CHAPTER 2

# RATES OF CHANGE AND LINEAR FUNCTIONS

## OVERVIEW

How does the U.S. population change over time? How do children's heights change as they age? Average rates of change provide a tool for measuring how change in one variable affects a second variable. When average rates of change are constant, the relationship is linear.

**After reading this chapter you should be able to**

- calculate and interpret average rates of change
- understand how representations of data can be biased
- recognize that a constant rate of change denotes a linear relationship
- construct a linear equation given a table, graph, or description
- derive by hand a linear model for a set of data

## 2.1 *Average Rates of Change*

In Chapter 1 we looked at how change in one variable could affect change in a second variable. In this section we'll examine how to measure that change.

### Describing Change in the U.S. Population over Time

We can think of the U.S. population as a function of time. Table 2.1 and Figure 2.1 are two representations of that function. They show the changes in the size of the U.S. population since 1790, the year the U.S. government conducted its first decennial census. Time, as usual, is the independent variable and population size is the dependent variable.

#### *Change in population*

Figure 2.1 clearly shows that the size of the U.S. population has been growing over the last two centuries, and growing at what looks like an increasingly rapid rate. How can the change in population over time be described quantitatively? One way is to pick two points on the graph of the data and calculate how much the population has changed during the time period between them.

Suppose we look at the change in the population between 1900 and 1990. In 1900 the population was 76.2 million; by 1990 the population had grown to 248.7 million. How much did the population increase?

$$\begin{aligned}\text{change in population} &= (248.7 - 76.2) \text{ million people}\\ &= 172.5 \text{ million people}\end{aligned}$$

This difference is portrayed graphically in Figure 2.2.

**Population of the United States: 1790–2000**

| Year | Population in Millions |
|---|---|
| 1790 | 3.9 |
| 1800 | 5.3 |
| 1810 | 7.2 |
| 1820 | 9.6 |
| 1830 | 12.9 |
| 1840 | 17.1 |
| 1850 | 23.2 |
| 1860 | 31.4 |
| 1870 | 39.8 |
| 1880 | 50.2 |
| 1890 | 63.0 |
| 1900 | 76.2 |
| 1910 | 92.2 |
| 1920 | 106.0 |
| 1930 | 123.2 |
| 1940 | 132.2 |
| 1950 | 151.3 |
| 1960 | 179.3 |
| 1970 | 203.3 |
| 1980 | 226.5 |
| 1990 | 248.7 |
| 2000 | 281.4 |

***Table 2.1***

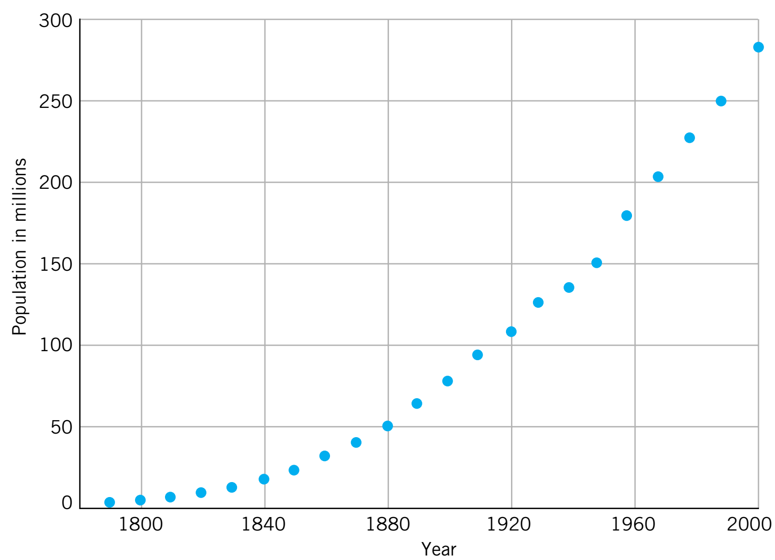



***Figure 2.1*** Population of the United States.
*Source:* U.S. Bureau of the Census, *Statistical Abstract of the United States: 2002.*

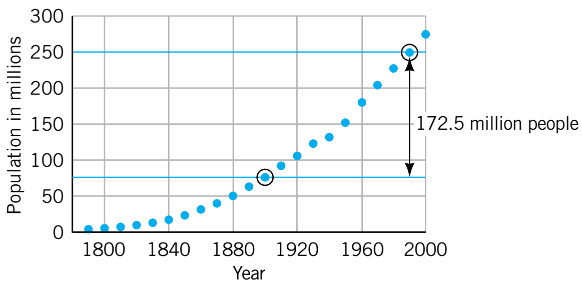



**Figure 2.2** Population change: 172.5 million people.

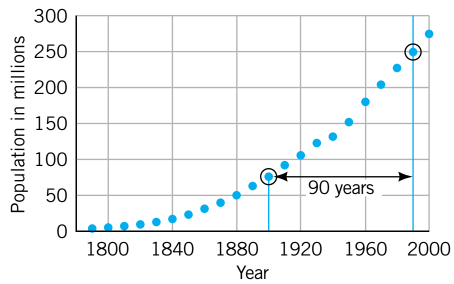



**Figure 2.3** Time change: 90 years.

### *Change in time*

Knowing that the population increased by 172.5 million tells us nothing about how rapid the change was; this change clearly represents much more dramatic growth if it happened over 20 years than if it happened over 200 years. In this case, the length of time over which the change in population occurred is

$$\begin{aligned} \text{change in years} &= (1990 - 1900) \text{ years} \\ &= 90 \text{ years} \end{aligned}$$

This interval is indicated in Figure 2.3.

### *Average rate of change*

To find the *average rate of change* in population per year from 1900 to 1990, divide the change in the population by the change in years:

$$\begin{aligned} \text{average rate of change} &= \frac{\text{change in population}}{\text{change in years}} \\ &= \frac{172.5 \text{ million people}}{90 \text{ years}} \\ &\approx 1.92 \text{ million people/year} \end{aligned}$$

In the phrase "million people/year" the slash represents division and is read as "per." So our calculation shows that "on average," the population grew at a rate of 1.92 million people per year from 1900 to 1990. Figure 2.4 depicts the relationship between time and population increase.

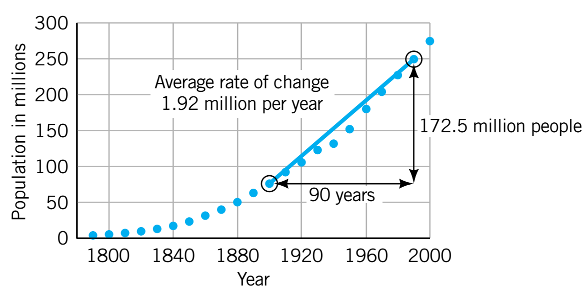



**Figure 2.4** Average rate of change: 1900–1990.

## Defining the Average Rate of Change

The notion of average rate of change can be used to describe the change in any variable with respect to another. If you have a graph that represents a plot of data points of the form $(x, y)$, then the average rate of change between any two points is the change in the $y$ value divided by the change in the $x$ value.

$$\text{The } \textit{average rate of change} \text{ of } y \text{ with respect to } x = \frac{\text{change in } y}{\text{change in } x}$$

If the variables represent real-world quantities that have units of measure (e.g., millions of people or years), then the average rate of change should be represented in terms of the appropriate units:

$$\text{units of the average rate of change} = \frac{\text{units of } y}{\text{units of } x}$$

For example, the units might be dollars/year (read as "dollars per year") or pounds/person (read as "pounds per person").

**EXAMPLE 1**

Between 1850 and 1950 the median age in the United States rose from 18.9 to 30.2, but by 1970 it had dropped to 28.0.

**a.** Calculate the average rate of change in the median age between 1850 and 1950.

**b.** Compare your answer in part (a) to the average rate of change between 1950 and 1970.

**Solution**

**a.** Between 1850 and 1950,

$$\begin{aligned}\text{average rate of change} &= \frac{\text{change in median age}}{\text{change in years}}\\ &= \frac{(30.2 - 18.9)\text{ years}}{(1950 - 1850)\text{ years}} = \frac{11.3\text{ years}}{100\text{ years}}\\ &= 0.113\text{ year/year}\end{aligned}$$

The units are a little confusing. But the results mean that between 1850 and 1950 the median age increased on overage by 0.113 each calendar year.

**b.** Between 1950 and 1970,

$$\begin{aligned}\text{average rate of change} &= \frac{\text{change in median age}}{\text{change in years}}\\ &= \frac{(28.0 - 30.2)\text{ years}}{(1970 - 1950)\text{ years}} = \frac{-2.2\text{ years}}{20\text{ years}}\\ &= -0.110\text{ year/year}\end{aligned}$$

Note that since the median age dropped in value between 1950 and 1970, the average rate is negative. The median age decreased by 0.110 year/year between 1950 and 1970, whereas the median age increased by 0.113 year/year beween 1850 and 1950.

## Limitations of the Average Rate of Change

*The average rate of change is an average*. Average rates of change have the same limitations as any average. Although the average rate of change of the U.S. population from 1900 to 1990 was 1.92 million people/year, it is highly unlikely that in each year the population grew by exactly 1.92 million. The number 1.92 million people/year is, as the name states, an average. Similarly, if the arithmetic average, or *mean*, height of students in your class is 67 inches, you wouldn't expect every student to be 67 inches tall. In fact, it may be the case that not even one student is 67 inches tall.

*The average rate of change depends on the end points*. If the data points do not all lie on a straight line, the average rate of change varies for different intervals. For instance, the average rate of change in population for the time interval 1840 to 1940 is 1.15 million people/year and from 1880 to 1980 is 1.77 million people/year. (See Table 2.2. *Note:* Here we abbreviate "million people" as "million.") You can see on the graphs that the line segment is much steeper from 1880 to 1980 than from 1840 to 1940 (Figures 2.5 and 2.6). Different intervals give different impressions of the rate of change in the U.S. population, so it is important to state which end points are used.

The average rate of change does not reflect all the fluctuations in population size that may occur between the end points. For more specific information, the average rate of change can be calculated for smaller intervals.

| Time Interval | Change in Time | Change in Population | Average Rate of Change |
|---|---|---|---|
| 1840–1940 | 100 yr | $132.2 - 17.1 = 115.1$ million | $\dfrac{115.1 \text{ million}}{100 \text{ yr}} \approx 1.15$ million/yr |
| 1880–1980 | 100 yr | $226.5 - 50.2 = 176.3$ million | $\dfrac{176.3 \text{ million}}{100 \text{ yr}} \approx 1.76$ million/yr |

**Table 2.2**

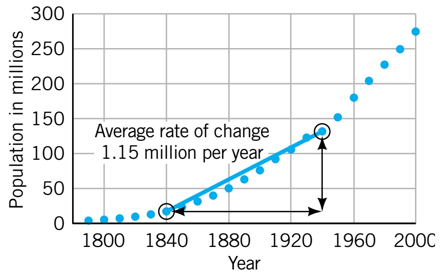



**Figure 2.5** Average rate of change: 1840–1940.

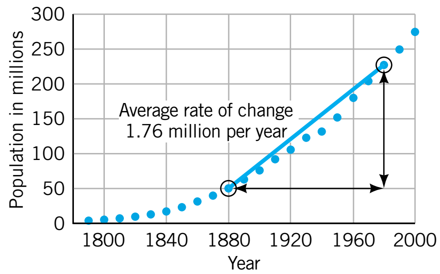



**Figure 2.6** Average rate of change: 1880–1980.

## Algebra Aerobics 2.1

1. Suppose your weight five years ago was 135 pounds and your weight today is 143 pounds. Find the average rate of change in your weight with respect to time.
2. Table 2.3 shows data on U.S. international trade as reported by the U.S. Bureau of the Census.
   - **a.** What is the average rate of change between 1990 and 2002 for:
     - **i.** Exports?
     - **ii.** Imports?
     - **iii.** The trade balance, the difference between what we sell abroad (exports) and buy from abroad (imports)?
   - **b.** What do these numbers tell us?

| Year | U.S. Exports (billions of $) | U.S. Imports (billions of $) | U.S. Trade Balance = Exports − Imports (billions of $) |
|---|---|---|---|
| 1990 | 537.2 | 618.4 | −81.2 |
| 2002 | 974.1 | 1392.1 | −418.0 |

**Table 2.3**

3. Table 2.4 indicates the number of deaths in motor vehicle accidents in the United States as listed by the U.S. Bureau of the Census.

Find the average rate of change:

**a.** From 1970 to 1980

**b.** From 1980 to 2000

Be sure to include units.

**Annual Deaths in Motor Vehicle Accidents (thousands)**

| 1970 | 1980 | 1990 | 2000 |
|---|---|---|---|
| 52.6 | 52.1 | 44.6 | 41.8 |

**Table 2.4**

4. A car is advertised to go from 0 to 60 mph in 5 seconds. Find the average rate of change (i.e., the average acceleration) over that time.
5. According to the U.S. Bureau of the Census workers' compensation payments in Florida rose from $362 million in 1980 to $1.976 billion in 1990. Find the average annual rate of change in payments.
6. A football player runs for 1056 yards in 2000 and for 978 yards in 2004. Find the average rate of change in his performance.
7. The African elephant is an endangered species, largely because poachers (people who illegally hunt elephants) kill elephants to sell the ivory from their tusks. In the African country of Kenya in the last 10 years, the elephant population has dropped from 150,000 to 30,000. Calculate the average rate of change and describe what it means.

## 2.2 *Change in the Average Rate of Change*

We can obtain an even better sense of patterns in the U.S. population if we look at how the average rate of change varies over time. One way to do this is to pick a fixed interval period for time and then calculate the average rate of change for each successive time period. Since we have the U.S. population data in 10-year intervals, we can calculate the average rate of change for each successive decade. The third column in Table 2.5 shows the results of these calculations. Each entry represents the average population growth *per year* (the average annual rate of change) during the previous decade. A few of these calculations are worked out in the last column of the table.

### *What is happening to the average rate of change over time?*

Start at the top of the third column and scan down the numbers. Notice that until 1910 the average rate of change increases every year. Not only is the population growing every decade until 1910, but it is growing at an increasing rate. It's like a car that is not only moving forward but also accelerating. A feature that was not so obvious in the original data is now evident: In the intervals 1910 to 1920, 1930 to 1940, and 1960 to

**SOMETHING TO THINK ABOUT**

*What might be some reasons for the slowdown in population growth from the 1960s through the 1980s?*

1990 we see an increasing population but a decreasing rate of growth. It's like a car decelerating—it is still moving forward but it is slowing down.

**Average Annual Rates of Change of U.S. Population: 1790-2000**

| Year | Population (millions) | Average Annual Rate for Prior Decade (millions/yr) | Sample Calculations |
|---|---|---|---|
| 1790 | 3.9 | Data not available | |
| 1800 | 5.3 | 0.14 | $0.14 = (5.3 - 3.9)/(1800 - 1790)$ |
| 1810 | 7.2 | 0.19 | |
| 1820 | 9.6 | 0.24 | |
| 1830 | 12.9 | 0.33 | |
| 1840 | 17.1 | 0.42 | $0.42 = (17.1 - 12.9)/(1840 - 1830)$ |
| 1850 | 23.2 | 0.61 | |
| 1860 | 31.4 | 0.82 | |
| 1870 | 39.8 | 0.84 | |
| 1880 | 50.2 | 1.04 | |
| 1890 | 63.0 | 1.28 | |
| 1900 | 76.2 | 1.32 | |
| 1910 | 92.2 | 1.60 | |
| 1920 | 106.0 | 1.38 | |
| 1930 | 123.2 | 1.72 | |
| 1940 | 132.2 | 0.90 | $0.90 = (132.2 - 123.2)/(1940 - 1930)$ |
| 1950 | 151.3 | 1.91 | |
| 1960 | 179.3 | 2.80 | |
| 1970 | 203.3 | 2.40 | |
| 1980 | 226.5 | 2.32 | |
| 1990 | 248.7 | 2.22 | |
| 2000 | 281.4 | 3.27 | |

***Table 2.5***

*Source:* U.S. Bureau of the Census, *Statistical Abstract of the United States: 2002.*

The graph in Figure 2.7, with years on the horizontal axis and average rates of change on the vertical axis, shows more clearly how the average rate of change fluctuates over time. The first point, corresponding to the year 1800, shows an average rate of change of 0.14 million people/year for the decade 1790 to 1800. The rate 1.72, corresponding to the year 1930, means that from 1920 to 1930 the population was increasing at a rate of 1.72 million people/year.

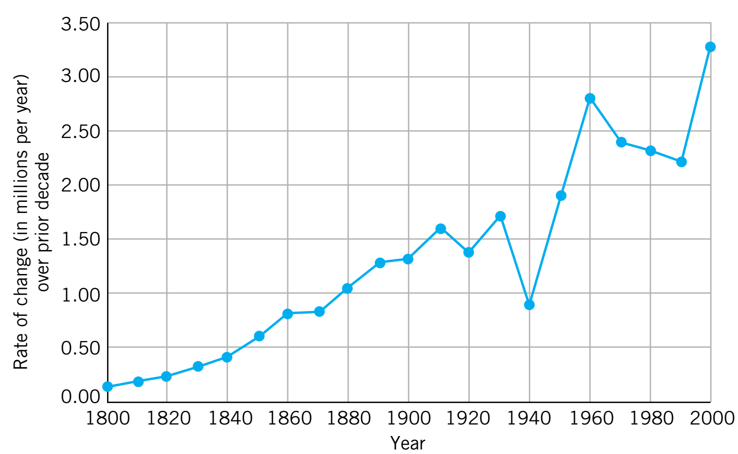



***Figure 2.7*** Average rates of change in the U.S. population by decade.

Kime2004_Ch02_p99_198.qxd 09/20/2004 10:17 PM Page 106



### *What does this tell us about the U.S. population?*

The pattern of growth was fairly steady up until about 1910. Why did it change? A possible explanation for the slowdown in the decade prior to 1920 might be World War I and the 1918 flu epidemic, which by 1920 had killed nearly 20,000,000 people, including about 500,000 Americans.

