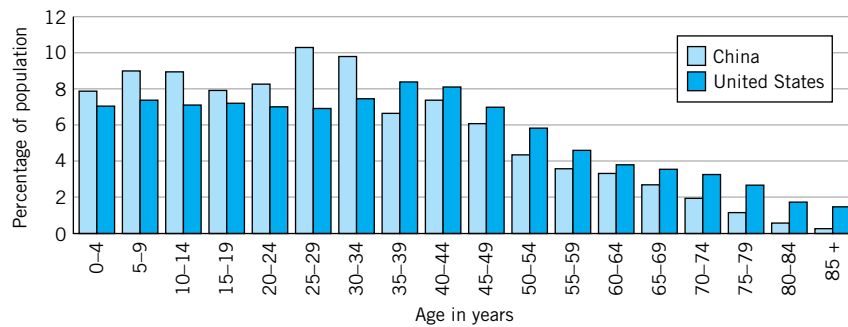
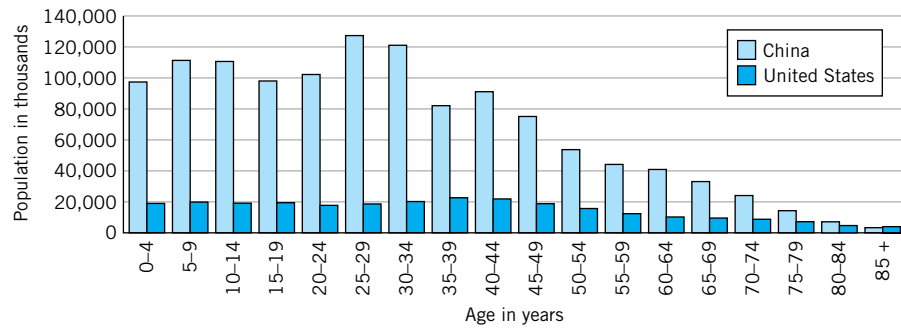


54. The following table and graphs (and related Excel and graph link file USCHINA) contain information about the populations of the United States and China. Write a 60-second summary comparing the two populations.

Age Distribution of the 1998 Populations (in 1000s) of China and the United States

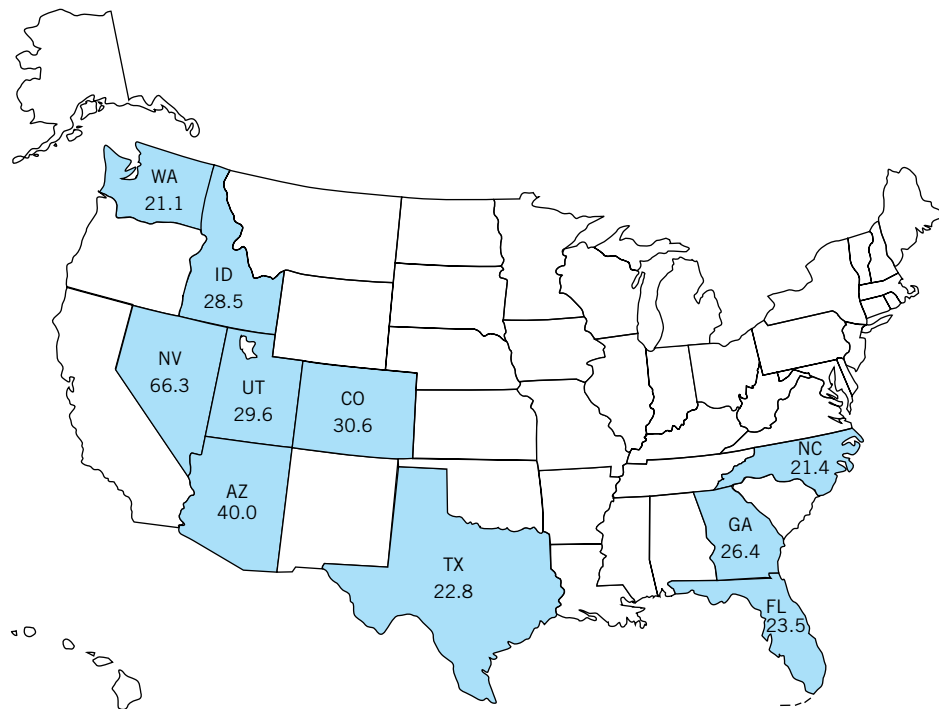
| Age Interval | China | | United States | |
|-------------------|------------------|---------------|----------------|---------------|
| | Total | Percent | Total | Percent |
| 0-4 | 97,396 | 7.87 | 19,020 | 7.04 |
| 5-9 | 111,211 | 8.99 | 19,912 | 7.37 |
| 10-14 | 110,638 | 8.94 | 19,184 | 7.10 |
| 15-19 | 98,012 | 7.92 | 19,473 | 7.20 |
| 20-24 | 102,095 | 8.25 | 17,768 | 6.57 |
| 25-29 | 127,336 | 10.29 | 18,680 | 6.91 |
| 30-34 | 121,072 | 9.79 | 20,209 | 7.48 |
| 35-39 | 82,084 | 6.64 | 22,638 | 8.38 |
| 40-44 | 91,118 | 7.37 | 21,891 | 8.10 |
| 45-49 | 75,137 | 6.07 | 18,855 | 6.98 |
| 50-54 | 53,743 | 4.34 | 15,728 | 5.82 |
| 55-59 | 44,169 | 3.57 | 12,408 | 4.59 |
| 60-64 | 40,913 | 3.31 | 10,256 | 3.79 |
| 65-69 | 33,124 | 2.68 | 9,575 | 3.54 |
| 70-74 | 24,084 | 1.95 | 8,781 | 3.25 |
| 75-79 | 14,322 | 1.16 | 7,195 | 2.66 |
| 80-84 | 7,127 | 0.58 | 4,712 | 1.74 |
| 85 and over | 3,335 | 0.27 | 4,006 | 1.48 |
| Total | 1,236,915 | 100.00 | 270,290 | 100.00 |
| Median age | 30.3 | | 35.5 | |
| Mean age | 27 | | 36.4 | |

Source: U.S. Bureau of the Census, www.census.gov



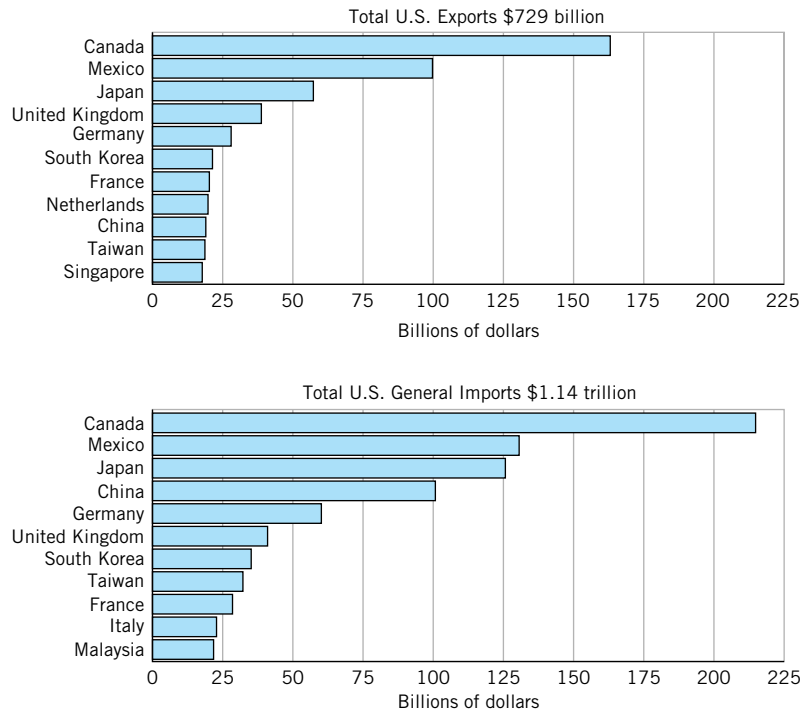


55. Read *The New York Times* op-ed article “A Fragmented War on Cancer” by Hamilton Jordan, who was President Jimmy Carter’s chief of staff. Jordan claims that we are on the verge of a cancer epidemic.
- Use what you have learned in Exercise 54 about the distribution of ages over time in the United States to refute his claim. Are there other arguments that refute his claim?
 - Read *The New York Times* letter to the editor by William M. London, Director of Public Health, American Council on Science and Health. London argues that Hamilton Jordan’s assertions are misleading. What questions are raised by the arguments of William London? What additional data would you need to evaluate his arguments?
 - Write a paragraph refuting Hamilton Jordan’s claim that we are on the verge of a cancer epidemic.
56. Construct a topic sentence that describes some aspect of the accompanying figure.



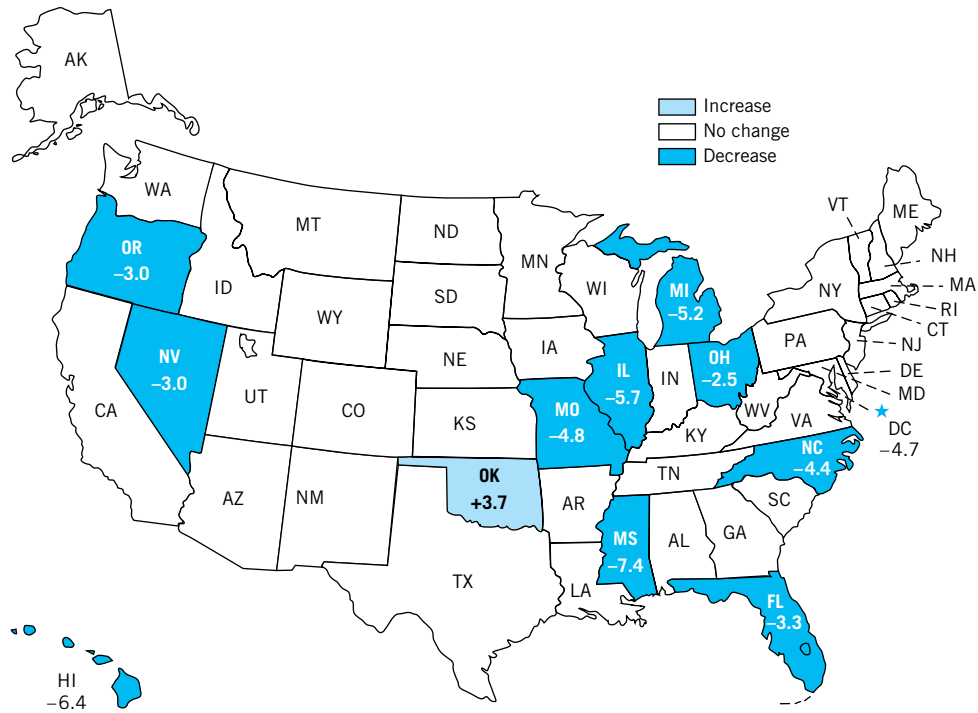
Ten fastest-growing states, percent population change: 1990 to 2002.
 Source: U.S. Bureau of the Census, *Statistical Abstract of the United States 2002*.