In Figure 2.7, the steepest decline in the average rate of change is between 1930 and 1940. One obvious suspect for the big slowdown in population growth in the 1930s is the Great Depression. Look back at Figure 2.1, the original graph that shows the overall growth in the U.S. population. The decrease in the average rate of change in the 1930s is large enough to show up in our original graph as a visible slowdown in population growth.

The average rate of change increases again between 1940 and 1960, then drops off from the 1960s through the 1980s. The rate increases once more in the 1990s. This latest surge in the growth rate is attributed partially to the "baby boom echo" (the result of baby boomers having children) and to a rise in birth rates and immigration.

## Algebra Aerobics 2.2

**1.** Table 2.6 and Figure 2.8 show estimates for world population between 1800 and 2050.

**a.** Fill in the third column of the table by calculating the average annual rate of change.

**World Population**

| Year | Total Population (millions) | Average Rate of Change |
|---|---|---|
| 1800 | 980 | n.a. |
| 1850 | 1260 | |
| 1900 | 1650 | |
| 1950 | 2520 | |
| 2000 | 6080 | |
| 2050 | 9104 (est.) | |

**Table 2.6**
*Source:* United National Population Division of the United Nations, *www.popin.org*

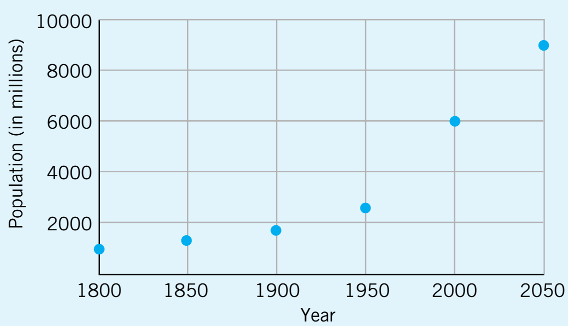



**Figure 2.8** World population.

**b.** Graph the average annual rate of change versus time.

**c.** During what period was the average annual rate of change the largest?

**d.** Describe in general terms what happened to the world population and its average rate of change between 1800 and 2050.

**2.** A graph illustrating a corporation's profits indicates a positive average rate of change between 1999 and 2000, another positive rate of change between 2000 and 2001, no rate of change between 2001 and 2002, and a negative rate of change between 2002 and 2003. Describe the graph and the company's financial situation over the years 1999–2003.

**3.** The following tables show educational data collected on 12- to 24-year-olds by the National Center for Educational Statistics. Table 2.7 shows the number of students who graduated from high school or completed a GED (a high school equivalency exam) during the preceding 12 months. Table 2.8 shows the number of students who enrolled in college the following October.

**High School Completers**

| Year | Number (thousands) | Average Rate of Change (thousands per year) |
|---|---|---|
| 1960 | 1679 | n.a. |
| 1970 | 2757 | |
| 1980 | 3089 | |
| 1990 | 2355 | |
| 2000 | 2756 | |

**Table 2.7**

**Enrolled in College**

| Year | Number (thousands) | Average Rate of Change (thousands per year) |
|---|---|---|
| 1960 | 758 | n.a. |
| 1970 | 1427 | |
| 1980 | 1524 | |
| 1990 | 1410 | |
| 2000 | 1745 | |

**Table 2.8**

**a.** Fill in the blank cells with the appropriate average rates of change for high school completers and those who enrolled in college.

**b.** Describe the average rates of change for high school completers.

**c.** Describe the average rates of change for those enrolled in college.

**d.** Compare and contrast the average rates of change for high school completers and those enrolled in college.

## 2.3 *The Average Rate of Change Is a Slope*

The reading "Slopes" describes many of the practical applications of slopes, from cowboy boots to handicap ramps.

### Calculating Slopes

On a graph, the average rate of change is the *slope* of the line connecting two points. The slope is an indicator of the steepness of the line.

If $(x_1, y_1)$ and $(x_2, y_2)$ are two points, then the change in $y$ equals $y_2 - y_1$ (see Figure 2.9). This difference is often denoted by $\Delta y$, read as "delta $y$," where $\Delta$ is the Greek letter capital D (think of D as representing difference): $\Delta y = y_2 - y_1$. Similarly, the change in $x$ (delta $x$) can be represented by $\Delta x = x_2 - x_1$. Then

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{change in } y}{\text{change in } x}$$

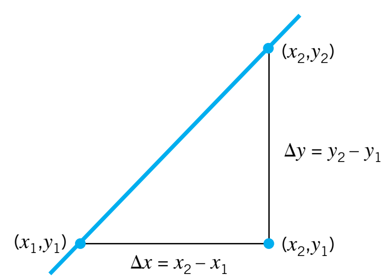



**Figure 2.9** Slope $= \Delta y/\Delta x$.

The average rate of change represents a *slope*. Given two points $(x_1, y_1)$ and $(x_2, y_2)$,

$$\text{average rate of change} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \text{slope}$$

### *Calculating slopes: It doesn't matter which point is first*

Given two points, $(x_1, y_1)$ and $(x_2, y_2)$, it doesn't matter which one we use as the first point when we calculate the slope. In other words, we can calculate the slope between $(x_1, y_1)$ and $(x_2, y_2)$ as

$$\frac{y_2 - y_1}{x_2 - x_1} \quad \text{or as} \quad \frac{y_1 - y_2}{x_1 - x_2}$$

The two calculations result in the same value.

We can show that the two forms are equivalent.

| | |
|---|---|
| Given | $\text{slope} = \dfrac{y_2 - y_1}{x_2 - x_1}$ |
| multiply by $\dfrac{-1}{-1}$ | $= \dfrac{-1}{-1} \cdot \dfrac{y_2 - y_1}{x_2 - x_1}$ |
| simplify | $= \dfrac{-y_2 + y_1}{-x_2 + x_1}$ |
| rearrange terms | $= \dfrac{y_1 - y_2}{x_1 - x_2}$ |

In calculating the slope, we do need to be consistent in the order in which the coordinates appear in the numerator and the denominator. If $y_1$ is the first term in the numerator, then $x_1$ must be the first term in the denominator.

**EXAMPLE 1** Plot the two points $(-2, -6)$ and $(7, 12)$ and calculate the slope of the line passing through them.

**Solution** Treating $(-2, -6)$ as $(x_1, y_1)$ and $(7, 12)$ as $(x_2, y_2)$ (Figure 2.10), then

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{12 - (-6)}{7 - (-2)} = \frac{18}{9} = 2$$

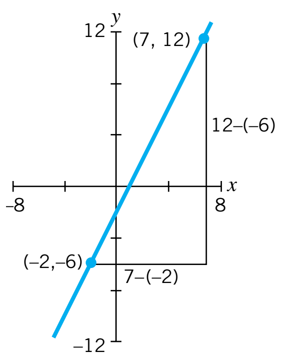



***Figure 2.10*** $(y_1 - y_2)/(x_1 - x_2) = (y_2 - y_1)/(x_2 - x_1)$.

We could also have used $-6$ and $-2$ as the first terms in the numerator and denominator, respectively:

$$\text{slope} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{-6 - 12}{-2 - 7} = \frac{-18}{-9} = 2$$

Either way we obtain the same answer.

**EXAMPLE 2** The percentage of the U.S. population living in rural areas decreased from 84.7% in 1850 to 21.0% in 2000. Plot the data, then calculate and interpret the average rate of change in the rural population over time.

**Solution** If we treat year as the independent and percentage as the dependent variable, our given data can be represented by the points (1850, 84.7%) and (2000, 21.0%). See Figure 2.11.

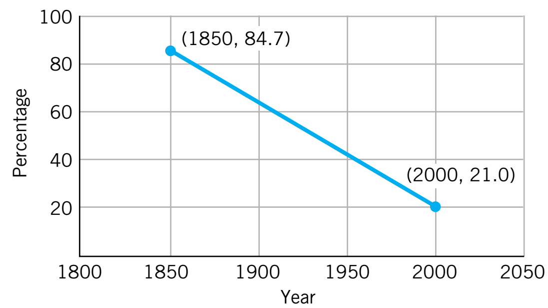



***Figure 2.11*** Percentage of the U.S. population living in rural areas.
*Source:* U.S. Bureau of the Census, *www.census.gov*

$$\begin{aligned}
\text{The average rate of change} &= \frac{\text{change in the percentage of rural population}}{\text{change in time}} \\
&= \frac{(21.0 - 84.7)\%}{(2000 - 1850)\text{ years}} \\
&= \frac{-63.7\%}{150\text{ years}} \\
&\approx -0.42\% \text{ per year}
\end{aligned}$$

**? SOMETHING TO THINK ABOUT**

*What kind of social and economic implications does a population shift of this magnitude have on society?*

The sign of the average rate of change is negative since the percentage of people living in rural areas was decreasing. (The negative slope of the graph in Figure 2.11 confirms this.) The value tells us that, on average, the percentage living in rural areas decreased by 0.42 percentage points (or about one-half of 1%) each year between 1850 and 2000. The change per year may seem small, but in a century and a half the rural population went from being the overwhelming majority (84.7%) to about one-fifth (21%) of the population.

If the slope, or average rate of change, of $y$ with respect to $x$ is *positive*, then the graph of the relationship rises up when read from left to right. This means that as $x$ increases in value, $y$ also increases in value.

If the slope is *negative*, the graph falls when read from left to right. As $x$ increases, $y$ decreases.

If the slope is *zero*, the graph is flat. As $x$ increases, there is no change in $y$.

**EXAMPLE 3** Given Table 2.9, of civil disturbances over time, plot and then connect the points, and (without doing any calculations) indicate on the graph when the average rate of change between adjacent data points is positive (+), negative (−), or zero (0).

**Solution** The data are plotted in Figure 2.12. Each line segment is labeled +, −, or 0, indicating whether the average rate of change between adjacent points is positive, negative, or zero. The largest positive average rate of change, or steepest upward slope, seems to be between the January-to-March and April-to-June counts in 1968. The largest negative average rate of change, or steepest downward slope, appears later in the same year (1968) between the July-to-September and October-to-December counts.

Civil disturbances between 1968 and 1972 occurred in cycles: The largest numbers generally occurred in the spring months and the smallest in the winter months. The peaks decrease over time. What was happening in America that might correlate with the peaks? This was a tumultuous period in our history. Many previously silent factions of society were finding their voices. Recall that in April 1968 Martin Luther King was assassinated and in January 1973 the last American troops were withdrawn from Vietnam.

**Civil Disturbances in U.S. Cities**

| Year | Period | Number of Disturbances |
|---|---|---|
| 1968 | Jan.–Mar. | 6 |
| | Apr.–June | 46 |
| | July–Sept. | 25 |
| | Oct.–Dec. | 3 |
| 1969 | Jan.–Mar. | 5 |
| | Apr.–June | 27 |
| | July–Sept. | 19 |
| | Oct.–Dec. | 6 |
| 1970 | Jan.–Mar. | 26 |
| | Apr.–June | 24 |
| | July–Sept. | 20 |
| | Oct.–Dec. | 6 |
| 1971 | Jan.–Mar. | 12 |
| | Apr.–June | 21 |
| | July–Sept. | 5 |
| | Oct.–Dec. | 1 |
| 1972 | Jan.–Mar. | 3 |
| | Apr.–June | 8 |
| | July–Sept. | 5 |
| | Oct.–Dec. | 5 |

**Table 2.9**

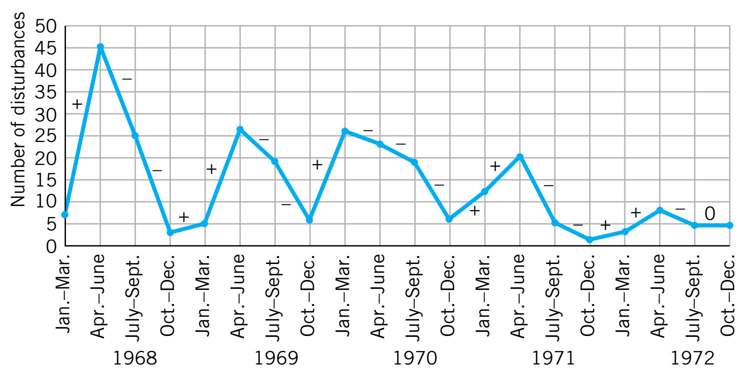



**Figure 2.12** Civil disturbances: 1968–1972.
*Source:* D. S. Moore and G. P. McCabe, *Introduction to the Practice of Statistics*. Copyright © 1989 by W.H. Freeman and Company. Used with permission.

## Algebra Aerobics 2.3

1. **a.** Plot each pair of points and then calculate the slope of the line that passes through them.
   **i.** (4, 1) and (8, 11) **iii.** (0, −3) and (−5, −1)
   **ii.** (−3, 6) and (2, 6)
   **b.** Recalculate the slopes in part (a), reversing the order of the points. Check that your answers are the same.
2. Specify the intervals on the graph in Figure 2.13 for which the average rate of change between adjacent data points is approximately zero.

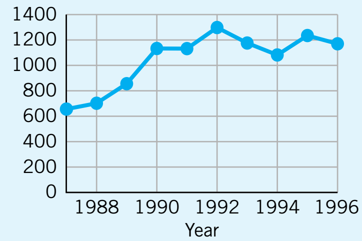



***Figure 2.14*** Tornadoes: 1987–1996.

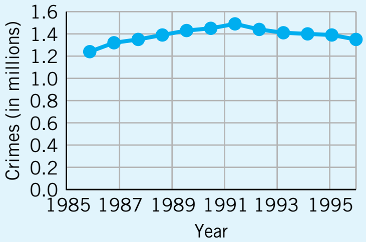



***Figure 2.13*** Violent crimes in the United States.

3. Specify the intervals on the graph in Figure 2.14 for which the average rate of change between adjacent data points appears positive, negative, or zero.
4. What is the missing $y$-coordinate that would produce a slope of 4, if a line were drawn through the points (3, −2) and (5, $y$)?
5. Find the slope of the line through the points (2, 9) and $(2 + h, 9 + 2h)$.
6. Consider points $P_1 = (0, 0)$, $P_2 = (1, 1)$, $P_3 = (2, 4)$, and $P_4 = (3, 9)$.
   **a.** Verify that these four points lie on the graph of $y = x^2$.
   **b.** Find the slope of the line segments connecting $P_1$ and $P_2$, $P_2$ and $P_3$, and $P_3$ and $P_4$.
   **c.** What do these slopes suggest about the graph of the function within those intervals?

## 2.4 Putting a Slant on Data

Whenever anyone summarizes a set of data, choices are being made. One choice may not be more “correct” than another. But these choices can convey, either accidentally or on purpose, very different impressions.

### Slanting the Slope: Choosing Different End Points

Within the same data set, one choice of end points may paint a rosy picture, while another choice may portray a more pessimistic outcome.

**EXAMPLE 1**

The data in Table 2.10 and the scatter plot in Figure 2.15 show the number of families below the poverty level in the United States from 1960 to 2002. How could we use the information to make the case that the poverty level has decreased? Has increased?

| Year | Number of Families below Poverty Level (in thousands) |
|---|---|
| 1960 | 8243 |
| 1965 | 6721 |
| 1970 | 5260 |
| 1975 | 5450 |
| 1980 | 6217 |
| 1985 | 7223 |
| 1990 | 7098 |
| 1995 | 7532 |
| 2000 | 6400 |
| 2002 | 7229 |

***Table 2.10***
*Source:* U.S. Bureau of the Census, *www.census.gov*

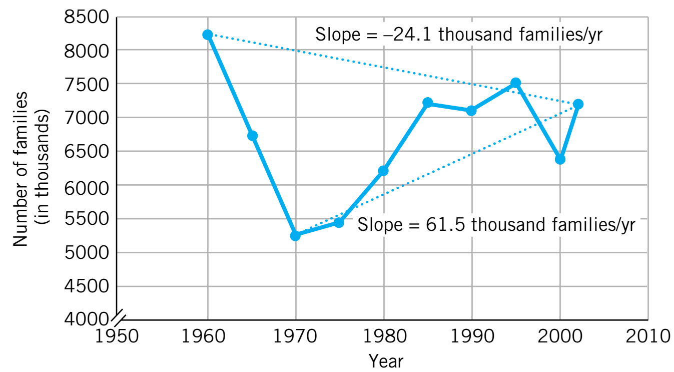



***Figure 2.15*** Number of families (in thousands) below poverty level.

**Solution** *Optimistic case:* To make an upbeat case that poverty numbers have decreased, we could choose as end points (1960, 8243) and (2002, 7229). Then

$$\begin{aligned}\text{average rate of change} &= \frac{\text{change in no. of families in poverty (000s)}}{\text{change in years}}\\ &= \frac{7229 - 8243}{2002 - 1960}\\ &= \frac{-1014}{42}\\ &= -24.1 \text{ thousand families/year}\end{aligned}$$

So between 1960 and 2002 the number of impoverished families *decreased* on average by 24.1 thousand (or 24,100) each year. We can see this reflected in Figure 2.15 in the negative slope of the line connecting (1960, 8243) and (2002, 7229).