57. Construct a topic sentence that describes a key aspect of the accompanying graphs.



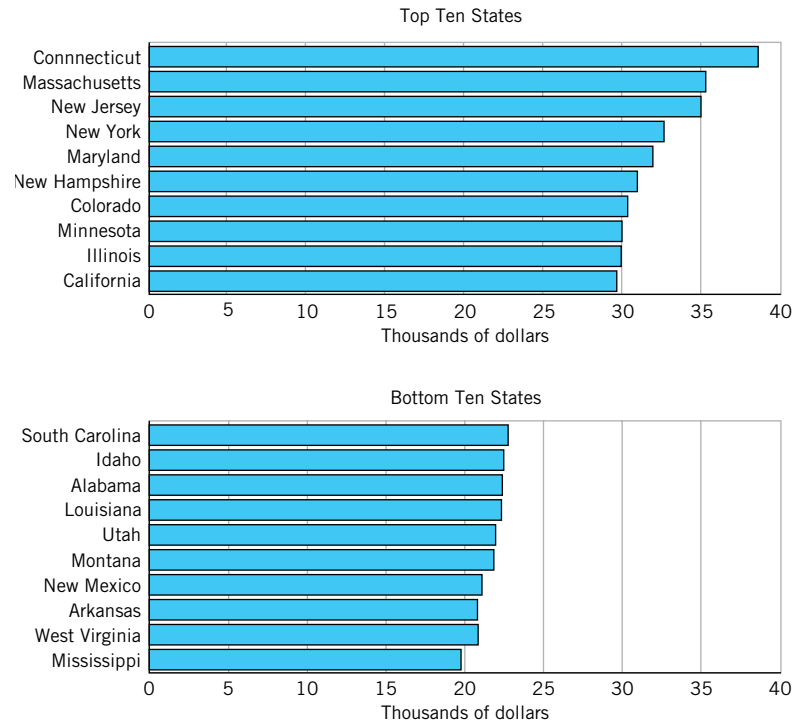
Top purchasers of U.S. exports and suppliers of U.S. general imports: 2001
 Source: U.S. Bureau of the Census, *Statistical Abstract 2002*.

58. Construct a 60-second summary for the accompanying figure showing change in household income.



Percent change in median household income.
 Source: U.S. Bureau of the Census, Current Population Survey, 2001, 2002, and 2003 Annual Social and Economic Supplements.

59. Construct a 60-second summary for the accompanying graphs.



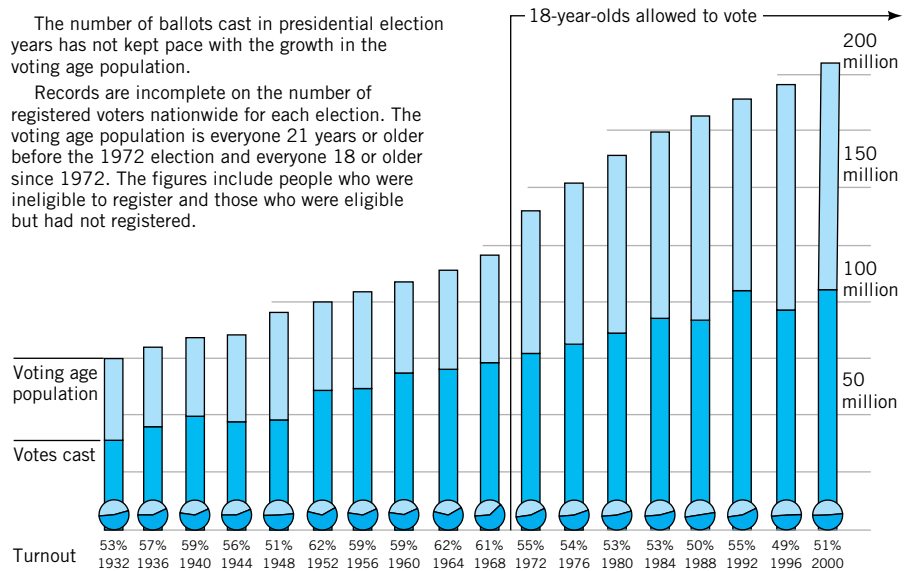
Personal income per capita, by state: 2001 (in constant 1996 dollars).
 Source: U.S. Bureau of the Census, *Statistical Abstract of the United States 2002*.

60. Explain in a 60-second summary why the “Vanishing Voter” project at Harvard University would state that “the number of ballots cast in the presidential election years has not kept pace with the growth in the voting age population.”

US Voting Age Population and Election Turnout

The number of ballots cast in presidential election years has not kept pace with the growth in the voting age population.

Records are incomplete on the number of registered voters nationwide for each election. The voting age population is everyone 21 years or older before the 1972 election and everyone 18 or older since 1972. The figures include people who were ineligible to register and those who were eligible but had not registered.



Source: The Vanishing Voter project at Harvard University; Federal Election Commission.



61. (Computer Required) Make a prediction about the distribution of income for males and females in the United States. Check your predictions using the course software “F1: Histograms” in *FAM 1000 Census Graphs* and/or using data from the U.S. Census Bureau at www.census.gov. Write a 60-second summary describing your results.

Exercises for Section 1.3

62. Which of the following tables describe a function? Explain your answers.

| | | | | | |
|----------------|----|----|---|---|---|
| a. Input value | -2 | -1 | 0 | 1 | 2 |
| Output value | -8 | -1 | 0 | 1 | 8 |

| | | | | | |
|----------------|----|----|---|---|---|
| b. Input value | 0 | 1 | 2 | 1 | 0 |
| Output value | -4 | -2 | 0 | 2 | 4 |

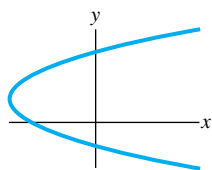
| | | | | | |
|----------------|----|---|---|----|----|
| c. Input value | 10 | 7 | 4 | 7 | 10 |
| Output value | 3 | 6 | 9 | 12 | 15 |

| | | | | | |
|----------------|---|---|---|----|----|
| d. Input value | 0 | 3 | 9 | 12 | 15 |
| Output value | 3 | 3 | 3 | 3 | 3 |

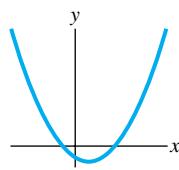
63. Determine whether each set of points represents a function. (*Hint*: It may be helpful to plot the points.)

- $(2, 6), (-4, 6), (1, -3), (4, -3)$
- $(2, -2), (3, -2), (4, -2), (6, -2)$
- $(2, -3), (2, 3), (-2, -3), (-2, 3)$
- $(-1, 2), (-1, 0), (-1, -1), (-1, -2)$

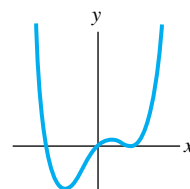
64. Which of the accompanying graphs describe a function? Explain your answer.



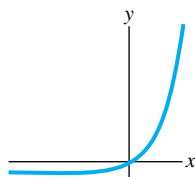
(a)



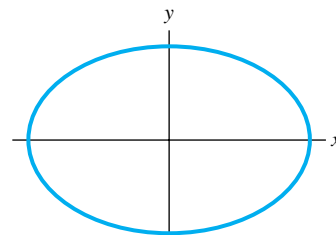
(b)



(c)

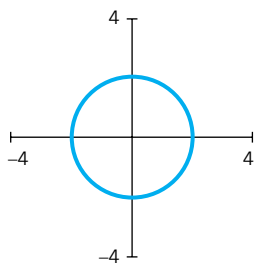


(d)

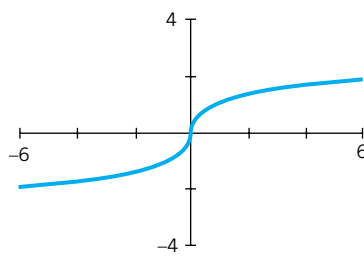


(e)

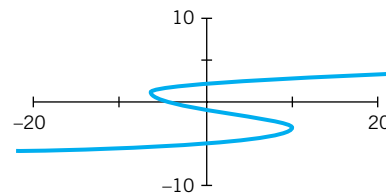
65. Which of the accompanying graphs describe a function? Explain your answer.