*Pessimistic case:* To make a depressing case that poverty numbers have risen, we could choose (1970, 5260) and (2002, 7229) as end points. Then

$$\begin{aligned}\text{average rate change of change} &= \frac{\text{change in no. of families in poverty (000s)}}{\text{change in years}}\\ &= \frac{7229 - 5260}{2002 - 1970}\\ &= \frac{1969}{32}\\ &= 61.5 \text{ thousand families/year}\end{aligned}$$

So between 1970 and 2002 the number of impoverished families *increased* on average by 61.5 thousand (or 61,500) per year. This is reflected in Figure 2.15 in the positive slope of the line connecting (1970, 5260) and (2002, 7229). Both average rates of change are correct, but they give very different impressions of the changing number of families living in poverty in America.

## Slanting the Data with Words and Graphs

If we wrap data in suggestive vocabulary and shape graphs to support a particular viewpoint, we can influence the interpretation of information. In Washington, D.C., this would be referred to as "putting a spin on the data."

See "C4: Distortion by Clipping and Scaling" in *Rates of Change.*

Take a close look at the following three examples. Each contains exactly the same underlying facts: the same average rate of change calculation and a graph with a plot of the same two data points (1990, 248.7) and (2000, 281.4), representing the U.S. population (in millions) in 1990 and in 2000.

### *The U.S. population increased by only 3.27 million/year between 1990 and 2000*

Stretching the scale of the horizontal axis relative to the vertical axis makes the slope of the line look almost flat and hence minimizes the impression of change (Figure 2.16).

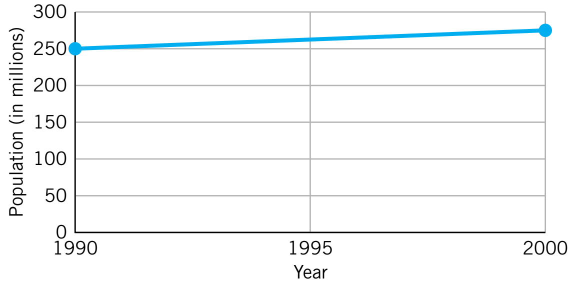



***Figure 2.16*** "Modest" growth in the U.S. population.

### *The U.S. population had an* explosive *growth of over 3.27 million/year between 1990 and 2000*

Cropping the vertical axis (which now starts at 240 instead of 0) and stretching the scale of the vertical axis relative to the horizontal axis makes the slope of the line look steeper and strengthens the impression of dramatic change (Figure 2.17).

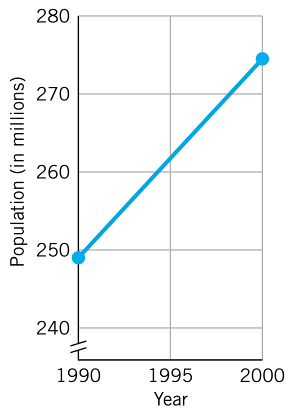



**Figure 2.17** "Explosive" growth in the U.S. population.

### *The U.S. population grew at a* reasonable *rate of 3.27 million/year during the 1990s*

Visually, the steepness of the line in Figure 2.18 seems to lie roughly halfway between the previous two graphs. *In fact, the slope of 3.27 million/year is precisely the same for all three graphs.*

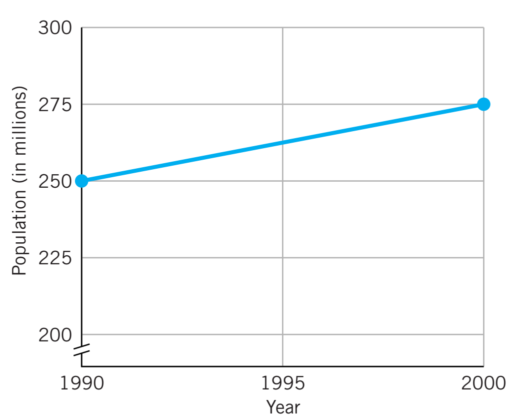



**Figure 2.18** "Reasonable" growth in the U.S. population.

Exploration 2.1 gives you a chance to put your own "spin" on data.

How could you decide upon a "fair" interpretation of the data? You might try to put the data in context by asking, How does the growth between 1990 and 2000 compare with other decades in the history of the United States? How does it compare with growth in other countries at the same time? Was this rate of growth easily accommodated, or did it strain national resources and overload the infrastructure?

A statistical claim is never completely free of bias. For every statistic that is quoted, others have been left out. This does not mean that you should discount all statistics. However, you will be best served by a thoughtful approach when interpreting the statistics to which you are exposed on a daily basis. By getting in the habit of asking questions and then coming to your own conclusions, you will develop good sense about the data you encounter.

## Algebra Aerobics 2.4

1. Use the data in Table 2.11, on immigration into the United States between 1901 and 2000, to answer the following questions.
   a. What two end points might you use to argue that the immigration level has declined slightly?
   b. What two end points might you use to argue that immigration levels have increased dramatically?

**Immigration: 1901–2000**

| Period | Number of Immigrants (thousands) |
|---|---|
| 1901–1910 | 8795 |
| 1911–1920 | 5736 |
| 1921–1930 | 4107 |
| 1931–1940 | 528 |
| 1941–1950 | 1035 |
| 1951–1960 | 2515 |
| 1961–1970 | 3322 |
| 1971–1980 | 4493 |
| 1981–1990 | 7338 |
| 1991–2000 | 9095 |

**Table 2.11**
*Source:* U.S. Bureau of the Census, *www.census.gov*

2. Assume you are the financial officer of a corporation whose stock earnings were $1.02 per share in 2001 and $1.06 per share in 2002. How could you make a case for dramatic growth?
3. Sketch a graph and compose a few sentences to forcefully advocate the views of the following persons.
   a. You are an antiwar journalist reporting on American casualties during a war. In week one there were 17, in week two 29, and in week three 26.
   b. You are the president's press secretary in charge of reporting war casualties [listed in part (a)] to the public.
4. The graph in Figure 2.19 appeared as part of an advertisement in the *Boston Globe* on June 27, 2003. Identify at least three strategies used to persuade you to buy gold.

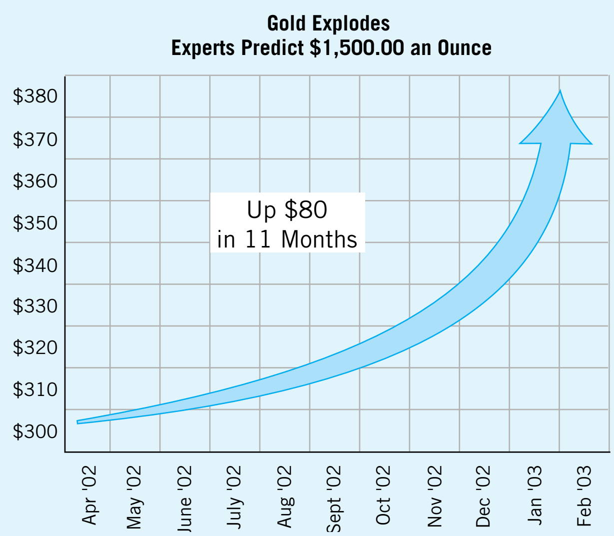



**Figure 2.19** Prices for an ounce of gold.

## 2.5 Linear Functions: When Rates of Change Are Constant

In many of our examples so far, the average rate of change has varied depending on the choice of end points. Now we will examine the special case when the average rate of change remains constant.

### What if the U.S. Population Had Grown at a Constant Rate? A Hypothetical Example

In Section 2.2 we calculated the average rate of change in the population between 1790 and 1800 as 0.14 million people/year. We saw that the average rate of change was different for different decades. What if the average rate of change had remained constant? What if in every decade after 1790 the U.S. population had continued to grow at the same rate of 0.14 million people/year? That would mean that starting with a population estimated at 3.9 million in 1790, the population would have grown by 0.14 million each year. The slopes of all the little line segments connecting adjacent population data points would be identical, namely 0.14 million people/year. The graph would be a straight line, indicating a constant average rate of change.

Experiment with varying the average velocities and then setting them all constant in "C3: Average Velocity and Distance" in *Rates of Change.*

On the graph of actual U.S. population data, the slopes of the line segments connecting adjacent points are increasing, so the graph curves upward. Figure 2.20 compares the actual and hypothetical results.

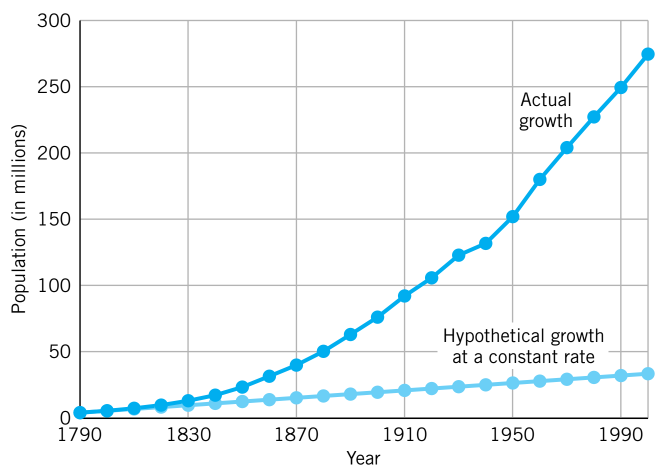



***Figure 2.20*** U.S. population: A hypothetical example.

Any function that has the same average rate of change on every interval has a graph that is a straight line. The function is called linear. This hypothetical example represents a linear function. When the average rate of change is constant, we can drop the word "average" and just say "rate of change."

| |
|---|
| A linear function has a constant rate of change. Its graph is a straight line. |

## Real Examples of a Constant Rate of Change

**EXAMPLE 1**

According to the standardized growth and development charts used by many American pediatricians, the median weight for girls during their first six months of life increases at an almost constant rate. Starting at 7.0 pounds at birth, female median weight increases by approximately 1.5 pounds per month. If we assume that the median weight for females, $W$, is increasing at a constant rate of 1.5 pounds per month, then $W$ is a linear function of age in months, $A$.

**a.** Generate a table that gives the median weight for females, $W$, for the first six months of life and create a graph of $W$ as a function of $A$.

**b.** Find an equation for $W$ as a function of $A$. What is an appropriate domain for this function?

**c.** Express the equation for part (b) using only units of measure.

**Solution** **a.** For female infants at birth ($A = 0$ months), the median weight is 7.0 lb ($W = 7.0$ lb). The rate of change, 1.5 pounds/month, means that as age increases by 1 month, weight increases by 1.5 pounds. See Table 2.12.

**Median Weight for Girls**

| Age (months) | Weight (lb) |
|---|---|
| 0 | 7.0 |
| 1 | 8.5 |
| 2 | 10.0 |
| 3 | 11.5 |
| 4 | 13.0 |
| 5 | 14.5 |
| 6 | 16.0 |

***Table 2.12***

*Source:* Data derived from the Ross Growth and Development Program, Ross Laboratories, Columbus, OH.

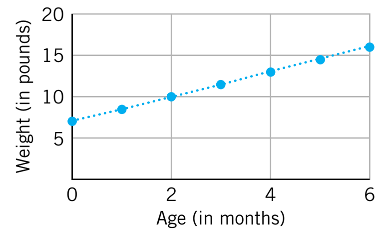



***Figure 2.21*** Median weight for girls.

The dotted line in Figure 2.21 shows the trend in the data.

**b.** To find a linear equation for $W$ (median weight in pounds) as a function of $A$ (age in months), we can study the table of values in Table 2.12.

$$\begin{aligned} W &= \text{initial weight} + \text{weight gained} \\ &= \text{initial weight} + \text{rate of growth} \cdot \text{number of months} \\ &= 7.0 \text{ lb} + 1.5 \text{ lb/month} \cdot \text{number of months} \end{aligned}$$

The equation would be

$$W = 7.0 + 1.5A$$

An appropriate domain for this function would be $0 \leq A \leq 6$.

**SOMETHING TO THINK ABOUT**

*If the median birth weight for baby boys is the same as for baby girls, but boys put on weight at a faster rate, which numbers in the model would change and which would stay the same? What would you expect to be different about the graph?*

**c.** Since our equation represents quantities in the real world, each term in the equation has units attached to it. The median weight $W$ is in pounds (lb), rate of change is in lb/month, and age, $A$, is in months. If we display the equation

$$W = 7.0 + 1.5A$$

showing only the units, we have

$$\text{lb} = \text{lb} + \left(\frac{\text{lb}}{\text{month}}\right)\text{month}$$

The rules for canceling units are the same as the rules for canceling numbers in fractions. So,

$$\text{lb} = \text{lb} + \left(\frac{\text{lb}}{\cancel{\text{month}}}\right)\cancel{\text{month}}$$

$$\text{lb} = \text{lb} + \text{lb}$$

This equation makes sense in terms of the original problem since pounds (lb) added to pounds (lb) should give us pounds (lb).

**EXAMPLE 2** You spend \$1200 on a computer and for tax purposes choose to depreciate it (or assume it decreases in value) to \$0 at a constant rate over a 5-year period.

**a.** Calculate the rate of change of the assumed value of the equipment over 5 years. What are the units?

**b.** Create a table and graph showing the value of the equipment over 5 years.

**c.** Find an equation for the value of the computer as a function of time in years. Why is this a linear function?

**d.** What is an appropriate domain for this function? What is the range?

**Solution**

**a.** After 5 years, your computer is worth \$0. If $V$ is the value of your computer in dollars and $t$ is the number of years you own the computer, then the rate of change of $V$ from $t = 0$ to $t = 5$ is

$$\begin{aligned}\text{rate of change} &= \frac{\text{change in value}}{\text{change in time}} = \frac{\Delta V}{\Delta t}\\ &= -\frac{\$1200}{5 \text{ years}} = -\$240/\text{year}\end{aligned}$$

Thus, the worth of your computer drops at a constant rate of \$240 per year. The rate of change in $V$ is negative because the worth of the computer decreases over time. The units for the rate of change are dollars per year.

**b.** Table 2.13 and Figure 2.22 show the depreciated value of the computer.

**Value of Computer Depreciated over 5 Years**

| Numbers of Years | Value of Computer (\$) |
|---|---|
| 0 | 1200 |
| 1 | 960 |
| 2 | 720 |
| 3 | 480 |
| 4 | 240 |
| 5 | 0 |

**Table 2.13**

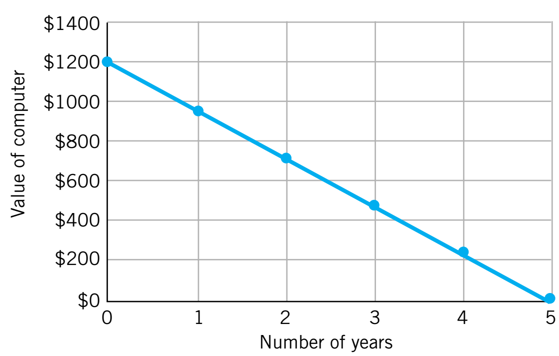



**Figure 2.22** Value of computer over 5 years.

**c.** To find a linear equation for $V$ as a function of $t$, think about how we found the table of values.

$$\begin{aligned}\text{value of computer} &= \text{initial value} + (\text{rate of decline}) \cdot (\text{number of years})\\ V &= \$1{,}200 + (-\$240/\text{year}) \cdot t\\ V &= 1200 - 240t\end{aligned}$$

This equation describes $V$ as a function of $t$ because for every value of $t$, there is one and only one value of $V$. It is a linear function because the rate of change is constant.

**d.** The domain is $0 \leq t \leq 5$ and the range is $0 \leq V \leq 1200$.

## The General Equation for a Linear Function

The equations in Examples 1 and 2 can be rewritten in terms of the output (dependent variable) and the input (independent variable).

$$\begin{aligned} \text{weight} &= \text{initial weight} + \text{rate of growth} \cdot \text{number of months} \\ \text{value of computer} &= \text{initial value} + \text{rate of decline} \cdot \text{number of years} \\ \underbrace{\text{output}}_{y} &= \underbrace{\text{initial value}}_{b} + \underbrace{\text{rate of change}}_{m} \cdot \underbrace{\text{input}}_{x} \end{aligned}$$

Thus, the general linear equation can be written in the form

$$y = b + mx$$

where we use the traditional mathematical choices of $y$ for the output (dependent variable) and $x$ for the input (independent variable). We let $m$ stand for the rate of change, thus

$$\begin{aligned} m &= \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} \\ &= \text{slope of the graph of the line} \end{aligned}$$

In our general equation, we let $b$ stand for the initial value. Why is $b$ called the initial value? The number $b$ is the value of $y$ when $x = 0$. If we let $x = 0$, then

$$\begin{aligned} \text{given} \qquad y &= b + mx \\ y &= b + m \cdot 0 \\ y &= b \end{aligned}$$

Explorations 2.2A and 2.2B (along with "L1: *m* & *b* Sliders" in *Linear Functions*) allow you to examine the effects of *m* and *b* on the graph of a linear function.

The point $(0, b)$ satisfies the equation and lies on the $y$-axis. The point $(0, b)$ technically the vertical intercept. However, since the coordinate $b$ tells us where the line crosses the $y$-axis, we often just refer to $b$ as $b$ is the *vertical* or *y-intercept* (see Figure 2.23).

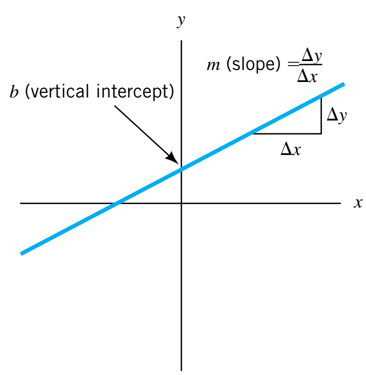



**Figure 2.23** Graph of $y = b + mx$.

> **A Linear Function**
>
> A function $y = f(x)$ is called *linear* if it can be represented by an equation of the form
>
> $$y = b + mx$$
>
> Its graph is a straight line where $m$ is the *slope*, the rate of change of $y$ with respect to $x$, so
>
> $$m = \frac{\Delta y}{\Delta x}$$
>
> $b$ is the *vertical* or *y-intercept*, and is the value of $y$ when $x = 0$.

The equation $y = b + mx$ could, of course, be written in the equivalent form $y = mx + b$. In mathematical models, $b$ is often the initial or starting value of the output, so it is useful to place it first in the equation.

**EXAMPLE 3** For each of the following equations, identify the value of $b$ and the value of $m$.

**a.** $y = -4 + 3.25x$ **c.** $y = -4x + 3.25$
**b.** $y = 3.25x - 4$ **d.** $y = 3.25 - 4x$

**Solution**
**a.** $b = -4$ and $m = 3.25$
**b.** $b = -4$ and $m = 3.25$
**c.** $b = 3.25$ and $m = -4$
**d.** $b = 3.25$ and $m = -4$

**EXAMPLE 4** In the following equations, $L$ represents the legal fees (in dollars) charged by four different law firms and $h$ represents the number of hours of legal advice.

$$L_1 = 500 + 200h$$
$$L_2 = 1000 + 150h$$
$$L_3 = 800 + 350h$$
$$L_4 = 500h$$

**a.** Which initial fee is the highest?
**b.** Which rate per hour is the highest?
**c.** If you need 5 hours of legal advice, which legal fee will be the highest?

**Solution**
**a.** $L_2$ has the highest initial fee of \$1000.
**b.** $L_4$ has the highest rate of \$500 per hour.
**c.** Evaluate each equation for $h = 5$ hours.

$$\begin{aligned} L_1 &= 500 + 200h \\ &= 500 + 200(5) \\ &= \$1500 \end{aligned}$$

$$\begin{aligned} L_2 &= 1000 + 150h \\ &= 1000 + 150(5) \\ &= \$1750 \end{aligned}$$

$$\begin{aligned} L_3 &= 800 + 350h \\ &= 800 + 350(5) \\ &= \$2550 \end{aligned}$$

$$\begin{aligned} L_4 &= 500h \\ &= 500(5) \\ &= \$2500 \end{aligned}$$

For 5 hours of legal advice, $L_3$ has the highest legal fee.

## Algebra Aerobics 2.5

**1.** From Figure 2.21 in Section 2.5, estimate the weight $W$ of a baby girl who is 4.5 months old. Then, use the equation $W = 7.0 + 1.5A$ to calculate the corresponding value for $W$. How close is your estimate?

**2.** From the same graph, estimate the age of a baby girl who weighs 11 pounds. Then use the equation to calculate the value for $A$.

**3.** Select any two points of the form $(A, W)$ from Table 2.12 in Section 2.5 that satisfy the equation $W = 7.0 + 1.5A$. Use these points to verify that the rate of change between them is 1.5.

**4. a.** If $C = 15P + 10$ describes the relationship between the number of persons ($P$) in a dining party and the total cost in dollars ($C$) of their meals, what is the unit of measure for 15? For 10?

**b.** The equation $W = 7.0 + 1.5A$ (modeling weight as a function of age) expressed in units of measure only is

$$\text{lb} = \text{lb} + \left(\frac{\text{lb}}{\text{month}}\right)\text{months}$$

Express $C = 15P + 10$ from part (a) in units of measure only.

**5.** Determine which (if any) of the following points satisfy the equation $y = 6.2 + 3x$.

**a.** $(5, 21.2)$ **b.** $(6.2, 0)$ **c.** $(-2, 0.2)$

**6.** Assume $S = 0.8Y + 19$ describes the projected relationship between $S$, sales of a company (in millions of dollars), for $Y$ years from today.

**a.** What are the units of 0.8 and what does it represent?

**b.** What are the units of 19 and what does it represent?

**c.** What would be the projected company sales in three years?

**7.** Assume $C = 0.45N$ represents the total cost $C$ (in dollars) of operating a car for $N$ miles.

**a.** What does 0.45 represent and what are its units?

**b.** Find the total cost to operate a car that has been driven 25,000 miles.

**8.** The relationship between the balance $B$ (in dollars) left on a mortgage loan and $N$, the number of monthly payments, is given by $B = 302{,}400 - 840N$.

**a.** What is the monthly mortgage payment?

**b.** What does 302,400 represent?

**c.** What is the balance on the mortgage after 10 years? 20 years? 30 years? (*Hint:* Remember there are 12 months in a year.)

**9.** Identify the slope, $m$, and the vertical intercept, $b$, of the line with the given equation.

**a.** $y = 5x + 3$

**b.** $y = 5 + 3x$

**c.** $y = 5x$

**d.** $y = 3$

**e.** $f(x) = 7.0 - x$

**f.** $h(x) = -11x + 10$

**g.** $y = 1 - \frac{2}{3}x$

**h.** $2y + 6 = 10x$

**10.** If $f(x) = 50 - 25x$:

**a.** Why does $f(x)$ describe a linear function?

**b.** Evaluate $f(0)$ and $f(2)$.

**c.** Use your answers in part (b) to verify that the slope is $-25$.

**11.** Identify the functions that are linear. For each linear function, identify the slope and the vertical intercept.

**a.** $f(x) = 3x + 5$

**b.** $f(x) = x$

**c.** $f(x) = 3x^2 + 2$

**d.** $f(x) = 4 - \frac{2}{3}x$

**12.** Write an equation for the line in the form $y = mx + b$ for the indicated values.

**a.** $m = 3$ and $b = 4$

**b.** $m = -1$ and passes through the origin

**c.** $m = 0$ and $b = -3$

**d.** $m = \frac{1}{2}$ and passes through the point $\left(2, \frac{1}{3}\right)$

**13.** Write the equation of the graph of each line in Figure 2.24 in $y = mx + b$ form. Use the $y$-intercept and the slope indicated on each graph.

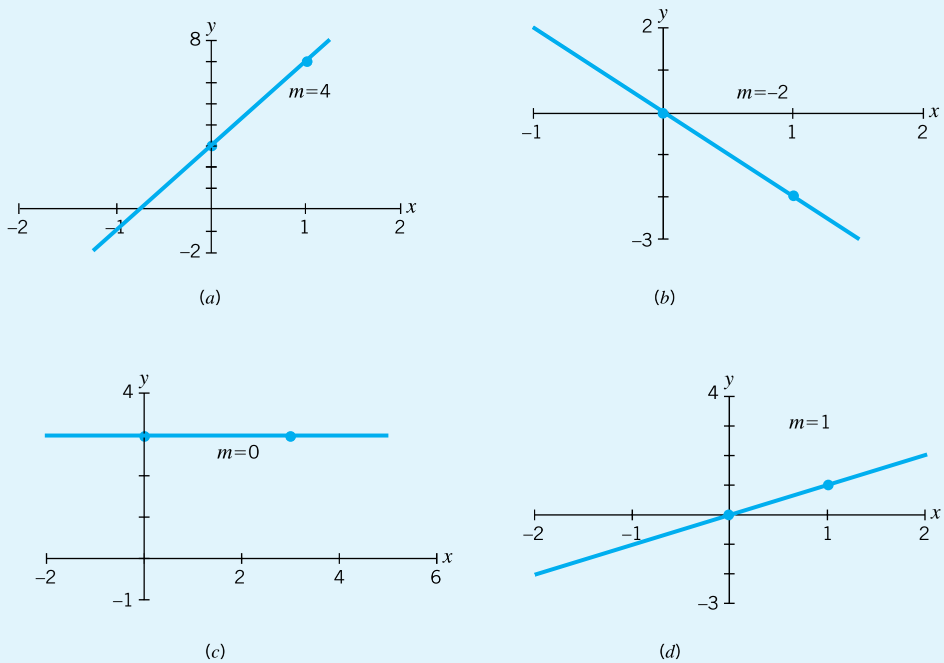



***Figure 2.24*** Four linear graphs.

## 2.6 *Visualizing Linear Functions*

The values for $b$ and $m$ in the general form of the linear equation, $y = b + mx$, tell us about the graph of the function. The value for $b$ tells us where to anchor the line on the $y$-axis. The value for $m$ tells us whether the line climbs or falls and how steep it is.

### The Effect of *b*

In the equation, $y = b + mx$, the number $b$ is the vertical intercept, so it anchors the line at the point $(0, b)$ (see Figure 2.25).

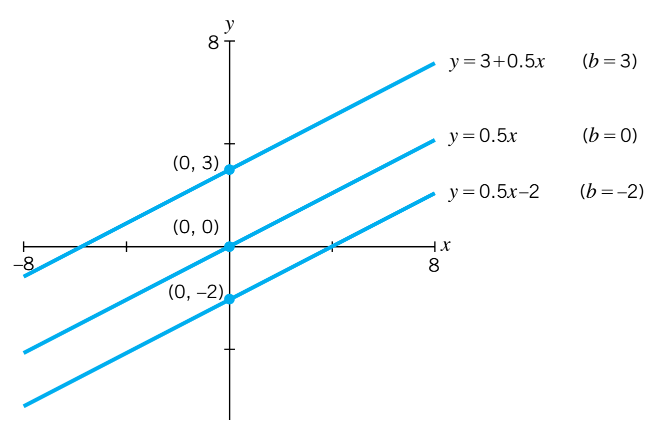



***Figure 2.25*** The effect of $b$, the vertical intercept.

**EXAMPLE 1** Explain how the graph of $y = 4 + 5x$ differs from the graphs of the following functions:

**a.** $y = 8 + 5x$ **b.** $y = 2 + 5x$ **c.** $y = 5x + 4$

**Solution** Although all of the graphs are straight lines with a slope of 5, they each have a different vertical intercept. The graph of $y = 4 + 5x$ has a vertical intercept at 4.

**a.** The graph of $y = 8 + 5x$ intersects the $y$-axis at 8, four units above the graph of $y = 4 + 5x$.

**b.** The graph of $y = 2 + 5x$ intersects the $y$-axis at 2, two units below the graph of $y = 4 + 5x$.

**c.** Since $y = 5x + 4$ and $y = 4 + 5x$ are equivalent equations, they have the same graph.

## The Effect of *m*

### *The sign of m*

The sign of $m$ in the equation $y = mx + b$ determines whether the line climbs (slopes up) or falls (slopes down) as we move left to right on the graph. If $m$ is positive, the line climbs from left to right (as $x$ increases, $y$ increases). If $m$ is negative, the line falls from left to right (as $x$ increases, $y$ decreases) (see Figure 2.26).

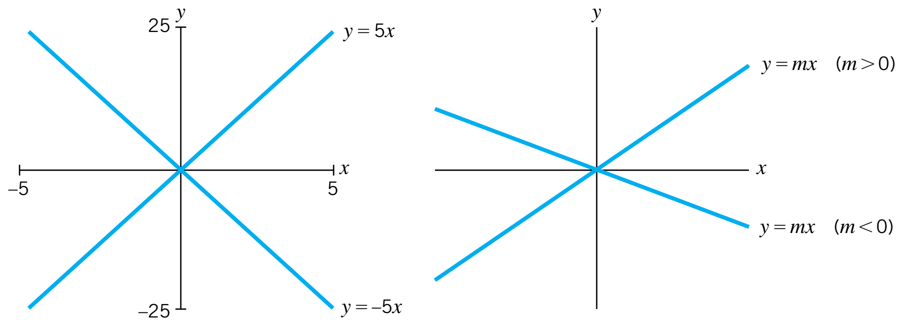



***Figure 2.26*** The effect of the sign of $m$.

**EXAMPLE 2** Match the following functions to the lines in Figure 2.27.

$$f(x) = 3 - 2x$$
$$g(x) = 3 + 2x$$
$$h(x) = -5 - 2x$$
$$j(x) = -5 + 2x$$

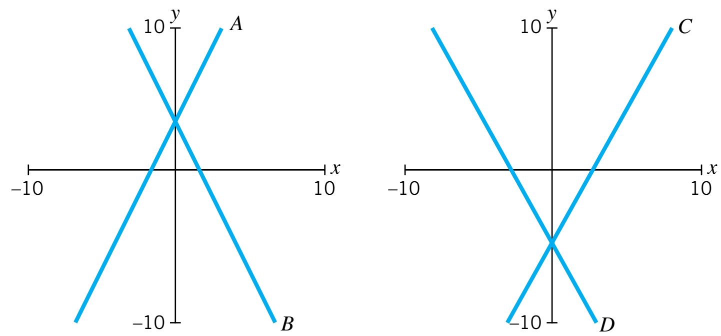



***Figure 2.27*** Matching graphs.

**Solution** $A$ is the graph of $g(x)$.
$B$ is the graph of $f(x)$.
$C$ is the graph of $j(x)$.
$D$ is the graph of $h(x)$.

### *The magnitude of m*

The magnitude (absolute value) of $m$ determines the steepness of the line. Recall that the absolute value of $m$ is the value of $m$ stripped of its sign; for example, $|-3| = 3$. The greater the magnitude ($|m|$), the steeper the line. This makes sense since $m$ is the slope or the rate of change of $y$ with respect to $x$. Notice how the steepness of each line in Figure 2.28 increases as the magnitude of $m$ increases. For example, the slope ($m = 5$) of $y = 7 + 5x$ is steeper than the slope ($m = 3$) of $y = 7 + 3x$.

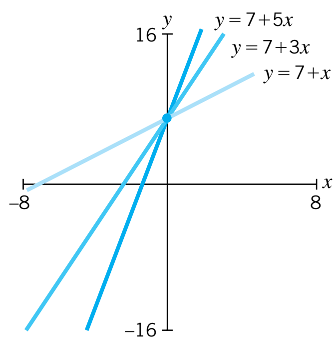



**Figure 2.28** Graphs with positive values for $m$.

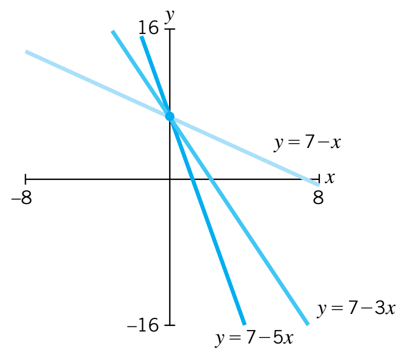



**Figure 2.29** Graphs with negative values for $m$.

**SOMETHING TO THINK ABOUT**
*What happens when $m = 0$?*

In Figure 2.29 we can see that the slope ($m = -5$) of $y = 7 - 5x$ is steeper than the slope ($m = -3$) of $y = 7 - 3x$ since $|-5| = 5 > |-3| = 3$. The lines $y = 7 - 5x$ and $y = 7 + 5x$ have the same steepness of 5 since $|-5| = |5| = 5$.

**EXAMPLE 3** Pair each graph in Figure 2.30 with a matching equation.

$f(x) = 9 - 0.4x$
$g(x) = -4 + x$
$h(x) = 9 - 0.2x$
$i(x) = -4 + 0.25x$
$j(x) = 9 + 0.125x$
$k(x) = 4 - 0.25x$

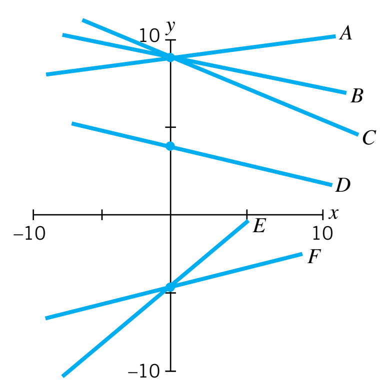



**Figure 2.30** Graphs of multiple linear functions.

**Solution** $A$ is the graph of $j(x)$.
$B$ is the graph of $h(x)$.
$C$ is the graph of $f(x)$.
$D$ is the graph of $k(x)$.
$E$ is the graph of $g(x)$.
$F$ is the graph of $i(x)$.

**EXAMPLE 4** Without graphing the following functions, how can you tell which graph will have the steepest slope?

**a.** $f(x) = 5 - 2x$ **b.** $g(x) = 5 + 4x$ **c.** $h(x) = 3 - 6x$

**Solution** The graph of the function $h$ will be steeper than the graphs of the functions $f$ and $g$ since the magnitude of $m$ is greater for $h$ than for $f$ or $g$. The greater the magnitude of $m$, the steeper the graph of the line. For $f(x)$, the magnitude of the slope is $|-2| = 2$. For $g(x)$, the magnitude of the slope is $|4| = 4$. For $h(x)$, the magnitude of the slope is $|-6| = 6$.