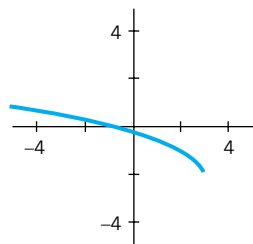
(a)



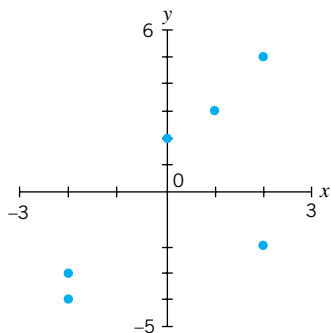
(b)



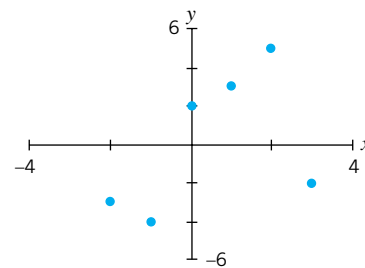
(c)



(d)



(e)



(f)

66. Consider the accompanying table, listing the weights (W) and heights (H) of five individuals. Based on this table, is height a function of weight? Is weight a function of height? Justify your answers.

| Weight W (lb) | Height H (in) |
|--------------------|--------------------|
| 120 | 54 |
| 120 | 55 |
| 125 | 58 |
| 130 | 60 |
| 135 | 56 |

67. The following weather conditions in the accompanying table were recorded in Pittsburgh over an 8-hour period. For this particular time period only, answer the following:

- Is temperature a function of time?
- Is time a function of temperature?
- Is humidity a function of temperature?
- Is wind a function of time?
- Is wind a function of temperature?
- Is dew point a function of temperature?

Wednesday Feb. 11

| | | Temp (°F) | Feels Like | Dew Point | Precip. | Humid. | Wind |
|---------|---------------|--------------|---------------|--------------|---------|--------|-----------------------------------|
| 12 P.M. | Partly cloudy | 27° | 19° | 17° | 0% | 65% | From the west at 7 mph |
| 1 P.M. | Partly cloudy | 28° | 21° | 17° | 0% | 62% | From the west at 7 mph |
| 2 P.M. | Partly cloudy | 29° | 22° | 17° | 0% | 60% | From the west at 7 mph |
| 3 P.M. | Partly cloudy | 30° | 24° | 17° | 0% | 58% | From the west at 6 mph |
| 4 P.M. | Partly cloudy | 30° | 25° | 17° | 0% | 58% | From the west southwest at 5 mph |
| 5 P.M. | Partly cloudy | 30° | 26° | 17° | 0% | 60% | From the west southwest at 4 mph |
| 6 P.M. | Partly cloudy | 29° | 29° | 18° | 0% | 62% | From the southwest at 2 mph |
| 7 P.M. | Partly cloudy | 28° | 28° | 18° | 0% | 66% | From the south southwest at 1 mph |

Source: www.weather.com

- 68.** The priority mail schedule as of June 30, 2002, is shown here for different U.S. zones and weights of parcels. Indicate whether each of the following statements is true or not for packages weighing less than or equal to 12 lbs. Explain your answers.
- Within each zone, the cost of mailing a parcel is a function of its weight.
 - Within each zone, the weight of a parcel is a function of the cost to mail it.

Priority Mail Schedule (Partial List)

| Weight Not Over (lb) | Zone | | | | | |
|-------------------------|-----------------------|--------|--------|--------|--------|--------|
| | Local, 1, 2, and 3 | 4 | 5 | 6 | 7 | 8 |
| 1 | \$3.85 | \$3.85 | \$3.85 | \$3.85 | \$3.85 | \$3.85 |
| 2 | 3.95 | 4.55 | 4.90 | 5.05 | 5.40 | 5.75 |
| 3 | 4.75 | 6.05 | 6.85 | 7.15 | 7.85 | 8.55 |
| 4 | 5.30 | 7.05 | 8.05 | 8.50 | 9.45 | 10.35 |
| 5 | 5.85 | 8.00 | 9.30 | 9.85 | 11.00 | 12.15 |
| 6 | 6.30 | 8.85 | 9.90 | 10.05 | 11.30 | 12.30 |
| 7 | 6.80 | 9.80 | 10.65 | 11.00 | 12.55 | 14.05 |
| 8 | 7.35 | 10.75 | 11.45 | 11.95 | 13.80 | 15.75 |
| 9 | 7.90 | 11.70 | 12.20 | 12.90 | 15.05 | 17.50 |
| 10 | 8.40 | 12.60 | 13.00 | 14.00 | 16.30 | 19.20 |
| 11 | 8.95 | 13.35 | 13.75 | 15.15 | 17.55 | 20.90 |
| 12 | 9.50 | 14.05 | 14.50 | 16.30 | 18.80 | 22.65 |

Source: United States Postal Service.

69. a. Find an equation that represents the relationship between x and y in each of the accompanying tables.

| x | y |
|-----|-----|
| 0 | 5 |
| 1 | 6 |
| 2 | 7 |
| 3 | 8 |
| 4 | 9 |

| x | y |
|-----|-----|
| 0 | 1 |
| 1 | 2 |
| 2 | 5 |
| 3 | 10 |
| 4 | 17 |

| x | y |
|-----|-----|
| 0 | 3 |
| 1 | 3 |
| 2 | 3 |
| 3 | 3 |
| 4 | 3 |

- b. Which of your equations represents y as function of x ? Justify your answers.
70. For each of the following tables find a function formula that takes the x values and produces the given y values.

a.

| x | y |
|-----|-----|
| 0 | 0 |
| 1 | 3 |
| 2 | 6 |
| 3 | 9 |
| 4 | 12 |

b.

| x | y |
|-----|-----|
| 0 | -2 |
| 1 | 1 |
| 2 | 4 |
| 3 | 7 |
| 4 | 10 |

c.

| x | y |
|-----|-----|
| 0 | 0 |
| 1 | -1 |
| 2 | -4 |
| 3 | -9 |
| 4 | -16 |

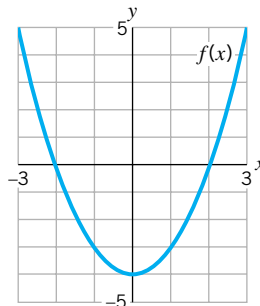
71. The basement of a large department store features discounted merchandise. Their policy is to reduce the previous month's price of the item by 10% each month for 5 months.
- Let S_1 be the sale price for the first month and P the original price. Express S_1 as a function of P . What is the price of a \$100 garment on sale for the first month?
 - Let S_2 be the sale price for the second month and P the original price. Express S_2 as a function of P . What is the price of a \$100 garment on sale for the second month?
 - Let S_3 be the sale price for the third month and P the original price. Express S_3 as a function of P . What is the price of a \$100 garment on sale for the third month?
 - Let S_5 be the sale price for the fifth month and P the original price. Express S_5 as a function of P . What is the final price of a \$100 garment on sale for the fifth month? By what total percentage has the garment now been reduced from its original price?
72. Write a formula to express each of the following sentences:
- The sale price is 20% off the original price. Assuming both cities are on a 12-hour clock, use S for sale price and P for original price to express S as a function of P .
 - The time in Paris is 6 hours ahead of New York. Assuming both cities are on a 12-hour clock, use P for Paris time and N for New York time to express P as a function of N . How would you adjust your formula if P comes out greater than 12?
 - For temperatures above 0°F the wind chill effect can be estimated by subtracting two-thirds of the wind speed (in miles per hour) from the outdoor temperature. Use C for the effective wind chill temperature, W for wind speed, and T for the actual outdoor temperature to write an equation expressing C in terms of W and T .
73. Determine whether y is a function of x in each of the following equations. If the equation does not define a function, find a value of x that is associated with two different y values.
- $y = x^2 + 1$
 - $y = 3x - 2$
 - $y = 5$
 - $y^2 = x$

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74. (Technology Required) For each equation below, write an equivalent equation that expresses z in terms of t . Is z a function of t ? Why or why not? Use technology to sketch the graph.
- a. $3t - 5z = 10$ b. $12t^2 - 4z = 0$ c. $2(t - 4) - (z + 1) = 0$
75. Assume that for persons who earn less than \$20,000 a year, income tax is 16% of their income.
- Generate a formula that describes income tax in terms of income for people earning less than \$20,000 a year.
 - What are you treating as the independent variable? The dependent variable?
 - Does your formula represent a function? Explain.
 - If it is a function, what is the domain? The range?
76. Suppose that the price of gasoline is \$2.09 per gallon.
- Generate a formula that describes the cost, c , of buying gas as a function of the number of gallons of gasoline, G , purchased.
 - What is the independent variable? The dependent variable?
 - Does your formula represent a function? Explain.
 - If it is a function, what is the domain? The range?
 - Generate a small table of values and a graph.
77. The cost of driving a car to work is estimated to be \$2.00 in tolls plus 32 cents per mile. Write an equation for computing the total cost C of driving M miles to work. Does your equation represent a function? What is the independent variable? What is the dependent variable? Generate a table of values and then graph the equation.
78. (Technology Required) For each equation, write the equivalent equation that expresses y in terms of x . Use technology to sketch each function and then estimate its domain.
- $3x + 5x - y = 3y$
 - $3x(5 - x) = x - y$
 - $x(x - 1) + y = 2x - 5$
 - $2(y - 1) = y + 5x(x + 1)$
79. If $f(x) = x^2 - x + 2$, find:
- $f(2)$
 - $f(-1)$
 - $f(0)$
 - $f(-5)$
80. If $g(x) = 2x + 3$, evaluate $g(0)$, $g(1)$, and $g(-1)$.
81. Look at the accompanying table.
- Find $p(-4)$, $p(5)$, and $p(1)$.
 - For what value(s) of n does $p(n) = 2$?

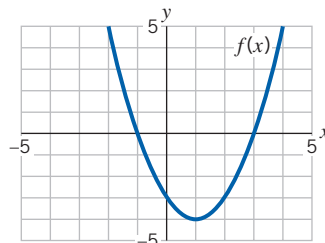
| n | $p(n)$ |
|-----|--------|
| -4 | 0.063 |
| -3 | 0.125 |
| -2 | 0.25 |
| -1 | 0.5 |
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |
| 4 | 16 |
| 5 | 32 |

82. Consider the function $y = f(x)$ graphed in the accompanying figure.



- a. Find $f(-3)$, $f(0)$, $f(1)$, and $f(2.5)$.
- b. Find two values of x such that $f(x) = 0$.

83. From the accompanying graph of $y = f(x)$:



- a. Find $f(-2)$, $f(-1)$, $f(0)$, and $f(1)$.
- b. Find two values of x for which $f(x) = -3$.
- c. Estimate the range of f . Assume that the arms of the graph extend upward indefinitely.

84. Find $f(3)$, if it exists, for each of the following functions:

a. $f(x) = (x - 3)^2$ b. $f(x) = \frac{1}{x}$ c. $f(x) = \frac{x + 1}{x - 3}$ d. $f(x) = \frac{2x}{x - 1}$

Determine the domain for each function.

85. If $f(x) = (2x - 1)^2$, evaluate $f(0)$, $f(1)$, and $f(-2)$.

86. Find the domain for each of the following functions:

a. $f(x) = 300.4 + 3.2x$ c. $j(x) = \frac{1}{x + 1}$ e. $f(x) = x^2 + 3$
 b. $g(x) = \frac{5 - 2x}{2}$ d. $k(x) = 3$

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87. Given $f(x) = 1 - 0.5x$ and $g(x) = x^2 + 1$, evaluate:

- a. $f(0), g(0)$ b. $f(-2), g(-3)$ c. $f(2), f(1)$ d. $f(3), g(3)$

88. Each of the following functions has a restricted domain. Find the domain for each function and explain why the restriction occurs.

a. $f(x) = \frac{3}{x+2}$

b. $g(x) = \sqrt{x-5}$

c. $h(x) = -\frac{1}{2x-3}$

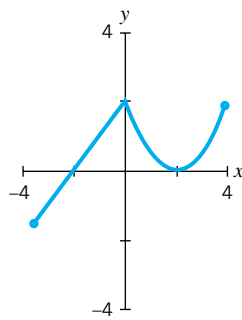
d. $k(x) = \frac{1}{x^2-4}$

e. $l(x) = \sqrt{x+3}$

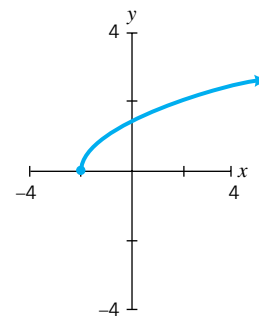
89. Using the functions for $f(x)$ and $k(x)$ in Exercise 88 find:

- a. $f(1) + k(0)$ b. $f(1)x - k(0)$ c. $f(1) \cdot k(0)$ d. $k(0)/f(1)$

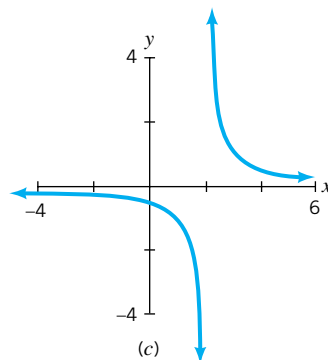
90. Which of the following graphs represent a function? For each function, estimate the domain and range from its graph. (Note: Arrows indicate that the graph continues indefinitely in the same direction.)



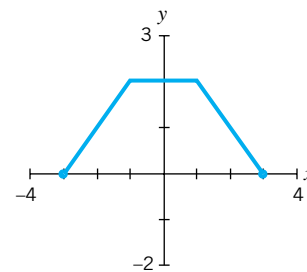
(a)



(b)

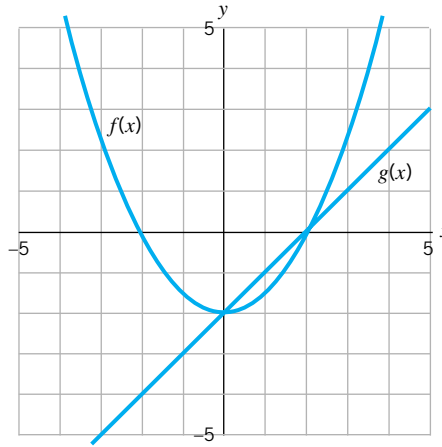


(c)



(d)

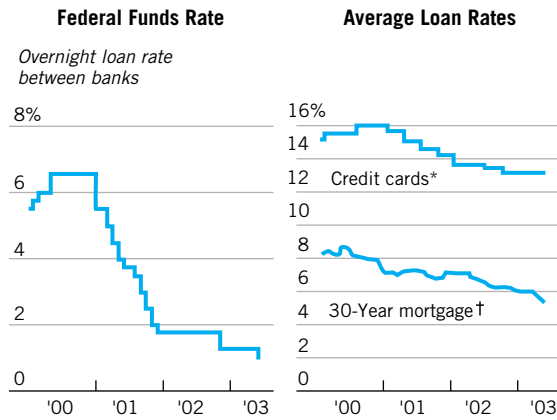
91. For the functions $f(x)$ and $g(x)$ shown on the accompanying graph, find the values of x that make the following true.



- a. $f(x) = 0$ b. $g(x) = 0$ c. $f(x) = g(x)$

Exercises for Section 1.4

92. The Federal Reserve is the central bank of the United States that sets monetary policy. The Federal Reserve oversees money supply, interest rates, and credit with the goal of keeping the U.S. economy and currency stable. The federal funds rate is the interest rate that banks with excess reserves at a Federal Reserve district bank charge other banks that need overnight loans. Look at the accompanying graphs for the time period between 2000 and 2003.



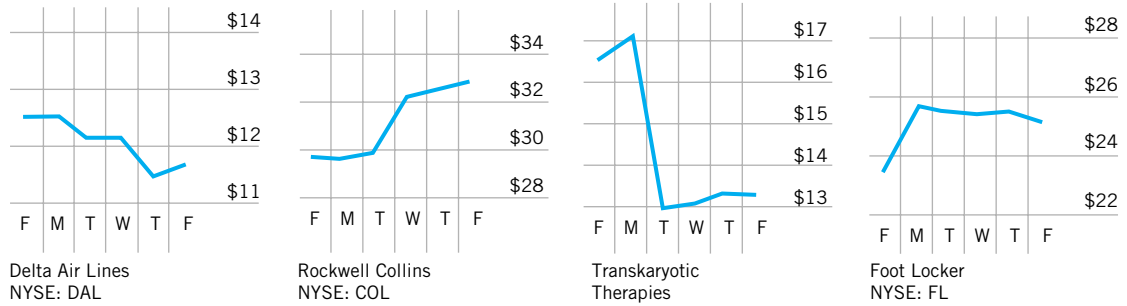
* Average annual percentage rate for all credit card accounts in banks in the United States that respond to the Federal Reserve's Survey.
 † National average for a 30-year fixed rate on a single-family home.

Source: www.federalreserve.gov

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- a. Describe the overall trends for this time period for the federal funds rate.
- b. Describe the overall trends for this time period for credit card rates.
- c. Describe the overall trends for this time period for the 30-year mortgage rates.
- d. Estimate the maximum federal funds rate for this time period. When did it occur?
- e. Approximate the minimum federal funds rate for this time period. When did it occur?
- f. Write a topic sentence that compares the federal funds rate with the consumer loan rates for credit cards and mortgages for this time period.

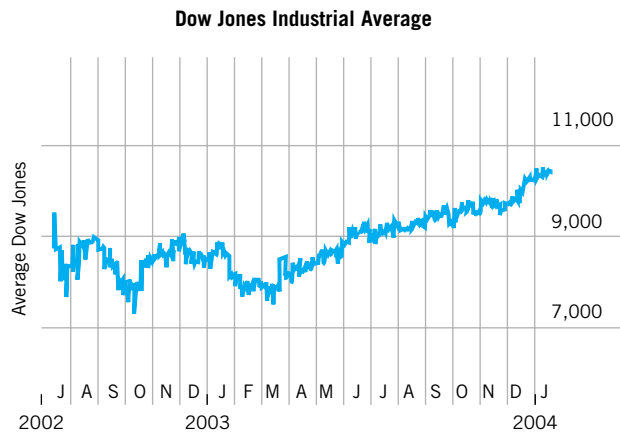
93. The accompanying graphs show the price of shares of stock of four companies over the one-week period January 9 to 16, 2004.



Source: *The New York Times*, Jan. 17, 2004.

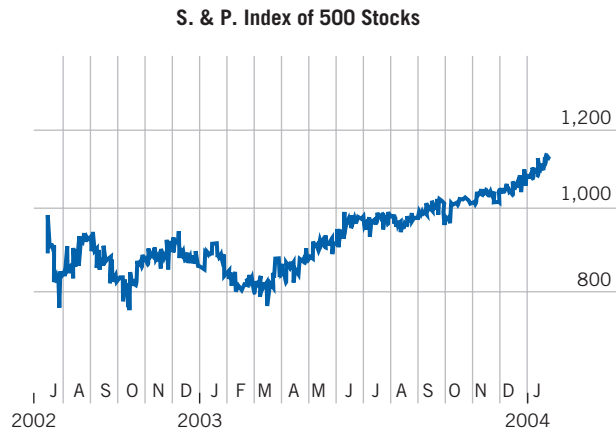
- a. Which stock increased, then had a drastic decrease at the beginning of the week, then increased only slightly?
- b. Which stock had an overall increasing trend for the week?
- c. Which stock had a decreasing trend for most of the week, then a slight increase?
- d. Which stock increased then remained fairly constant?

94. The Dow Jones Industrial Average is an index of the New York Stock Exchange (NYSE). It is derived from the values for 30 hand-picked stocks traded on the NYSE. The accompanying graph shows the Dow Jones Industrial Average from January 2002 to January 2004.

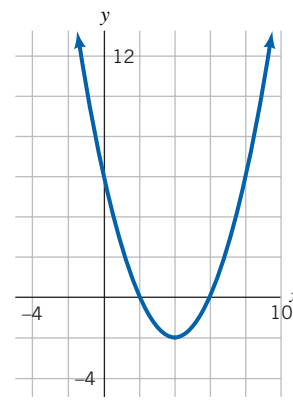
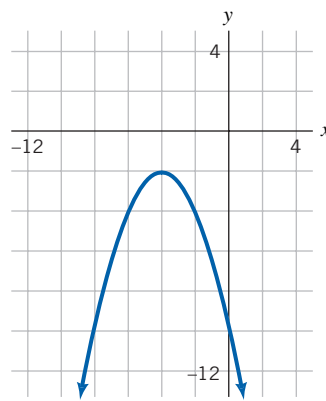


Source: www.dowjones.com

- a. Over which time period did the Dow Jones show a long, steady increase?
 - b. Over which time period(s) was the shape of the Dow Jones graph roughly concave down?
95. The Standard and Poor's (S&P) 500 is a stock market index constructed from the stock values of 400 industrial, 20 transportation, 40 utility, and 40 financial companies. The accompanying graph shows the S&P index of 500 stocks from January 2002 to January 2004.