**EXAMPLE 5** Which of the graphs in Figure 2.31 has the steeper slope?

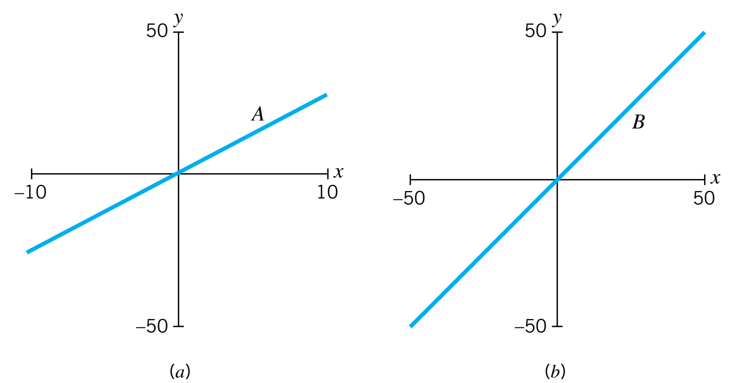



***Figure 2.31*** Comparing slopes.

**Solution** Remember from Section 2.4 that the steepness of a linear graph is not related to your visual impression, but to the numerical magnitude of the slope. The scales of the horizontal axis are different for the graphs in Figure 2.31, so the impression of relative steepness is deceiving. The slope of line $A$ is 2.5 units and the slope of line $B$ is 1 unit, so line $A$ has a steeper slope than line $B$.

**The Graph of a Linear Equation**

Given the general linear equation, $y = b + mx$, whose graph is a straight line:

The $y$-intercept, $b$, tells us where the line crosses the $y$-axis.

The slope, $m$, tells us how fast the line is climbing or falling. The larger the magnitude of $m$, the steeper the graph.

If the slope, $m$ is positive, the line climbs from left to right. If $m$ is negative, the line falls from left to right.

## Algebra Aerobics 2.6

1. Place these numbers in order from smallest to largest.

$$|-12|, |-7|, |-3|, |-1|, 0, 4, 9$$

2. Match the graph with the functions.
   **a.** $f(x) = 2 + 3x$  **c.** $h(x) = \frac{1}{2}x - 2$
   **b.** $g(x) = -2 - 3x$  **d.** $k(x) = 2 - 3x$

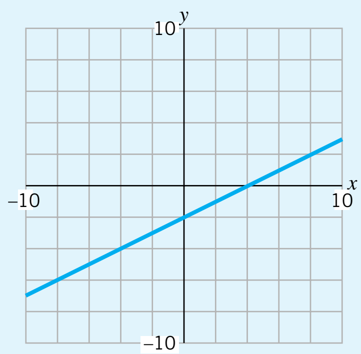
3. Without graphing the function, explain how the graph $y = 6x - 2$ differs from:
   **a.** $y = 6x$  **c.** $y = -2 + 3x$
   **b.** $y = 2 + 6x$  **d.** $y = -2 - 2x$

4. Without graphing, order the graphs of the functions from least steep to steepest.
   **a.** $y = 100 - 2x$  **c.** $y = -3x - 5$
   **b.** $y = 1 - x$  **d.** $y = 3 - 5x$

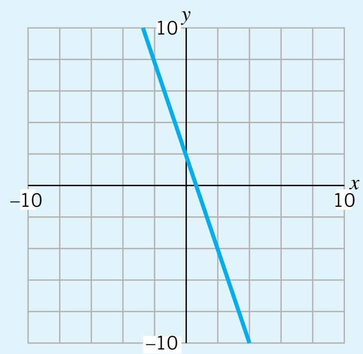
5. On an $x$-$y$ coordinate system, draw a line with a positive slope and label it $f(x)$.
   **a.** Draw a line $g(x)$ with the same slope but a $y$-intercept three units above $f(x)$.
   **b.** Draw a line $h(x)$ with the same slope but a $y$-intercept four units below $f(x)$.
   **c.** Draw a line $k(x)$ with the same steepness as $f(x)$ but with a negative slope.

6. Which function has the steepest slope? Create a table of values for each function and graph the function to show that this is true.
   **a.** $f(x) = 3x - 5$
   **b.** $g(x) = 7 - 8x$

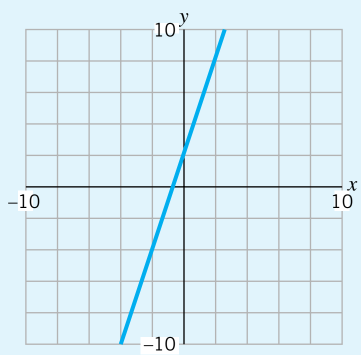
7. Create three functions with a $y$-intercept of 4 and three different negative slopes. Indicate which function has the steepest slope.



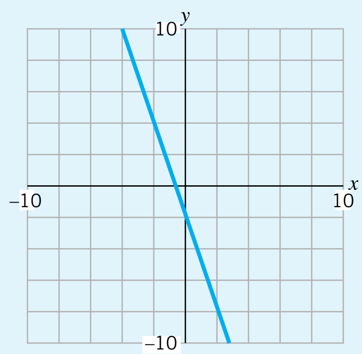

Graph *A*




Graph *C*




Graph *B*




Graph *D*

## 2.7 *Finding Graphs and Equations of Linear Functions*

### Finding the Graph

**EXAMPLE 1** **Given the equation**

A forestry study measured the diameter of the trunk of a red oak tree over 5 years. They created the linear model $D = 1 + 0.13Y$, where $D$ = diameter in inches and $Y$ = number of years from the beginning of the study.

The program "L4: Finding 2 Points on a line" in *Linear Functions* will give you practice in finding solutions to equations.

**a.** What do the numbers 1 and 0.13 represent in this context?

**b.** Sketch a graph of the function model.

**Solution**

**a.** The number 1 represents a starting diameter of 1 inch. The number 0.13 represents the annual growth rate of the oak's diameter (change in diameter/change in time), 0.13 inches per year.

**b.** The linear equation $D = 1 + 0.13Y$ tells us that 1 is the vertical intercept, so the point (0, 1) lies on the graph. The graph represents solutions to the equation. So to find a second point, we can evaluate $D$ for any other value of $Y$. If we set $Y = 1$, then $D = 1 + (0.13 \cdot 1) = 1 + 0.13 = 1.13$. So (1, 1.13) is another point on the line. Since two points determine a line, we can sketch our line through (0, 1) and (1, 1.13) (see Figure 2.32).

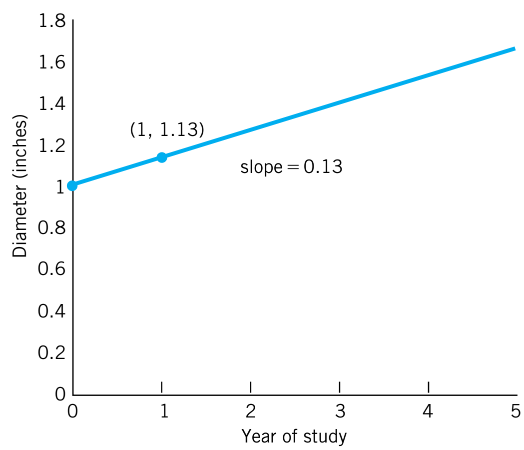



***Figure 2.32*** The diameter of a red oak over time.

**EXAMPLE 2** **Given a point off the *y*-axis and the slope**

Given a point (2, 3) and a slope of $m = -5/4$, describe at least two ways you could find a second point to plot a line with these characteristics without constructing the equation.

**Solution** Plot the point (2, 3).

**a.** If we write the slope as $(-5)/4$, then a change of 4 in $x$ corresponds to a change of $-5$ in $y$. So starting at (2, 3), moving horizontally four units to the right (adding 4 to the $x$-coordinate) and, then moving vertically five units down (subtracting 5 from the $y$-coordinate) gives us a second point on the line at $(2 + 4, 3 - 5) = (6, -2)$. Now we can plot our second point $(6, -2)$ and draw the line through it and our original point, (2, 3) (see Figure 2.33).

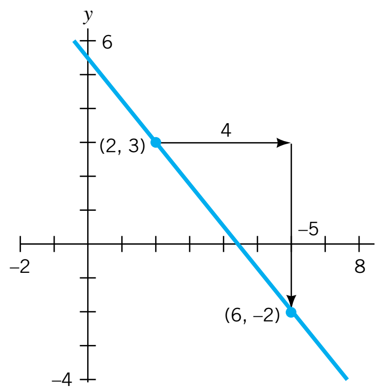



***Figure 2.33*** Graph of the line through (2, 3) with slope $-5/4$.

**b.** If we are modeling real data, we are more likely to convert the slope to decimal form. In this case $m = (-5)/4 = -1.25$. We can generate a new point on the line by starting at (2, 3) and moving 1 unit into the right and down (since $m$ is negative) $-1.25$ units to get the point $(2 + 1, 3 - 1.25) = (3, 1.75)$.

**EXAMPLE 3** **Given a general description**

A recent study reporting on the number of smokers showed:

**a.** A linear increase in Georgia
**b.** A linear decrease in Utah
**c.** A nonlinear decrease in Hawaii
**d.** A nonlinear increase in Oklahoma

Generate four rough sketches that could represent these situations.

**Solution** See Figure 2.34.

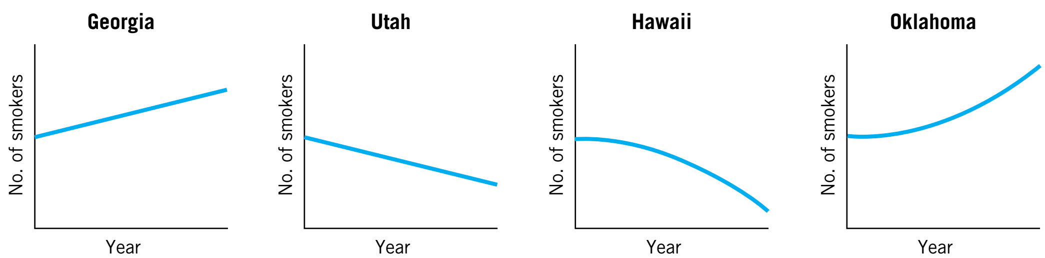



***Figure 2.34*** The change in the number of smokers over time in four states.

The program "L.3: Finding a Line Through 2 Points" in *Linear Functions* will give you practice in this skill.

## Finding the Equation

To determine the equation of any particular linear function $y = b + mx$, we only need to find the specific values for $m$ and $b$.

**EXAMPLE 4** **From a graph**

Find the equation of the linear function graphed in Figure 2.35.

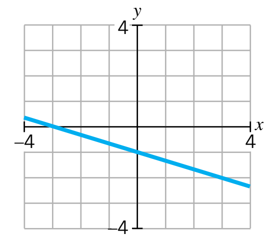



***Figure 2.35*** Graph of a linear function.

**Solution** We can use any two points on the graph to calculate $m$, the slope. If, for example, we take $(-3, 0)$ and $(3, -2)$, then

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{-2-0}{3-(-3)} = \frac{-2}{6} = \frac{-1}{3}$$

From the graph we can estimate the $y$-intercept as $-1$. So $b = -1$.

Hence the equation is $y = -1 - \frac{1}{3}x$.

**EXAMPLE 5** **From a verbal description**

A long-distance phone carrier charges a monthly base fee of \$5.00 plus \$0.07 per minute for long-distance calls. Construct an equation to model your monthly phone bill.

**Solution** In making the transition from words to an equation, it's important to first identify which is the independent and which the dependent variable. We usually think of the phone bill, $B$, as a function of the number of minutes you talk, $N$. If you haven't used any phone minutes, then $M = 0$ and your bill $B = \$5.00$. So \$5.00 is the vertical intercept. The number \$0.07 is the rate of change of the phone bill with respect to number of minutes talked. The rate of change is constant, making the relationship linear. So the slope is \$0.07/minute and the equation is

$$B = 5.00 + 0.07N$$

**EXAMPLE 6** The top speed a snowplow can travel on dry pavement is 40 miles per hour speed, which decreases by 0.8 miles per hour with each inch of snow on the highway.

**a.** Construct an equation describing the relationship between snowplow speed and snow depth.

**b.** Determine a reasonable domain and then graph the function.

**Solution** **a.** If we think of the snow depth, $D$, as determining the snowplow speed, $P$, then we need an equation of the form $P = b + mD$. If there is no snow, then the snowplow can travel at its maximum speed of 40 mph; that is, when $D = 0$, then $P = 40$. So the point (0, 40) lies on the line, making the vertical intercept $b = 40$. The rate of change, $m$, (change in snowplow speed)/(change in snow depth) $= -0.8$ mph per inch of snow. So the desired equation is

$$P = 40 - 0.8D$$

**b.** Consider the snowplow as only going forward (i.e. not backing up). Then the snowplow speed does not go below 0 mph. So if we let $P = 0$ and solve for $D$, we have

$$\begin{aligned} 0 &= 40 - 0.8D \\ 0.8D &= 40 \\ D &= 50 \end{aligned}$$

So when the snow depth reaches 50 inches, the plow is no longer able to move. A reasonable domain then would be $0 \leq D \leq 50$. (See Figure 2.36.)

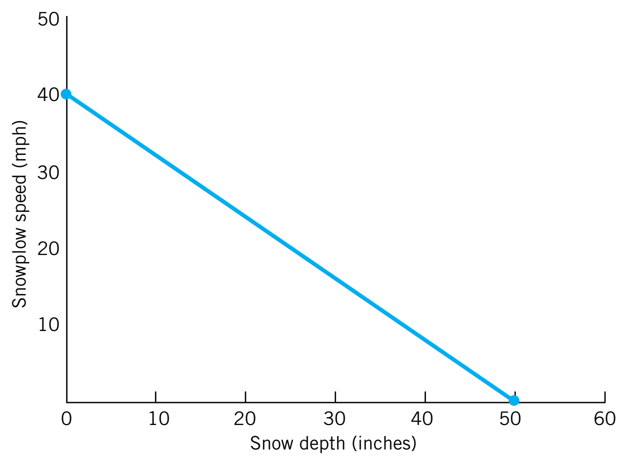



***Figure 2.36*** Snowplow speed versus snow depth.

**EXAMPLE 7** Pediatric growth charts suggest a linear relationship between age (in years) and median height (in inches) for children between 2 and 12 years. Two-year-olds have a median height of 35 inches, and 12-year-olds have a median height of 60 inches.

**a.** Generate the average rate of change of height with respect to age. (Be sure to include units.) Interpret your result in context.

**b.** Generate an equation to describe height as a function of age. What is an appropriate domain?

**c.** What would this model predict as the median height of 8-year-olds?

**Solution** **a.** Average rate of change $= \frac{\text{change in height}}{\text{change in age}} = \frac{60 - 35}{12 - 2} = \frac{25}{10} = 2.5$ inches/year

The chart suggests that, on average, children between the ages of 2 and 12 grow 2.5 inches each year.

**b.** If we think of height, $H$, depending on age, $A$, then we want an equation of the form $H = b + m \cdot A$. From part (a) we know $m = 2.5$, so our equation is $H = b + 2.5A$. To find $b$, we can substitute the values for any known point into the equation. When $A = 2$, then $H = 35$. Substituting in, we get

$$\begin{aligned} H &= b + m \cdot A \\ 35 &= b + (2.5 \cdot 2) \\ 35 &= b + 5 \\ 30 &= b \end{aligned}$$

So the final form of our equation is

$$H = 30 + 2.5A$$

where the domain is $2 \le A \le 12$.

**c.** When $A = 8$ years, our model predicts that the median height is $H = 30 + (2.5 \cdot 8) = 30 + 20 = 50$ inches.

## EXAMPLE 8 From a table

**a.** Determine if the data in Table 2.14 represent a linear relationship between values of blood alcohol concentration and number of drinks consumed for a 160-pound person. (One drink is defined as 5 oz of wine, 1.25 oz of 80-proof liquor, or 12 oz of beer.)

**b.** If the relationship is linear, determine the corresponding equation.

| $D$, Number of Drinks | $A$, Blood Alcohol Concentration |
|---|---|
| 2 | 0.047 |
| 4 | 0.094 |
| 6 | 0.141 |
| 10 | 0.235 |

**Table 2.14**

**Solution** **a.** We can generate a third column in the table that represents the average rate of change between consecutive points (see Table 2.15). Since the average rate of change of $A$ with respect to $D$ remains constant at 0.0235, these data represent a linear relationship.

| $D$ | $A$ | Average Rate of Change |
|---|---|---|
| 2 | 0.047 | n.a. |
| 4 | 0.094 | $\frac{0.094 - 0.047}{4 - 2} = \frac{0.047}{2} = 0.0235$ |
| 6 | 0.141 | $\frac{0.141 - 0.094}{6 - 4} = \frac{0.047}{2} = 0.0235$ |
| 10 | 0.235 | $\frac{0.235 - 0.141}{10 - 6} = \frac{0.094}{4} = 0.0235$ |

**Table 2.15**

**b.** The rate of change is the slope, so the corresponding linear equation will be of the form

$$A = b + 0.0235D \tag{1}$$

To find $b$, we can substitute any of the original paired values for $D$ and $A$, for example, (4, 0.094), into Equation (1) to get

$$\begin{aligned} 0.094 &= b + (0.0235 \cdot 4) \\ 0.094 &= b + 0.094 \\ 0.094 - 0.094 &= b \\ 0 &= b \end{aligned}$$

So the final equation is

$$A = 0 + 0.0235D$$

or just

$$A = 0.0235D$$

So when $D$, the number of drinks, is 0, $A$, the blood alcohol concentration, is 0, which makes sense.

## Algebra Aerobics 2.7

For Problems 1 and 2, find an equation, make an appropriate table, and sketch the graph of:

1. A line with slope 1.2 and vertical intercept $-4$.
2. A line with slope $-400$ and vertical intercept 300 (be sure to think about scales on both axes).
3. Write an equation for the line graphed in Figure 2.37.