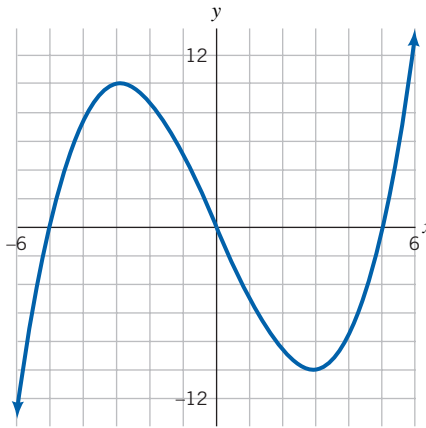
- a. Over which time period(s) was the shape of the S&P 500 graph roughly concave down?
 - b. From March 2003 to January 2004, how would you describe the trend of the S&P 500?
 - c. How does the S&P 500 relate to the trend in the Dow Jones for the time period January 2002 to January 2004? (See Exercise 94.)
96. For each of the functions,



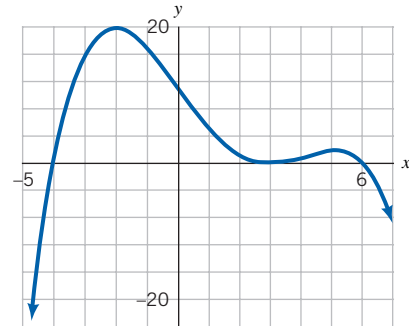
Source: www.standardandpoors.com

- a. Over which interval is the function decreasing?
- b. Over which interval is the function increasing?
- c. Does the function appear to have a minimum? If so, where?
- d. Does the function appear to have a maximum? If so, where?
- e. Describe the concavity.

97. For each of the functions,



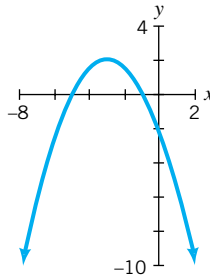
(i)



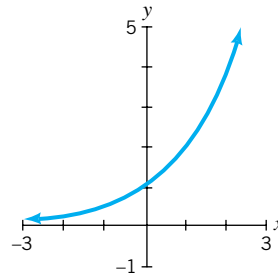
(ii)

- a. Over which interval(s) is the function positive?
- b. Over which interval(s) is the function negative?
- c. Over which interval(s) is the function decreasing?
- d. Over which interval(s) is the function increasing?
- e. Does the function appear to have a minimum? If so, where?
- f. Does the function appear to have a maximum? If so, where?

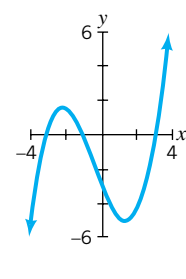
98. Choose which graph(s) match the description: As x increases, the graph is:



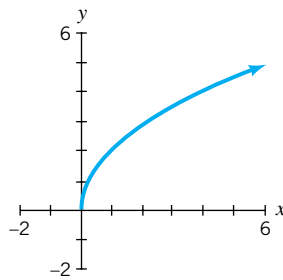
(i)



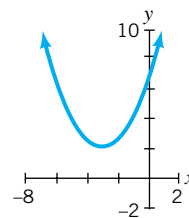
(iii)



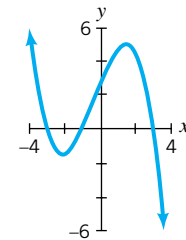
(v)



(ii)

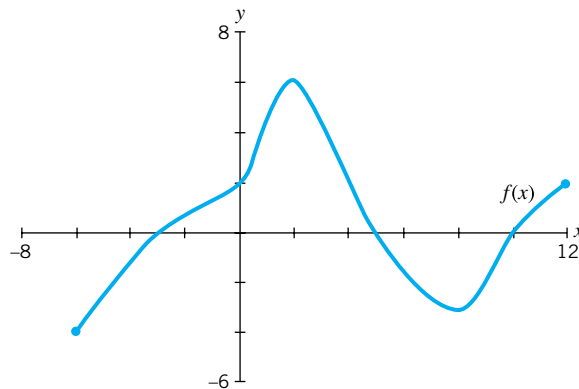


(iv)

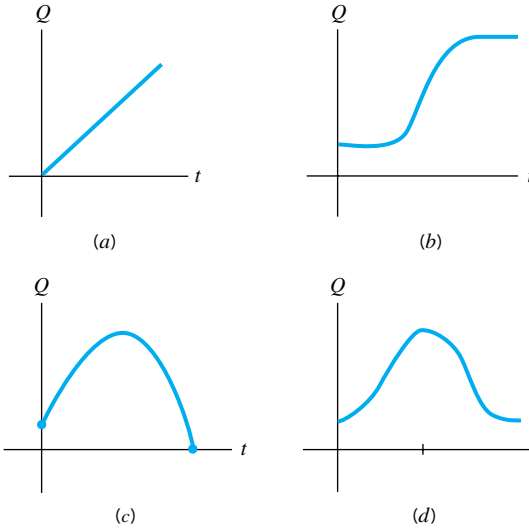


(vi)

- a. Increasing and concave up
 - b. Increasing and concave down
 - c. Concave up and appears to have a minimum value at $(-3, 2)$
 - d. Concave down and appears to have a maximum value at $(-3, 2)$
 - e. First concave down and then concave up
 - f. First concave up and then concave down
99. Examine each of the graphs in Exercise 98. Assume each graph describes a function $f(x)$. The arrows indicate that the graph extends indefinitely in the direction shown.
- a. For each function estimate the domain and range.
 - b. For each function estimate the x interval(s) where $f(x) > 0$.
 - c. For each function estimate the x interval(s) where $f(x) < 0$.
100. Look at the graph of $y = f(x)$ in the accompanying figure.



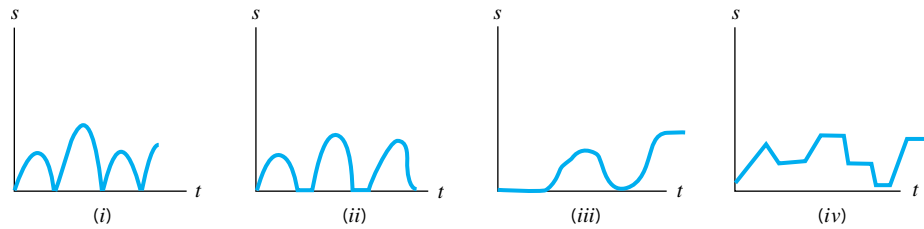
- a. Find $f(-6)$, $f(2)$, and $f(12)$.
 - b. Find $f(0)$. What would you call this point?
 - c. For what value(s) of x is $f(x) = 0$? What would you call these point(s)?
 - d. Is $f(8) > 0$ or is $f(8) < 0$?
 - e. How many times would the line $y = 1$ intersect the graph of $f(x)$?
 - f. What are the domain and range of $f(x)$?
 - g. What is the maximum? The minimum?
101. Use the graph of Exercise 100 to answer the following questions about $f(x)$.
- a. Over which interval is $f(x) < 0$?
 - b. Over which interval is $f(x) > 0$?
 - c. Over which interval is $f(x)$ increasing?
 - d. Over which interval is $f(x)$ decreasing?
 - e. How would you describe the concavity of $f(x)$ over the interval $[0, 5]$ for x ? Over $[5, 10]$ for x ?
 - f. Find a value for x when $f(x) = 4$.
 - g. $f(-8) = ?$
102. Match each graph with the best description of the function. Assume that the horizontal axis represents time, t .



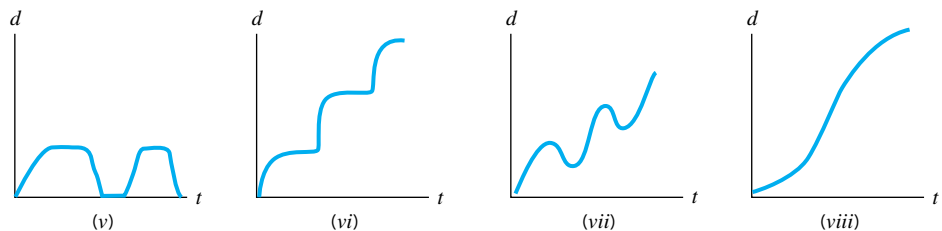
- i. The height of a ball thrown straight up is a function of time.
- ii. The distance a truck travels at a constant speed is a function of time.
- iii. The number of daylight hours is a function of the day of the year.
- iv. The temperature of a pie baking in an oven is a function of time.

103. This exercise is to be done with a partner. In parts (a) and (b) choose the “best” graph to describe the situation, and then explain to your partner why the other graphs are not good choices.

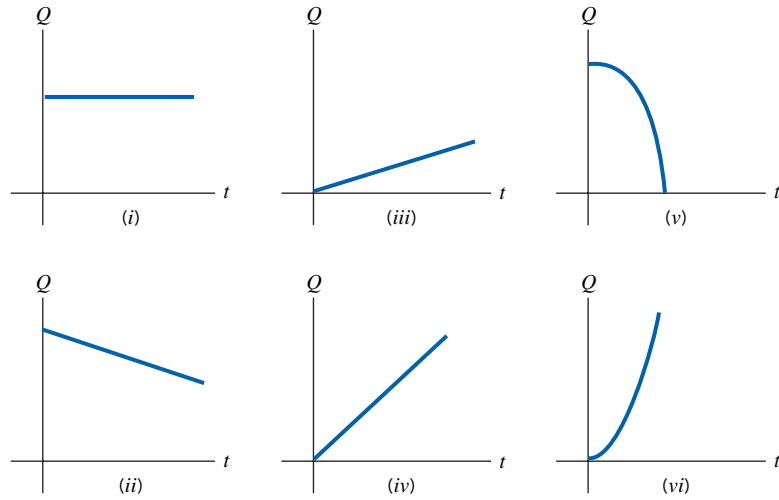
- a. A student in a large urban area takes a local bus whose route ends at the college. Time, t , is on the horizontal axis and speed, s , is on the vertical axis.