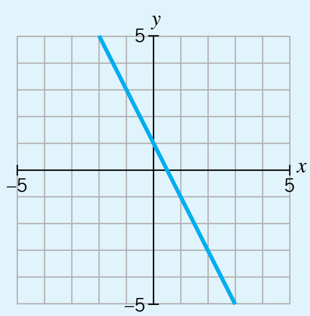



*Figure 2.37* Graph of a linear function.

4. **a.** Find an equation to represent the current salary after $x$ years of employment if the starting salary is \$12,000 with annual increases of \$3,000.
   **b.** Create a small table of values and sketch a graph.
5. **a.** Plot the following data.

| Years of Education | Hourly Wage |
|---|---|
| 8 | \$5.30 |
| 10 | \$8.50 |
| 13 | \$13.30 |

   **b.** Is the relationship between hourly wage and years of education linear? Why or why not?
   **c.** If it is linear, construct a linear equation to model it.
6. Select two points whose coordinates satisfy the equation $y = 6.2 + 3x$, and use these points to calculate the slope of the line.
7. Complete this statement regarding the graph of the line with equation $y = 6.2 + 3x$: Beginning with any point on the graph of the line, we could find another point by moving up ___ units for each unit that we move horizontally to the right.
8. Given the equation $y = 8 + 4x$, complete the following statements.
   **a.** Beginning with the vertical intercept, if we move one unit horizontally to the right, then we need to move up ___ units vertically to stay on the line and arrive at point (__, __).
   **b.** In general, if we move ten units horizontally to the right, then we must move up ___ units vertically to stay on the line.
   **c.** Beginning with the vertical intercept, if we move one unit horizontally to the left, then we need to move down ___ units vertically to stay on the line and arrive at point (__, __).
   **d.** In general, if we move one hundred units horizontally to the left, then we must move down ___ units vertically to stay on the line.
9. Given the equation $y = 4 + 5x$, describe how you could graph the line without creating a table by using its vertical intercept and its slope.

**10.** The relationship between the number of payments $P$ made and the balance $B$ (in dollars) of a \$10,800 car loan can be represented by Table 2.16.

**a.** Based on the table, develop a linear equation for the amount of the car loan balance $y$ as a function of the number of monthly payments $x$.

**b.** What is the monthly car payment?

**c.** What is the balance after twenty-four payments?

**d.** How many months are needed to produce a balance of zero?

| $P$, Number of Monthly Payments | $B$, Amount of Loan Balance (\$) |
|---|---|
| 0 | 10,800 |
| 1 | 10,500 |
| 2 | 10,200 |
| 3 | 9,900 |
| 4 | 9,600 |
| 5 | 9,300 |
| 6 | 9,000 |

**Table 2.16**

**11.** The relationship between the number of tickets purchased for a movie and the revenue generated from that movie is indicated in Table 2.17.

**a.** Based on this table construct a linear equation for the relationship between revenue, $R$, and the number of tickets purchased, $T$.

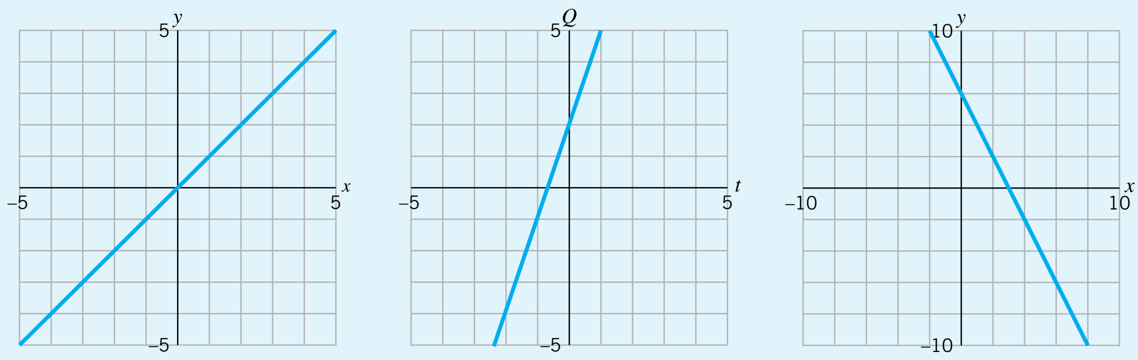
**b.** What is the cost per ticket?

**c.** Find the revenue generated by 120 ticket purchases.

| Number of Tickets Purchased, $T$ | Revenue, $R$ (\$) |
|---|---|
| 0 | 0 |
| 10 | 75 |
| 20 | 150 |
| 30 | 225 |
| 40 | 300 |

**Table 2.17**

**12.** Examine each graph below, identify two points on each line, determine the slope, then write an equation for the line.




(*a*) (*b*) (*c*)

## 2.8 *Special Cases*

### Direct Proportionality

The simplest relationship between two variables is when one variable is equal to a constant multiple of the other. For instance, in the previous example $A = 0.0235D$; blood alcohol concentration $A$ equals a constant, 0.0235, times $D$, the number of drinks. We say that *$A$ is directly proportional to $D$.*

#### *How to recognize direct proportionality*

Linear functions of the form

$$y = mx$$

describe a relationship where $y$ is directly proportional to $x$ (where $b = 0$). If two variables are directly proportional to each other, the graph will be a straight line that passes through the point (0, 0), the origin. Figure 2.38 shows the graphs of two relationships in which $y$ is directly proportional to $x$, namely $y = 2x$ and $y = -x$.

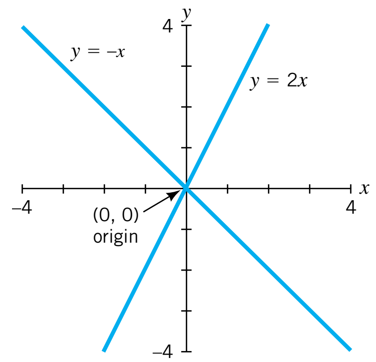



**Figure 2.38** Graphs of two relationships in which $y$ is directly proportional to $x$. Note that both graphs are lines that go through the origin.

**SOMETHING TO THINK ABOUT**

*If y is directly proportional to x, is x directly proportional to y?*

**Direct Proportionality**

In a linear equation of the form

$$y = mx \qquad m \neq 0$$

the relationship between $x$ and $y$ is described by saying

$y$ is *directly proportional to x*, or

*$y$ varies directly with $x$.*

**EXAMPLE 1**

Suppose you go on a road trip, driving at a constant speed of 60 miles per hour. Write an equation relating distance ($d$) and time traveled ($t$). Does it represent direct proportionality? What happens to $d$ if $t$ doubles? If $t$ triples?

**Solution**

If $d$ = distance (in miles) and $t$ = time traveled (in hours), then the average rate of change of distance with respect to time is 60 miles per hour. If we assume the initial distance is 0, then $d = 60t$. If your traveling speed is constant, then the distance you travel is directly proportional to the time you spend traveling. If $t$ doubles, $d$ doubles; if $t$ triples, $d$ triples.

**EXAMPLE 2**

You are traveling to Canada and need to exchange American dollars for Canadian dollars. On that day the exchange rate is 1 American dollar for 1.34 Canadian dollars.

**a.** Construct an equation converting American to Canadian dollars. Does it represent direct proportionality?

**b.** Suppose the Exchange Bureau charges a \$2 flat fee to change money. Alter your equation from part (a) to include the service fee. Does the new equation represent direct proportionality?

**Solution** **a.** If we let $A$ = American dollars and $C$ = Canadian dollars, then the equation

$$C = 1.34A$$

describes the conversion from American (the input) to Canadian (the output). The amount of Canadian money you receive is directly proportional to the amount of American money you exchange.

**b.** If there is a \$2 service fee, you would have to subtract \$2 from the American money you have before converting to Canadian. The new equation is

$$C = 1.34(A - 2)$$

or equivalently,

$$C = 1.34A - 2.68$$

where 2.68 is the service fee in Canadian dollars. Then $C$ is no longer directly proportional to $A$.

**EXAMPLE 3** In 1997 a prominent midwestern university decided to change its tuition cost. Previously there was a ceiling on tuition (which included fees). Currently the university uses what it calls a linear model, charging \$106 per credit hour for in-state students and \$369 per credit hour for out-of-state students.

**a.** Is it correct to call this pricing scheme a linear model for in-state students? For out-of-state students? Why?

**b.** Generate equations and graphs for the cost of tuition for both in-state and out-of-state students. If we limit costs to one semester during which the usual maximum credit hours is 15, what would be a reasonable domain?

**c.** In each case is the tuition directly proportional to the number of credit hours?

**Solution** **a.** Yes, both relationships are linear since the rate of change is constant in each case: \$106/credit hour for in-state students and \$369/credit hour for out-of-state students.

**b.** Let $N$ = number of credit hours, $C_i$ = cost for an in-state student, and $C_o$ = cost for an out-of-state student. In each case if the number of credit hours is zero $(N = 0)$, then the cost would be zero $(C_i = 0 = C_o)$. Hence both lines would pass through the origin (0, 0), making the vertical intercept 0 for both equations. So the results would be of the form

$$C_i = 106N \quad \text{and} \quad C_o = 369N$$

which are graphed in Figure 2.39. A reasonable domain would be $0 \le N \le 15$.

**c.** In both cases the tuition is directly proportional to the number of credit hours. The graphs verify this since both are lines going through the origin.

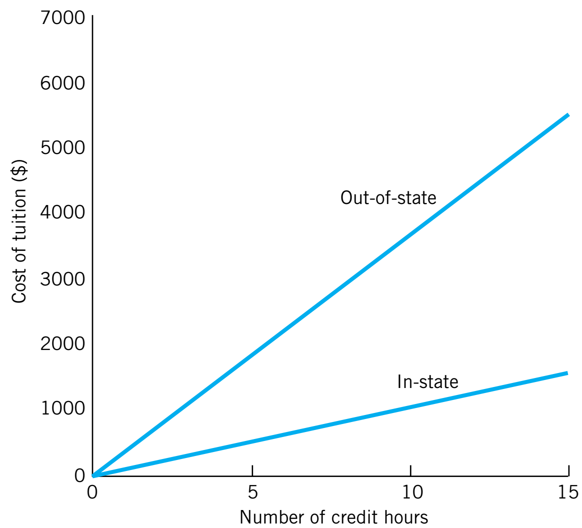



**Figure 2.39** Tuition for in-state and out-of-state students at a midwestern university.

## Algebra Aerobics 2.8a

1. Construct an equation and draw the graph of the line that passes through the origin and has the given slope.
   **a.** $m = -1$ **b.** $m = 0.5$
2. For each of Tables 2.18 and 2.19, determine whether the $x$ and $y$ are directly proportional to each other. Represent each relationship with an equation.

**a.**

| $x$ | $y$ |
|---|---|
| −2 | 6 |
| −1 | 3 |
| 0 | 0 |
| 1 | −3 |
| 2 | −6 |

**Table 2.18**

**b.**

| $x$ | $y$ |
|---|---|
| 0 | 5 |
| 1 | 8 |
| 2 | 11 |
| 3 | 14 |
| 4 | 17 |

**Table 2.19**

3. Recently, the exchange rate was \$1.00 U.S. to 0.86 euros.
   **a.** Find a linear function that converts U.S. dollars to euros.
   **b.** Find a linear function that converts U.S. dollars to euros with a service fee of \$2.50.
   **c.** Which function represents a directly proportional relationship and why?
4. The euro was worth 1.2 US dollars in May 2003. Find an equation that expresses this relationship. If you want to purchase a plane ticket from Paris to Miami that costs \$500 U.S., how many euros would you have to spend on the same ticket?
5. The total cost $C$ for football tickets is directly proportional to the number of tickets purchased $N$. If two tickets cost \$50, construct the formula relating $C$ and $N$. What would the total cost of ten tickets be?
6. Write a formula to describe each situation.
   **a.** $y$ is directly proportional to $x$, and $y$ is 4 when $x$ is 12.
   **b.** $d$ is directly proportional to $t$, and $d$ is 300 when $t$ is 50.

### Horizontal and Vertical Lines

The slope, $m$, of any horizontal line is 0. So the general form for the equation of a horizontal line is

$$y = b + 0x$$

or just

$$y = b$$

For example, Table 2.20 and Figure 2.40 show points that satisfy the equation of the horizontal line $y = 1$. If we calculate the slope between any two points in the table—for example, $(-2, 1)$ and $(2, 1)$—we get

$$\text{slope} = \frac{1 - 1}{-2 - 2} = \frac{0}{-4} = 0$$

| $x$ | $y$ |
|---|---|
| $-4$ | 1 |
| $-2$ | 1 |
| 0 | 1 |
| 2 | 1 |
| 4 | 1 |

**Table 2.20**

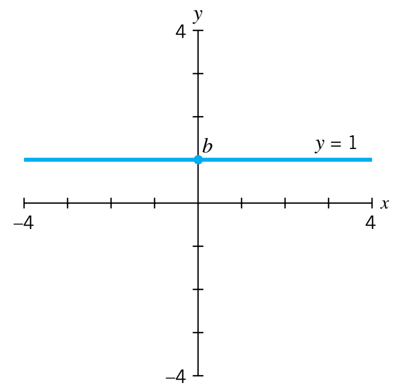



**Figure 2.40** Graph of the horizontal line $y = 1$.

For a vertical line the slope, $m$, is undefined, so we can't use the standard $y = b + mx$ format. The graph of a vertical line (as in Figure 2.41) fails the vertical line test, so $y$ is not a function of $x$. However, every point on a vertical line does have the same horizontal coordinate, which equals the coordinate of the horizontal intercept. Therefore, the general equation for a vertical line is of the form

$$x = c \quad \text{where } c \text{ is a constant (the horizontal intercept)}$$

For example, Table 2.21 and Figure 2.41 show points that satisfy the equation of the vertical line $x = 1$.

| $x$ | $y$ |
|---|---|
| 1 | $-4$ |
| 1 | $-2$ |
| 1 | 0 |
| 1 | 2 |
| 1 | 4 |

**Table 2.21**

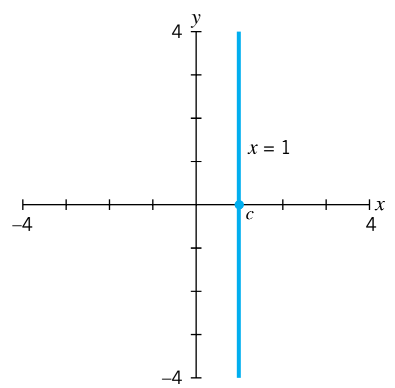



**Figure 2.41** Graph of the vertical line $x = 1$.

Kime2004_Ch02_p99_198.qxd 09/20/2004 10:17 PM Page 138



Note that if we tried to calculate the slope between two points, say $(1, -4)$ and $(1, 2)$, on the vertical line $x = 1$ we would get

$$\text{slope} = \frac{-4 - 2}{1 - 1} = \frac{-6}{0} \quad \text{which is undefined}$$

The general equation of a *horizontal line* is

$$y = b$$

where $b$ is a constant (the vertical intercept) and the slope is 0.

The general equation of a *vertical line* is

$$x = c$$

where $c$ is a constant (the horizontal intercept) and the slope is undefined.

**EXAMPLE 4** Find the equation for each line in Figure 2.42.

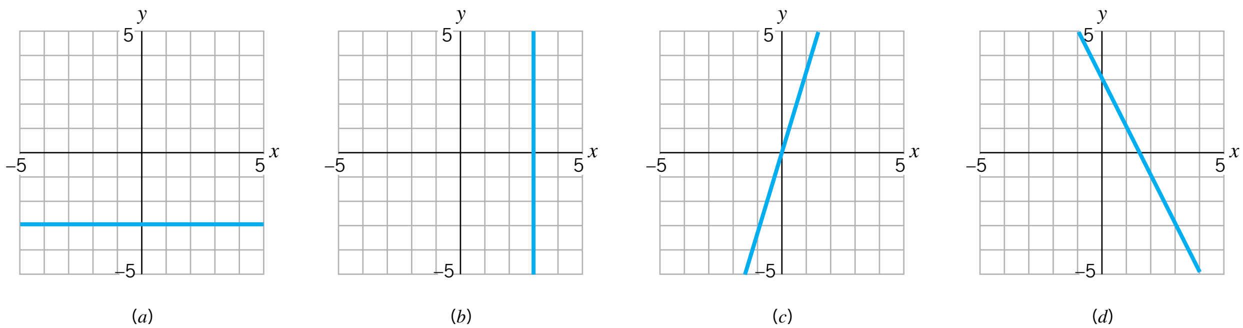



**Figure 2.42** Four linear graphs.

**Solution**
**a.** $y = -3$, a horizontal line
**b.** $x = 3$, a vertical line
**c.** $y = 3x$, a direct proportion, slope $= 3$
**d.** $y = -2x + 3$, a line with slope $-2$ and $y$-intercept 3

## Parallel and Perpendicular Lines

Parallel lines have the same slope. So if the two equations $y = b_1 + m_1x$ and $y = b_2 + m_2x$ describe two parallel lines, then $m_1 = m_2$. For example, the two lines $y = 2.0 - 0.5x$ and $y = -1.0 - 0.5x$ each have a slope of $-0.5$ and thus are parallel (see Figure 2.43).