- b. These graphs depict the distance the student in the bus traveled. Time, t , is on the horizontal axis and distance, d , is on the vertical axis.

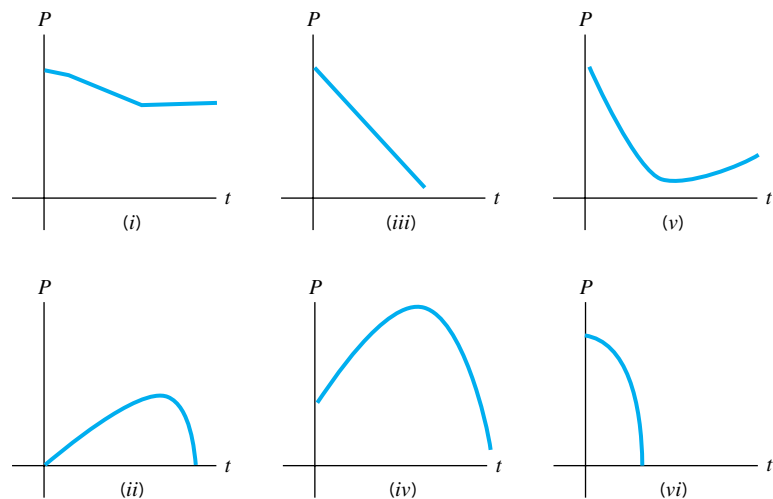


104. The phrases below describe how a quantity Q changes over time t . Match the phrase with the most suitable graph. A phrase may describe more than one graph.



- a. Increases, beginning from an initial value of zero
- b. Decreases from an initial value, which is not zero
- c. Continues unchanged from an initial value, which is not zero
- d. Increases gradually from an initial value of zero
- e. Increases rapidly, beginning with an initial value of zero
- f. Increases at an increasing rate
- g. Decreases gradually from an initial value, which is not zero
- h. Decreases rapidly from an initial value, which is not zero

105. The following phrases describe how a variable P changes over time t . Match the phrase with the most suitable graph. A phrase may describe more than one graph.



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- a. Decreases from an initial value, which is not zero
- b. Increases slowly then decreases precipitously
- c. Decreases from an initial value and then remains constant
- d. Decreases from an initial value and then increases gradually
- e. Increases dramatically to a maximum value, then decreases rapidly
- f. Is always concave down

106. (Technology Required) Use technology to graph each function. Then approximate the x intervals where the function is concave up, and then where it is concave down

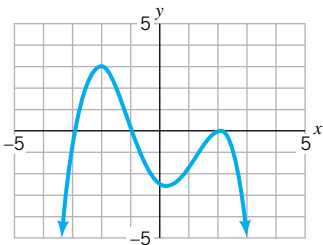
a. $f(x) = x^3$ b. $g(x) = x^3 - 4x$

107. (Technology Required) Use technology to graph each function. Then approximate the x intervals where the function is concave up, and then where it is concave down.

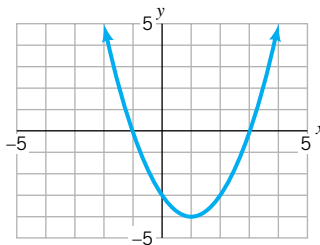
a. $h(x) = x^4$
 b. $k(x) = x^4 - 24x + 50$ (*Hint:* Use an interval of $[-10, 10]$ for x and $[-100, 100]$ for y .)

108. a. In the accompanying graphs, estimate the coordinates where of the maximum and minimum points (if any) of the function.

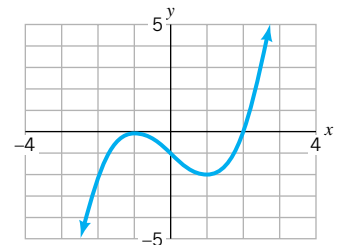
b. Specify the interval(s) over which each function is increasing.



(a)

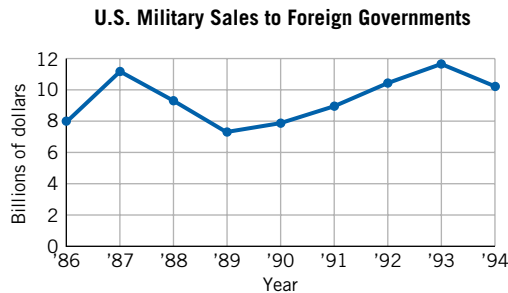


(b)



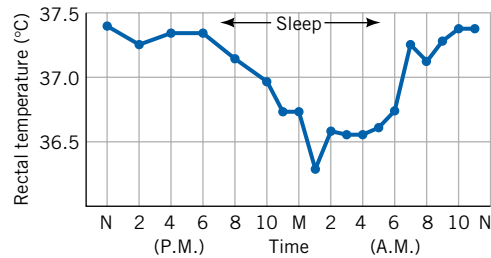
(c)

109. Consider the accompanying graph of U.S. military sales to foreign governments during the 80's and 90's.



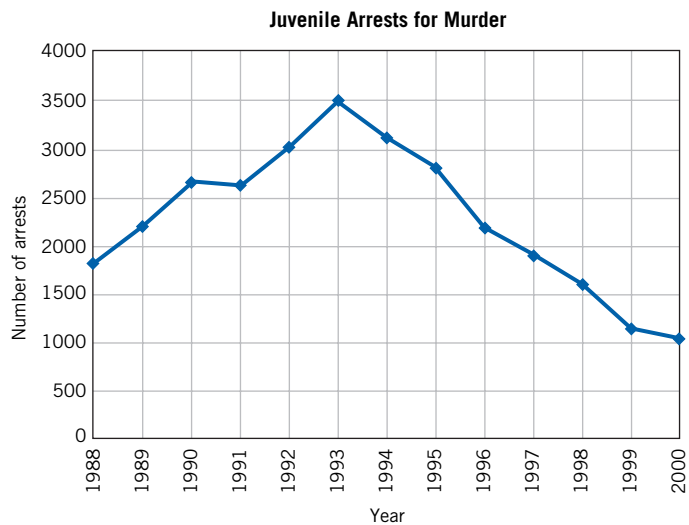
Source: U.S. Bureau of the Census, *Statistical Abstract of the United States 1996*.

- a. Between what years did sales increase?
 - b. Between what years did sales decrease?
 - c. Estimate the maximum value for sales.
 - d. Estimate the minimum value for sales.
110. Sketch a plausible graph for each of the following and label the axes.
- a. The amount of snow in your backyard each day from December 1 to March 1.
 - b. The temperature during a 24-hour period in your home town during one day in July.
 - c. The amount of water inside your fishing boat if your boat leaks a little and your fishing partner bails out water every once in a while.
 - d. The total hours of daylight each day of the year.
 - e. The temperature of an ice-cold drink left to stand.
111. The accompanying graph shows the 24-hour temperature cycle of a normal man. The man was confined to bed to minimize temperature fluctuations caused by activity.
- a. Estimate the man's maximum temperature.
 - b. Estimate his minimum temperature.
 - c. Give a short general description of the 24-hour temperature cycle.



Source: V. B. Mountcastle, *Medical Physiology*, vol. 2, 14th ed. (St. Louis: Mosby-Year Book).

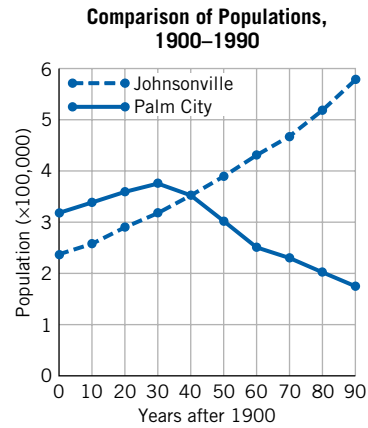
112. Consider the accompanying chart of juvenile arrests for murder.



Source: U.S. Bureau of the Census, *Statistical Abstract of the United States 2002*.

- a. Over what years did the number of arrests show a decrease?
- b. Approximate the number of juvenile arrests for murder in 1988 and 1993.
- c. Estimate the ratio of the number of arrests in 1993 to the number of arrests in 1988.
- d. What can be said generally about the number of juveniles arrests for murder between 1988 and year 2000 based on this chart?

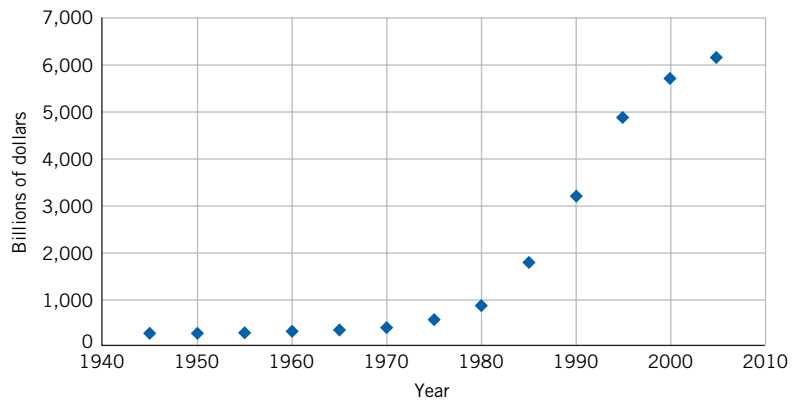
113. Examine the accompanying graph, which shows the populations of two towns.



- a. What is the range of population size for Johnsonville? For Palm City?
- b. During what years did the population of Palm City increase?
- c. During what years did the population of Palm City increase?
- d. When were the populations equal?



114. In Section 1.2 we examined the annual federal budget *surplus* or *deficit*. The federal *debt* takes into account the cumulative effect of all the deficits and surpluses for each year together with any interest or payback of principal. The graph shows the accumulated gross federal debt from 1945 to 2005 (est.). (See related Excel or graph link file FEDDEBT.)



U.S. accumulated gross federal debt (in billions).

- a. Identify the intervals on which the function is increasing or decreasing.
- b. Identify any maximum or minimum values of the function.
- c. Create a topic sentence for this graph for a news paper article.

115. (Technology Recommended) The accompanying table shows the progress of national regulations in controlling carbon monoxide emissions.

**National Ambient Air Pollutant Concentrations,
1985–1996**

| Year | Carbon Monoxide* (ppm) |
|------|------------------------|
| 1985 | 6.97 |
| 1987 | 6.69 |
| 1988 | 6.38 |
| 1989 | 6.34 |
| 1990 | 5.87 |
| 1991 | 5.55 |
| 1992 | 5.18 |
| 1993 | 4.88 |
| 1994 | 5.10 |
| 1995 | 4.50 |
| 1996 | 4.20 |
| 2001 | 3.21 |

*Air quality standard is 0 parts per million (ppm).

Source: U.S. Bureau of the Census, *Statistical Abstract of the United States 1998*, and Environmental Protection Agency.

- Identify which sets of data would be logical choices for the independent variable and dependent variable.
- Verify whether or not your choice for dependent variable is a function of the independent variable.

If one variable is a function of the other, then:

- Express in words how the dependent variable relates to the independent variable.
- Generate a graph.
- Identify the intervals on which the function is increasing or decreasing.
- Identify any maximum or minimum values of the function.

116. The formula $A = 25W$

where A is ampicillin dosage in milligrams and W is weight in kilograms, represents the *minimum* effective pediatric daily drug dosage of ampicillin as a function of a child's weight. (You may recall from Algebra Aerobics 1.3a, Problem 7, that $D = 50W$ represents the *maximum* recommended dosage. Both formulas apply only to children weighing up to 10 kilograms.)

- Identify which variables would be logical choices for the independent and dependent variables.
- Verify whether or not your choice for dependent variable is a function of the independent variable.

If one variable is a function of the other, then:

- Express in words how the dependent variable relates to the independent variable.
- Generate a table of values and a graph.
- Identify the intervals on which the function is increasing or decreasing.
- Identify any maximum or minimum values of the function.

117. Consider the accompanying table.

| Y | P |
|------|--------|
| 1990 | \$1.4 |
| 1991 | \$2.3 |
| 1992 | \$0 |
| 1993 | −\$0.5 |
| 1994 | \$1.4 |
| 1995 | \$1.2 |

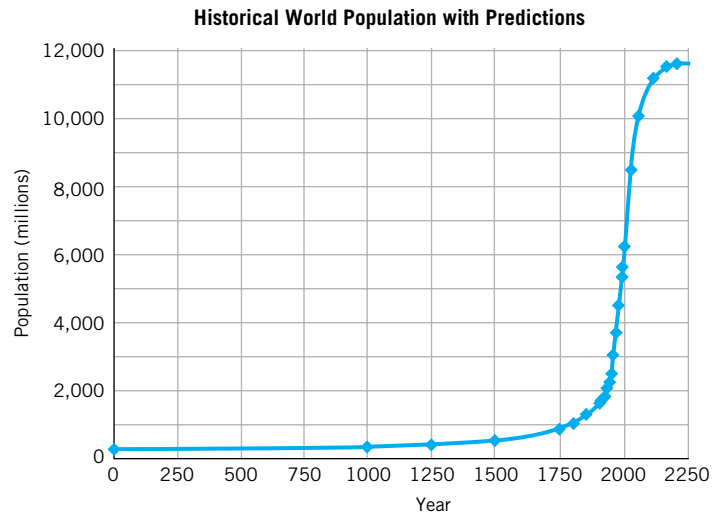
- Is P a function of Y ?
 - What is the domain? What is the range?
 - What is the maximum value of P ? In what year did this occur?
 - During what intervals was P increasing? Decreasing?
 - Now consider P as the independent variable and Y as the dependent variable. Is Y a function of P ?
118. (Technology Required) Using technology, graph each function over the intervals $[-20, 20]$ for x and $[-6, 6]$ for y .

$$y_1 = x^2 - 3x + 2 \qquad y_2 = 0.5x^3 - 2x - 1$$

For each function,

- Determine the maximum value of y on each interval.
 - Determine the minimum value of y on each interval.
119. The accompanying table gives data on the historical population of the world, with future predictions, and the related figure gives the graph of the data. Assume population is a function of year.

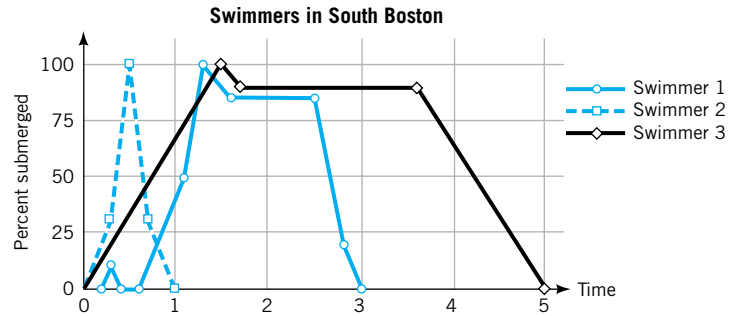
| Year | Population (millions) | Year | Population (millions) |
|------|-----------------------|------|-----------------------|
| 0 | 300 | 1950 | 2,520 |
| 1000 | 310 | 1960 | 3,020 |
| 1250 | 400 | 1970 | 3,700 |
| 1500 | 500 | 1980 | 4,450 |
| 1750 | 790 | 1990 | 5,300 |
| 1800 | 980 | 1994 | 5,630 |
| 1850 | 1,260 | 2000 | 6,230 |
| 1900 | 1,650 | 2025 | 8,470 |
| 1910 | 1,750 | 2050 | 10,020 |
| 1920 | 1,860 | 2100 | 11,190 |
| 1930 | 2,070 | 2150 | 11,540 |
| 1940 | 2,300 | 2200 | 11,600 |



Source: www.worldonline.nl/invd/world/whist2.html

- a. What is the domain of this function?
 - b. Over what interval can the population be said to be relatively constant?
 - c. Over what interval would say the world population had dramatic growth?
 - d. Does the dramatic growth slow down?
 - e. Write a topic sentence for a report for the United Nations.
120. Make a graph showing what you expect would be the relative ups and downs throughout the year of sales (in dollars) of:
 - a. Turkey
 - b. Candy
 - c. Bathing suits in your state
 - d. Textbooks at your school bookstore
 121. A student breaks her ankle and is taken to a doctor, who puts a cast on her leg and tells her to keep the foot off the ground altogether. After two weeks she is given crutches and can begin to walk around more freely, but then she falls and is resigned to keeping stationary again for a while. After 6 weeks she is given a walking cast in which she can begin to put her foot on the ground again. She is now able to limp around using crutches. Her walking speed slowly progresses. At 12 weeks she hits a plateau and, seeing no increase in her mobility, starts physical therapy. She rapidly improves. At 16 weeks the cast is removed and she can walk freely. Make a graph of the student's mobility level during her recovery.
 122. Every January 1 a hardy group called the L Street Brownies celebrates the New Year by going for a swim at the L Street Beach in South Boston. The water is always very cold, and swimmers adopt a variety of strategies for getting into it. The graph shows the progress of three different friends who join in the event, with percentage of body submerged on the vertical axis and time

on the horizontal axis. Match the graphs to the descriptions below of how each of the friends manages to get completely submerged in the icy ocean.



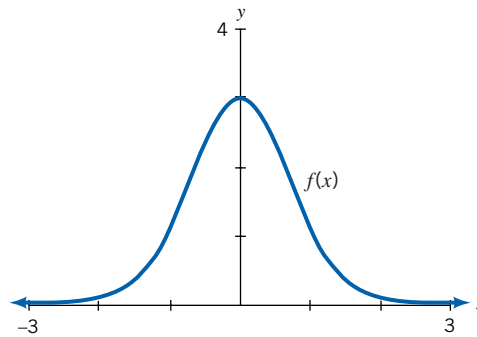
- a. Ali has done this before and confidently walks in until his head is underwater; then he puts his head out and swims around a few minutes; then he walks out.
- b. Ben dashes until the water is up to his knee, trips over a hidden rock, and falls in completely; he stands up and, since he is now totally wet, runs back out of the water.
- c. Cat puts one foot in, takes it out again, and shivers. She makes up her mind to get it over with, runs until she is up to her waist, dives in, swims back as close to the water's edge as she can get, stands up, and steps out of the water.

123. (Technology Required) Use technology to graph the following functions and then complete each sentence

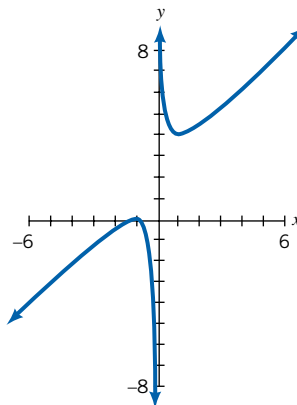
- a. As x approaches positive infinity, y approaches _____.
- b. As x approaches negative infinity, y approaches _____.

$$y_1 = x^3 \quad y_2 = x^2 \quad y_3 = \frac{1}{x + 3} \quad y_4 = \frac{1}{x} + 2$$

124. Describe the behavior of $f(x)$ in the accompanying figure over the interval $(-\infty, +\infty)$ for x , using such words as “increases,” “decreases,” “concavity,” “maximum/minimum,” and “approaches infinity.”



125. Describe the behavior of $g(x)$ in the accompanying figure over the interval $(-\infty, +\infty)$ for x , using such words as “increases,” “decreases,” “concavity,” “maximum/minimum,” and “approaches infinity.”