**SOMETHING TO THINK ABOUT**

*Describe the equation for any line perpendicular to the horizontal line $y = b$.*

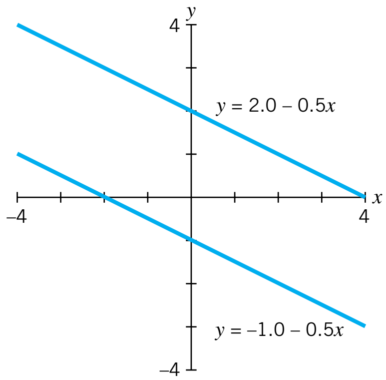



***Figure 2.43*** Two parallel lines have the same slope.

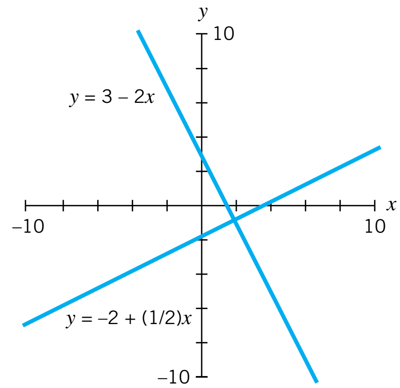



***Figure 2.44*** Two perpendicular lines have slopes that are negative reciprocals.

Two lines are perpendicular if their slopes are negative reciprocals. If $y = b_1 + m_1x$ and $y = b_2 + m_2x$ describe two perpendicular lines, then $m_1 = -1/m_2$. For example, in Figure 2.44 the two lines $y = 3 - 2x$ and $y = -2 + \frac{1}{2}x$ have slopes of $-2$ and $\frac{1}{2}$, respectively. Since $-2$ is the negative reciprocal of $\frac{1}{2}$ (i.e., $-\dfrac{1}{(\frac{1}{2})} = -1 \div \frac{1}{2} = -1 \cdot \frac{2}{1} = -2$), the two lines are perpendicular.

### *Why does this relationship hold for perpendicular lines?*

Consider a line whose slope is given by $v/h$. Now imagine rotating the line 90 degrees clockwise to generate a second line perpendicular to the first (Figure 2.45). What would the slope of this new line be?

The positive vertical change, $v$, becomes a positive horizontal change. The positive horizontal change, $h$, becomes a negative vertical change. The slope of the original line is $v/h$, and the slope of the line rotated 90 degrees clockwise is $-h/v$. Note that $-h/v = -1/(v/h)$, which is the original slope inverted and multiplied by $-1$.

In general, the slope of a perpendicular line is the negative reciprocal of the slope of the original line. If the slope of a line is $m_1$, then the slope, $m_2$, of a line perpendicular to it is $-1/m_1$.

This is true for any pair of perpendicular lines for which slopes exist. It does not work for horizontal and vertical lines since vertical lines have undefined slopes.

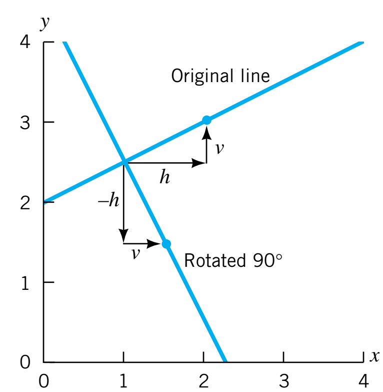



***Figure 2.45*** Perpendicular lines $m_2 = -1/m_1$.

Parallel lines have the same slope.

Perpendicular lines have slopes that are negative reciprocals of each other.

**EXAMPLE 5** Determine from the equations which pairs of lines are parallel, perpendicular, or neither.

**a.** $y = 2 + 7x$ and $y = 7x + 3$

**b.** $y = 6 - x$ and $y = 6 + x$

**c.** $y = 5 + 3x$ and $y = 5 - 3x$

**d.** $y = 3x + 13$ and $3y + x = 2$

**Solution**

**a.** The two lines are parallel since they share the same slope, 7.

**b.** The two lines are perpendicular since the negative reciprocal of $-1$ (the slope of first line) equals $-(1/(-1)) = -(-1) = 1$, the slope of the second line.

**c.** The lines are neither parallel nor perpendicular.

**d.** The lines are perpendicular. The slope of the first line is 3. If we solve the second equation for $y$, we get

$$\begin{aligned} 3y + x &= 2 \\ 3y &= 2 - x \\ y &= 2/3 - (1/3)x \end{aligned}$$

So the slope of the second line is $-(1/3)$, the negative reciprocal of 3.

## Algebra Aerobics 2.8b

1. In each case, find an equation for the horizontal line that passes through the given point.
   **a.** $(3, -5)$ **b.** $(5, -3)$ **c.** $(-3, 5)$ **d.** $(-5, 3)$

2. In each case, find an equation for the vertical line that passes through the given point.
   **a.** $(3, -5)$ **b.** $(5, -3)$ **c.** $(-3, 5)$ **d.** $(-5, 3)$

3. Construct the equation of the line that passes through the points.
   **a.** $(0, -7)$, $(3, -7)$, $(-1, -7)$, and $(350, -7)$
   **b.** $(-4.3, 0)$, $(-4.3, 8)$, $(-4.3, -1000)$, and $(-4.3, 280)$

4. Find the equation of the line that is parallel to $y = 4 - x$ and that passes through the origin.

5. Find the equation of the line that is parallel to $W = 360C + 2500$ and passes through the point where $C = 4$ and $W = 1000$.

6. Find the slope of a line perpendicular to each of the following.
   **a.** $y = 4 - 3x$
   **b.** $y = x$
   **c.** $y = 3.1x - 5.8$
   **d.** $y = -\frac{3}{5}x + 1$

7. **a.** Find an equation for the line that is perpendicular to $y = 2x - 4$ and passes through $(3, -5)$.
   **b.** Find the equations of two other lines that are perpendicular to $y = 2x - 4$ but do not pass through the point $(3, -5)$.
   **c.** How do the three lines from parts (a) and (b) that are perpendicular to $y = 2x - 4$ relate to each other?
   **d.** If technology is available, check your answers by graphing the equations.

8. Find the slope of the line $Ax + By = C$ assuming that $y$ is a function of $x$. (*Hint:* Solve the equation for $y$.)

9. Use the result of the previous exercise to determine the slope of each line described by the following linear equations (again assuming $y$ is a function of $x$).
   **a.** $2x + 3y = 5$ **d.** $x = -5$
   **b.** $3x - 4y = 12$ **e.** $x - 3y = 5$
   **c.** $2x - y = 4$ **f.** $y = 4$

10. Solve the equation $2x + 3y = 5$ for $y$, identify the slope, then find an equation for the line that is parallel to the line $2x + 3y = 5$ and passes through the point $(0, 4)$.

11. Solve the equation $3x + 4y = -7$ for $y$, identify the slope, then find an equation for the line that is perpendicular to the line of $3x + 4y = -7$ and passes through $(0, 3)$.

12. Solve the equation $4x - y = 6$ for $y$, identify the slope, then find an equation for the line that is perpendicular to the line to $4x - y = 6$ and passes through $(2, -3)$.

## 2.9 *Constructing Linear Models for Data*

According to Edward Tufte in *Data Analysis of Politics and Policy,* "Fitting lines to relationships is the major tool of data analysis." Of course, when we work with actual data searching for an underlying linear relationship, the data points will rarely fall exactly in a straight line. However, we can model the trends in the data with a linear equation.

Linear relationships are of particular importance, not because most relationships are linear, but because straight lines are easily drawn and analyzed. A human can fit a straight line by eye to a scatter plot almost as well as a computer. This paramount convenience of linear equations as well as their relative ease of manipulation and interpretation means that lines are often used as first approximations to patterns in data.

### Fitting a Line to Data: The Kalama Study

Children's heights were measured monthly over several years as a part of a study of nutrition in developing countries. Table 2.22 and Figure 2.46 show data collected on the mean heights of 161 children in Kalama, Egypt.

**Mean Heights of Kalama Children**

| Age (months) | Height (cm) |
|---|---|
| 18 | 76.1 |
| 19 | 77.0 |
| 20 | 78.1 |
| 21 | 78.2 |
| 22 | 78.8 |
| 23 | 79.7 |
| 24 | 79.9 |
| 25 | 81.1 |
| 26 | 81.2 |
| 27 | 81.8 |
| 28 | 82.8 |
| 29 | 83.5 |

***Table 2.22***
*Source:* D. S. Moore and G. P. McCabe, *Introduction to the Practice of Statistics.* Copyright © 1989 by W.H. Freeman and Company. Used with permission.

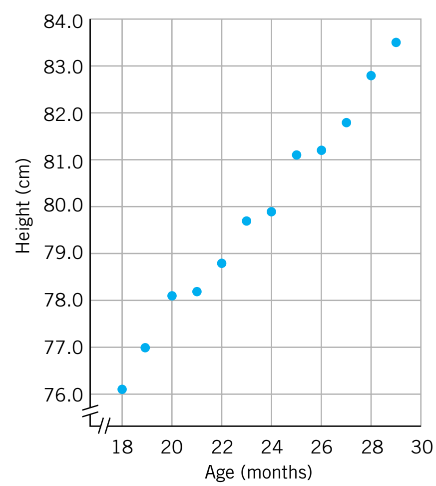



***Figure 2.46*** Mean heights of children in Kalama, Egypt.

#### *Sketching a line through the data*

Although the data points do not lie exactly on a straight line, the overall pattern seems clearly linear. Rather than generating a line through two of the data points, try eyeballing a line that approximates all the points. A ruler or a piece of black thread laid down through the dots will give you a pretty accurate fit.

In the Extended Exploration on education and earnings following this chapter, we will use technology to find a "best-fit" line. Figure 2.47 shows a line sketched that approximates the data points. This line does not necessarily pass through any of the original points.

### Finding the slope

Estimating the coordinates of two points *on the line,* say (20, 77.5) and (26, 81.5), we can calculate the slope, $m$, or rate of change, as

$$\begin{aligned} m &= \frac{(81.5 - 77.5)\text{ cm}}{(26 - 20)\text{ months}} \\ &= \frac{4.0\text{ cm}}{6\text{ months}} \\ &\approx 0.67\text{ cm/month} \end{aligned}$$

So our model predicts that for each additional month an "average" Kalama child will grow about 0.67 centimeter.

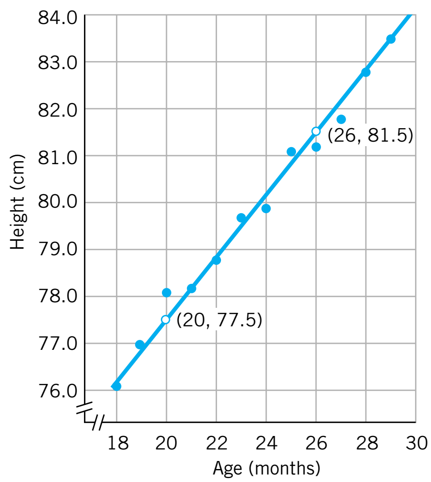



**Figure 2.47** Estimated coordinates of two points on the line.

### Constructing the equation

Since the slope of our linear model is 0.67 cm/month, then our equation is of the form

$$H = b + 0.67A \tag{1}$$

where $A$ = age in months and $H$ = mean height in centimeters.

How can we find $b$, the vertical intercept? We have to resist the temptation to estimate $b$ directly from the graph. As is frequently the case in social science graphs, both the horizontal and the vertical axes are cropped. Because the horizontal axis is cropped, we can't read the vertical intercept off the graph. We'll have to calculate it.

Since the line passes through (20, 77.5) we can

| | |
|---|---|
| substitute (20, 77.5) in Equation (1) | $77.5 = b + (0.67)(20)$ |
| simplify | $77.5 = b + 13.4$ |
| solve for $b$ | $b = 64.1$ |

Having found $b$, we complete the linear model.

$$H = 64.1 + 0.67A$$

where $A$ = age in months and $H$ = height in centimeters. It offers a compact summary of the data.

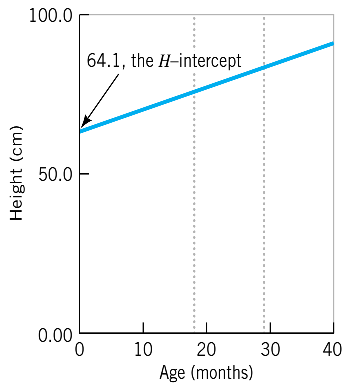



**Figure 2.48** Graph of $H = 64.1 + 0.67A$ that includes the origin (0, 0). Dotted lines show the region that models the Kalama data.

What is the domain of this model? In other words, for what inputs does our model apply? The data were collected on children age 18 to 29 months. We don't know its predictive value outside these ages, so

the domain consists of all values of $A$ for which $18 \le A \le 29$

### *The vertical intercept may not be in the domain*

Although the $H$-intercept is necessary to write the equation for the line, it lies outside of the domain.

Compare Figure 2.47 with Figure 2.48. They both show graphs of the same equation $H = 64.1 + 0.67A$. In Figure 2.47 both axes are cropped, while Figure 2.48 includes the origin (0, 0). In Figure 2.48 the vertical intercept is now visible, and the dotted lines bound the region that applies to our model. So a word of warning when reading graphs: Always look carefully to see if the axes have been cropped.

## Reinitializing the Independent Variable

When we model real data, it often makes sense to reinitialize the independent variable in order to have a reasonable vertical intercept. This is especially true for time series, as shown in the following example, where the independent variable is the year.

**EXAMPLE 3** **Time series**

How can we find an equation that models the trend in smoking in the United States?

**Solution** The U.S. Bureau of the Census web site provided the data reproduced in Table 2.23 and graphed in Figure 2.49. Although in some states smoking increased, the overall trend is a steady decline in the percentage of adult smokers in the United States between 1965 and 2000.

| Year | Percentage of Adults Who Smoke |
|---|---|
| 1965 | 41.9 |
| 1974 | 37.0 |
| 1979 | 33.3 |
| 1983 | 31.9 |
| 1985 | 29.9 |
| 1990 | 25.3 |
| 1995 | 24.6 |
| 2000 | 23.1 |

**Table 2.23**

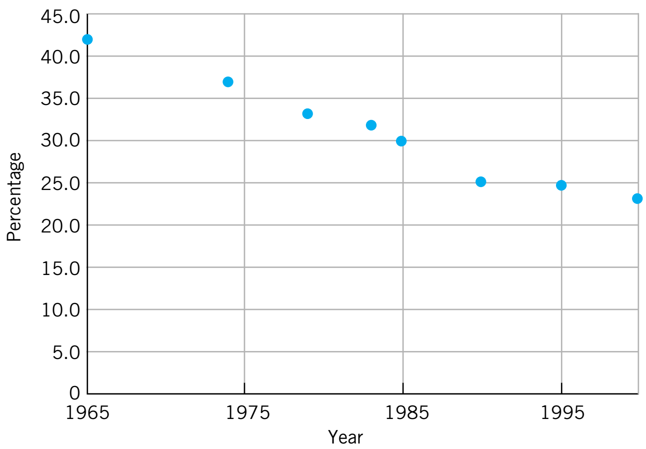



**Figure 2.49** Percentage of adults who smoke.

The relationship appears fairly linear. So the equation of a best-fit line could provide a fairly accurate description of the data. Since the horizontal axis is cropped, starting at the year 1965, the real vertical intercept would occur 1965 units to the left, at 0 A.D.! If you drew a big enough graph, you'd find that the vertical intercept would occur at approximately (0, 1220). This nonsensical extension of the model outside its known values would say that in 0 A.D., 1220% of the adult population smoked. A better strategy would be to define the independent variable as the number of years *since* 1965. Table 2.24 shows the reinitialized values for the independent variable, and Figure 2.50 gives a sketched-in best-fit line.

| Year | Number of Years Since 1965 | Percentage of Adults Who Smoke |
|---|---|---|
| 1965 | 0 | 41.9 |
| 1974 | 9 | 37.0 |
| 1979 | 14 | 33.3 |
| 1983 | 18 | 31.9 |
| 1985 | 20 | 29.9 |
| 1990 | 25 | 25.3 |
| 1995 | 30 | 24.6 |
| 2000 | 35 | 23.1 |

**Table 2.24**

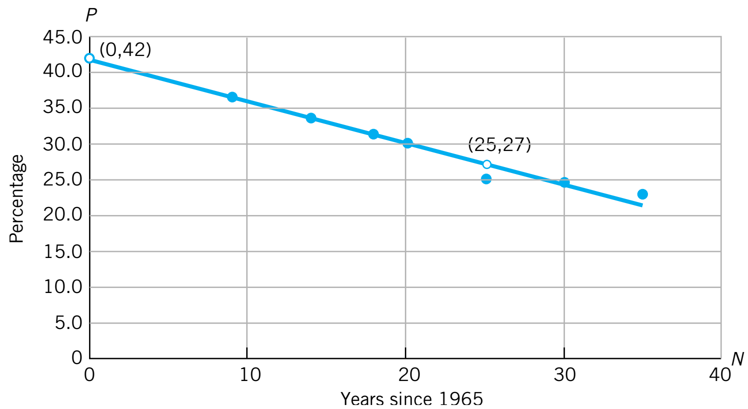



**Figure 2.50** Percentage of adult smokers since 1965 with estimated best-fit line.

We can estimate the coordinates of two points, (0, 42) and (25, 27), on our best-fit line. Using them, we have

$$\text{Vertical intercept} = 42 \quad \text{and} \quad \text{the slope} = \frac{42 - 27}{0 - 25} = \frac{15}{-25} = -0.6$$

If we let $N$ = the number of years since 1965 and $P$ = percentage of adult smokers, then the equation for our best-fit line is

$$P = 42 - 0.6N$$

where the domain is $0 \leq N \leq 35$ (see Figure 2.50). This model says that, starting in 1965, when about 42% of U.S. adults smoked, the percentage of the adult smokers has declined on average by 0.6 percentage points a year for 35 years.