126. (Technology Required) This exercise is to be done with a partner. Name the partners person #1 and person #2.
- Person #1, using technology, graphs the function $f(x) = 0.5(x - 3)(x + 2)^2$, but does not show the graph to person #2.
 - Person #1 describes to person #2 the behavior of the graph of $f(x)$ so that he/she can sketch it on a piece of paper.
 - Switch roles; now person #2, using technology, graphs $g(x) = -0.5(x - 3)(x + 2)^2$, but does not show the graph to person #1.
 - Person #2 describes to person #1 the behavior of the graph of $g(x)$ so that he/she can sketch it on a piece of paper.
 - Compare the accuracy of the graphs and compare the shapes of the two graphs. What do $f(x)$ and $g(x)$ have in common? How do they differ?

EXPLORATION 1.1

Collecting, Representing, and Analyzing Data

Objectives

- explore issues related to collecting data.
- learn techniques for organizing and graphing data using a computer (with a spreadsheet program) or a graphing calculator.
- describe and analyze the overall shape of single-variable data using frequency and relative frequency histograms.
- use the mean and median to represent single-variable data.

Material/Equipment

- class questionnaire
- measuring tapes in centimeters and inches
- optional measuring devices: eye chart, flexibility tester, measuring device for blood pressure
- computer with spreadsheet program or graphing calculator with statistical plotting features
- data from class questionnaire or other small data set formatted either as spreadsheet or graph link file
- overhead projector and projection panel for computer or graphing calculator
- transparencies for printing or drawing graphs for overhead projector (optional)

Related Readings



(On the web at www.wiley.com/college/kimeclark)
 “U.S. Government Definitions of Census Terms”
 “Health Measurements”

Related Software



“F1: Histograms,” in *FAM 1000 Census Graphs*

Procedure

This exploration may take two class periods.

Day One

In a Small Group or with a Partner

1. Pick (or your instructor will assign you) one of the undefined variables on the questionnaire. Spend about 15 minutes coming up with a workable definition and a strategy for measuring that variable. Be sure there is a way in which a number or single letter can be used to record each individual's response on the questionnaire.
2. Consult the reading “Health Measurements” if you decide to collect health data.

Class Discussion

After all of the definitions are recorded on the board, discuss your definition with the class. Is it clear? Does everyone in the class fall into one of the categories of your definition? Can anyone

think of someone who might not fit into any of the categories? Modify the definition until all can agree on some wording. As a class, decide on the final version of the questionnaire and record it in your class notebook.

In a Small Group or with a Partner

Help each other when necessary to take measurements and fill out the entire questionnaire. Questionnaires remain anonymous, and you can leave blank any question you can't or don't want to answer. Hand in your questionnaire to your instructor by the end of class.

Exploration-Linked Homework

Read "U.S. Government Definitions of Census Terms" for a glimpse into the federal government's definitions of the variables you defined in class. How do the "class" definitions and the "official" ones differ?

Day Two

Class Demonstration

1. If you haven't used a spreadsheet or graphing calculator before, you'll need a basic technical introduction. (*Note:* If you are using a TI-82, TI-83, or TI-84 graphing calculator, there are basic instructions in the Graphing Calculator Manual on www.wiley.com.college/kimeclark.) Then you'll need an electronic version of the data set from which you will choose one variable for the whole class to study (e.g., age from the class data).
 - a. If you're using a spreadsheet:
 - Copy the column with the data onto a new spreadsheet and graph the data. What does this graph tell you about the data?
 - Sort the data and plot it again. Is this graph any better at conveying information about the data?
 - b. If you're using a graphing calculator:
 - Discuss window sizes, changing interval sizes and statistical plot procedures.
2. Select an interval size and then construct a frequency histogram and a relative frequency histogram. If possible, label one of these carefully and print it out. If you have access to a laser printer, you can print onto an overhead transparency.
3. Calculate the mean and median using spreadsheet or graphing calculator functions.

In a Small Group or with a Partner

Choose another variable from your data. Pick an interval size, and then generate both a frequency histogram and a relative frequency histogram. If possible, make copies of the histograms for both your partner and yourself. Calculate the mean and median.

Discussion/Analysis

With your partner(s), analyze and jot down patterns that emerge from the data. What are some limitations of the data? What other questions are raised and how might they be resolved? In your notebook, record jottings for a 60-second summary that would describe your results.

Exploration-Linked Homework

Prepare a verbal 60-second summary to give to the class. If possible, use an overhead projector with a transparency of your histogram or a projector linked to your graphing calculator. If not, bring in a paper copy of your histogram. Construct a written 60-second summary. (See Section 1.2 for some writing suggestions.)

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ALGEBRA CLASS QUESTIONNAIRE**(You may leave any category blank)**

1. Age (in years)
2. Sex (female = 1, male = 2)
3. Your height (in inches)
4. Distance from your naval to the floor (in centimeters)
5. Estimate your average travel time to school (in minutes)
6. What is your most frequent mode of transportation to school?
(F = by foot, C = car, P = public transportation, B = bike, O = other)

The following variables will be defined in class. We will discuss ways of coding possible responses and then use the results to record our personal data.

7. The number of people in your household
8. Your employment status
9. Your ethnic classification
10. Your attitude toward mathematics

Health Data

11. Your pulse rate before jumping (beats per minute)
12. Your pulse rate after jumping for 1 minute (beats per minute)
13. Blood pressure: systolic (mm Hg)
14. Blood pressure: diastolic (mm Hg)
15. Flexibility (in inches)
16. Vision, left eye
17. Vision, right eye

Other Data

EXPLORATION 1.2

Picturing Functions

Objective

- develop an intuitive understanding of functions.

Material/Equipment

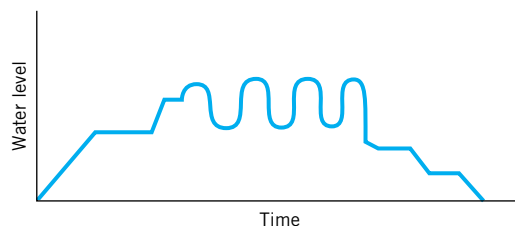
None required

Procedure

Part I

Class Discussion

Bridget, the 6-year-old daughter of a professor at the University of Pittsburgh, loves playing with her rubber duckie in the bath at night. Her mother drew the accompanying graph for her math class. It shows the water level (measured directly over the drain) in Bridget's tub as a function of time.



Pick out the time period during which:

- The tub is being filled
- Bridget is entering the tub
- She is playing with her rubber duckie
- She leaves the tub
- The tub is being drained

With a Partner

Create your own graph of a function that tells a story. Be as inventive as possible. Some students have drawn functions that showed the decibel levels during a phone conversation of a boyfriend and girlfriend, number of hours spent doing homework during one week, and amount of money in one's pocket during the week.

Class Discussion

Draw your graph on the blackboard and tell its story to the class.

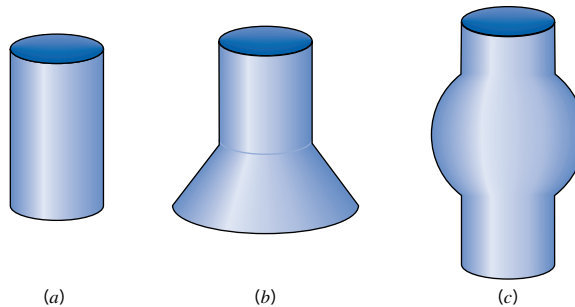
Part II*With a Partner*

Generate a plausible graph for each of the following:

1. Time spent driving to work as a function of the amount of snow on the road. (*Note:* The first inch or so may not make any difference; the domain may be only up to about a foot of snow since after that you may not be able to get to work.)
2. The hours of nighttime as a function of the time of year.
3. The temperature of ice cream taken out of the freezer and left to stand.
4. The distance that a cannonball (or javelin or baseball) travels as a function of the angle of elevation at which it is launched. The maximum distance is attained for angles of around 45° from the horizontal.
5. Assume that you leave your home walking at a normal pace, realize you have forgotten your homework and run back home, and then run even faster to school. You sit for a while in a classroom and then walk leisurely home. Now plot your distance from home as a function of time.

Bonus Question

Assume that water is pouring at a constant rate into each of the containers shown. The height of water in the container is a function of the volume of liquid. Sketch a graph of this function for each container.

**Discussion/Analysis**

Are your graphs similar to those generated by the rest of the class? Can you agree as a class as to the basic shape of each of the graphs? Are there instances in which the graphs could look quite different?

EXPLORATION 1.3

Deducing Formulas to Describe Data

Objective

- find and describe patterns in data.
- deduce functional formulas from data tables.
- extend patterns using functional formulas.

Material/Equipment

None required

Procedure

Class Discussion

1. Examine data tables (a) and (b). In each table, look for a pattern in terms of how y changes when x changes. Explain in your own words how to find y in terms of x .

a.

| x | y |
|-----|-----|
| 0 | 0.0 |
| 1 | 0.5 |
| 2 | 1.0 |
| 3 | 1.5 |
| 4 | 2.0 |

b.

| x | y |
|-----|-----|
| 0 | 5 |
| 1 | 8 |
| 2 | 11 |
| 3 | 14 |
| 4 | 17 |

2. Assuming that the pattern continues indefinitely, use the rule you have found to extend the data table to include negative numbers for x .
3. Check your extended data tables. Did you find only one value for y given a particular value for x ?
4. Use a formula to describe the pattern that you have found. Do you think this formula describes a function? Explain.

On Your Own

1. For each of the data tables (c) to (h), explain in your own words how to find y in terms of x . Then extend each table using the rules you have found.

c.

| x | y |
|-----|-----|
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |
| 4 | 16 |

d.

| x | y |
|-----|-----|
| 0 | 0 |
| 1 | 1 |
| 2 | 8 |
| 3 | 27 |
| 4 | 64 |

e.

| x | y |
|-----|-----|
| 0 | 0 |
| 1 | 2 |
| 2 | 12 |
| 3 | 36 |
| 4 | 80 |

[Hint: For table (e) think about some combination of data in tables (c) and (d).]