What this model doesn't tell us is that (according to the U.S. Bureau of the Census) the total number of smokers during this time has remained fairly constant, at 50 million.

## Interpolation and Extrapolation: Making Predictions

We can use this linear model on smokers to make predictions. We can *interpolate* or estimate new values between known ones. For example, in our smoking example we have no data for the year 1970. Using our equation we can estimate that in 1970 (when $N = 5$), $P = 42 - 0.6 \cdot 5 = 39\%$ of adults smoked. Like any other point on the best-fit line, this prediction is only an estimate and may, of course, be different from the actual percentage of smokers (see Figure 2.51).

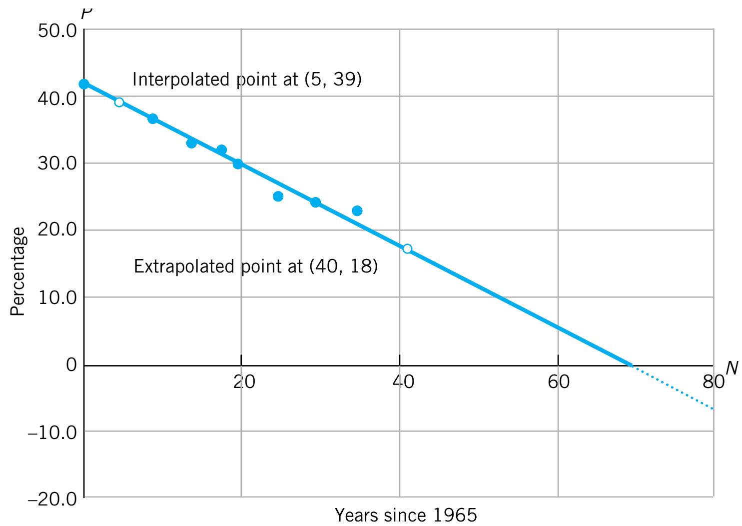



**Figure 2.51** Interpolation and extrapolation of percentage of smokers.

We can also use our model to *extrapolate* or to predict beyond known values. For example, our model predicts that in 2005 (when $N = 40$) $P = 42 - 0.6 \cdot 40 = 18\%$ of adults will smoke. Extrapolation much beyond known values is risky. For 2035 (where $N = 70$) our model predicts that 0% will smoke, which seems unlikely. After 2035 our model would give the impossible answer that a negative percentage of adults will smoke.

## Algebra Aerobics 2.9

**1.** Figure 2.52 shows the number of students in the United States who were 35 or older between the years of 1980 and 1998. The data appear roughly linear.

**a.** Sketch a line through the data that best approximates their pattern.

**b.** Estimate the coordinates of two points on your line and use them to find the slope.

**c.** If $x$ = number of years *since* 1980 and $y$ = number (in thousands) of U.S. students 35 or older, what would the coordinates of your two points in part (b) be in terms of $x$ and $y$?

**d.** Construct a linear equation using the $x$ and $y$ defined in part (c).

**e.** What does your model tell you about older students in the United States?

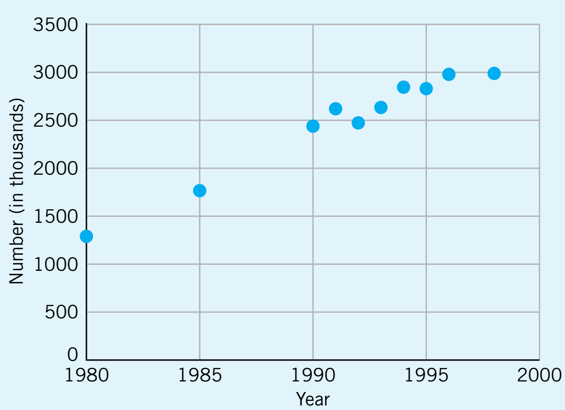



**Figure 2.52** U.S. population 35 years old and above enrolled in school.
*Source:* U.S. National Center for Education Statistics, *Digest of Education Statistics,* annual.

**2.** Figure 2.53 illustrates fall enrollment in Mathematics for Liberal Arts courses at a community college in Florida. The plotted data points (1998, 185), keep coordinates together (1999, 173), (2000, 178) (2001, 164), and (2002, 152) are of the form (year, enrollment).

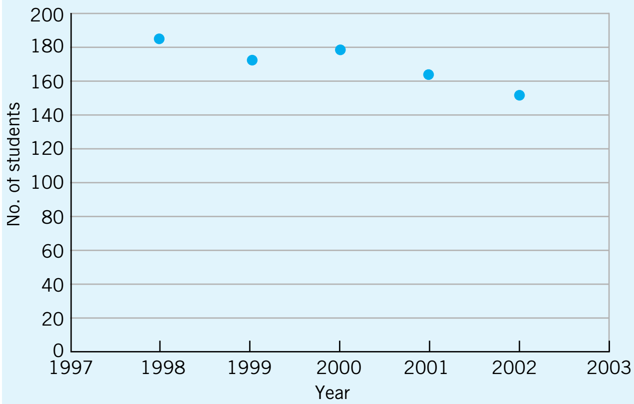



***Figure 2.53*** Enrollment in Liberal Arts Math.

**a.** Sketch a best-fit line through the data. (This line need not pass through any of the five data points.)

**b.** Using 0 to represent year 1997, 1 to represent 1998, and so on, write the coordinates of any two points that lie on the line that you drew. Use these coordinates to find the slope of the line. What does this tell you about enrollment in this course?

**c.** Give the approximate vertical intercept of the line that you drew (using the reinitialized value for year). What does this suggest about enrollments?

**d.** Using your values for $m$ and $b$, write an equation for your line.

**e.** Use your equation to estimate the enrollment in Liberal Arts Math in the fall of 2003 and in the fall of 2004.

**f.** Use your equation to estimate the year in which the enrollment would have been 215 students.

# CHAPTER SUMMARY

## The Average Rate of Change

The average rate of change of $y$ with respect to $x = \dfrac{\text{change in } y}{\text{change in } x}$.

$$\text{The units of the average rate of change} = \frac{\text{units of } y}{\text{units of } x}$$

For example, the units might be dollars/year (read as "dollars per year") or pounds/person (read as "pounds per person").

### The Average Rate of Change Is a Slope

The average rate of change between two points is the slope of the straight line connecting the points. Given two points $(x_1, y_1)$ and $(x_2, y_2)$,

$$\text{average rate of change} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \text{slope}$$

If the slope, or average rate of change, of $y$ with respect to $x$ is *positive,* then the graph of the relationship rises when read from left to right. This means that as $x$ increases in value, $y$ increases in value. If the slope is *negative,* the graph falls when read from left to right. As $x$ increases, $y$ decreases. If the slope is *zero,* the graph is flat. As $x$ increases, there is no change in $y$.

### Linear Functions and Their Graphs

A *linear function* has a constant average rate of change. It can be described by an equation of the form

$$\underbrace{\text{Output}}_{y} = \underbrace{\text{is initial value}}_{b} + \underbrace{\text{rate of change}}_{m} \cdot \underbrace{\text{input}}_{x}$$

or

$$y = b + mx$$

where $b$ is the vertical intercept and $m$ is the slope, or rate of change of $y$ with respect to $x$.

### The Graph of a Linear Function

The graph of the linear function $y = b + mx$ is a straight line.

The $y$-intercept, $b$, tells us where the line crosses the $y$-axis.

The slope, $m$, tells us how fast the line is climbing or falling. The larger the magnitude (or absolute value) of $m$, the steeper the graph.

If the slope, $m$, is positive, then the line climbs from left to right. If $m$ is negative, the line falls from left to right.

### Special Cases of Linear Functions

*Direct proportionality:* $y$ is *directly proportional to (or varies directly with)* $x$ if

$$y = mx \quad \text{where the constant } m \neq 0$$

This equation represents a linear function in which the $y$-intercept is 0, so the graph passes through (0, 0), the origin.

*Horizontal line:* A line of the form $y = b$, with slope 0.

*Vertical line:* A line of the form $x = c$, with slope undefined.

*Parallel lines:* Two lines that have the same slope.

*Perpendicular lines:* Two lines whose slopes are negative reciprocals.

### Fitting Lines to Data

We can visually position a line to fit data whose graph exhibits a linear pattern. The equation of a best-fit line offers an approximate but compact description of the data. Lines are of special importance in describing patterns in data because they are easily drawn and manipulated and give a quick first approximation of trends.

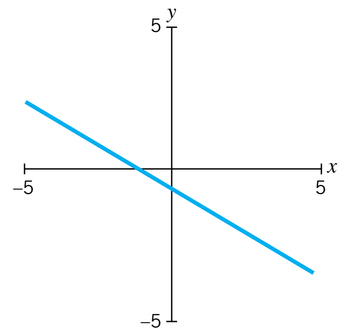
## CHECK YOUR UNDERSTANDING

**I.** Is each of the statements in Problems 1–30 true or false? Give an explanation for your answer.

In Problems 1–4 assume that $y$ is a function of $x$.

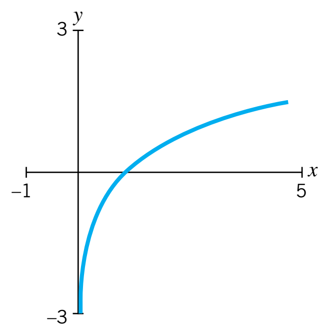
1. The graph of $y - 5x = 5$ is decreasing.
2. The graph of $2x + 3y = -12$ has a negative slope and negative vertical intercept.
3. The graph of $x - 3y + 9 = 0$ is steeper than the graph of $3x - y + 9 = 0$.

4. The accompanying figure is the graph of $5x - 3y = -2$.




5. Health care costs between the years 1990 and 2004 would most likely have a positive average rate of change.

6. If we choose any two points on the graph in the accompanying figure, the average rate of change between them would be positive.




7. The average rate of change between two points $(t_1, Q_1)$ and $(t_2, Q_2)$ is the same as the slope of the line joining these two points.

8. The average rate of change of a variable $M$ between the years 1990 and 2000 is the slope of the line joining two points of the form $(M_1, 1990)$ and $(M_2, 2000)$.

9. To calculate the average rate of change of a variable over an interval, you must have two distinct data points.

10. A set of data points of the form $(x, y)$ that do not fall on a straight line will generate varying average rates of change depending on the choice of endpoints.

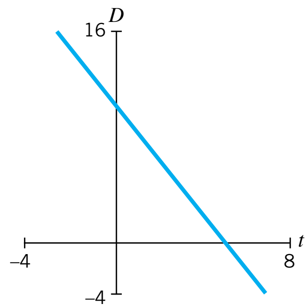
11. If the average rate of change of women's salaries from 2000 to 2003 is \$1000/year, then women's salaries increased by exactly \$1000 from 2000 to 2001.

12. If the average rate of change is positive, the acceleration (or rate of change of the average rate of change) may be positive, negative, or zero.

13. If the average rate of change is constant, then the acceleration (or rate of change of the average rate of change) is zero.

14. The average rate of change between $(W_1, D_1)$ and $(W_2, D_2)$ can be written as either $(W_1 - W_2)/(D_1 - D_2)$ or $(W_2 - W_1)/(D_2 - D_1)$.

**15.** If we choose any two points on the line in the accompanying figure, the average rate of change between them would be the same negative number.




**16.** The average rate of change between $(t_1, Q_1)$ and $(t_2, Q_2)$ can be written as either $(Q_1 - Q_2)/(t_1 - t_2)$ or $(Q_2 - Q_1)/(t_2 - t_1)$.

**17.** On a linear graph, it does not matter which two distinct points on the line you use to calculate the slope.

**18.** If the distance a sprinter runs (measured in meters) is a function of the time (measured in minutes), then the units of the average rate of change are minutes per meter.

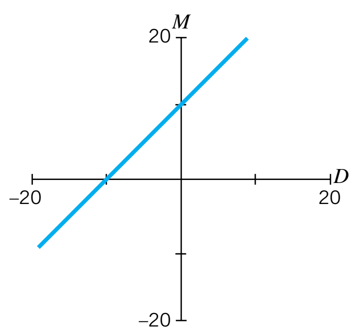
**19.** Every linear function crosses the horizontal axis exactly one time.

**20.** If a linear function in the form $y = mx + b$ has slope $m$, then increasing the $x$-value by one unit changes the $y$ value by $m$ units.

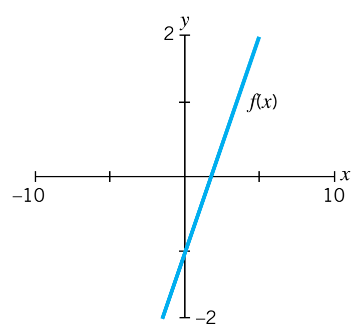
**21.** If the units of the dependent variable $y$ are pounds and the units of the independent variable $x$ are square feet, then the units of the slope are pounds per square feet.

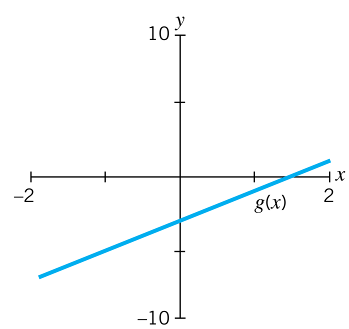
**22.** If the average rate of change between any two data points is increasing as you move from left to right, then the function describing the data is linear and is increasing.

**23.** The function in the accompanying figure has a slope that is increasing as you move from left to right.




**24.** The slope of the function $f(x)$ is of greater magnitude than the slope of the function $g(x)$ in the accompanying figures.



**25.** You can calculate a slope of a line through any two points on the plane.

**26.** If $f(x) = y$ is decreasing throughout then the $y$ values decrease as the $x$ values decrease.

**27.** Having a slope of zero is the same as having an undefined slope.

**28.** Vertical lines are linear functions.

**29.** Two nonvertical lines that are perpendicular must have slopes of opposite sign.

**30.** All linear functions can be written in the form $y = b + mx$.

**II.** In Problems 31–40, give an example of a function or functions with the specified properties. Express your answer using equations, and specify the independent and dependent variables.

**31.** Linear and decreasing with positive vertical intercept

**32.** Linear and horizontal with vertical intercept 0

**33.** Linear and with positive horizontal intercept and negative vertical intercept

**34.** Linear and does not pass through the first quadrant (where $x > 0$ and $y > 0$)

**35.** Linear with average rate of change of 37 minutes/lap

**36.** Linear describing the value of a stock that is currently at \$19.25 per share and is increasing exactly \$0.25 per quarter

**37.** Two linear functions that are parallel, such that moving one of the functions horizontally to the right two units gives the graph of the other

**38.** Four distinct linear functions all passing through the point (0, 4)

**39.** Two linear functions where the slope of one is $m$ and the slope of the second is $-1/m$, where $m$ is a negative number

**40.** Five data points for which the average rates of change between consecutive points are positive and are increasing at a decreasing rate

**III.** Is each of the statements in Problems 41–50 true or false? If a statement is true, explain how you know. If a statement is false, give a counterexample.

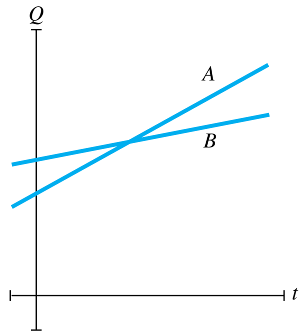
**41.** If the average rate of change between any two points of a data set is constant, then the data are linear.

**42.** If the slope of a linear function is negative, then the average rate of change decreases.

**43.** For any two distinct points, there is a linear function whose graph passes through them.

**44.** To write the equation of a specific linear function, one needs to know only the slope.

**45.** Function $A$ in the accompanying figure is increasing at a faster rate than function $B$.




**46.** The graph of a linear function is a straight line.

**47.** Vertical lines are not linear functions because they cannot be written in the form $y = b + mx$, as they have undefined slope.

**48.** There exist linear functions that slant upward moving from left to right but have negative slope.

**49.** A constant average rate of change means that the slope of the graph of a function is zero.

**50.** All linear functions in $x$ and $y$ describe a relationship where $y$ is directly proportional to $x$.

# EXERCISES

## Exercises for Section 2.1

**1.** If $r$ is measured in inches, $s$ in pounds, and $t$ in minutes, identify the units for the following average rates of change:

**a.** $\dfrac{\text{change in } r}{\text{change in } s}$ **b.** $\dfrac{\text{change in } t}{\text{change in } r}$ **c.** $\dfrac{\text{change in } s}{\text{change in } r}$

**2.** Assume that $R$ is measured in dollars, $S$ in ounces, $T$ in dollars per ounce, and $V$ in ounces per dollars. Write a product of two of these terms whose resulting units will be in:

**a.** Dollars **b.** Ounces

**3.** Your car's gas tank is full and you take a trip. You travel 212 miles, then you fill your gas tank up again and it takes 10.8 gallons. If you designate your change in distance as 212 miles and your change in gallons as 10.8, what is the average rate of change of gasoline used, measured in miles per gallon?

**4.** The gas gauge on your car is broken, but you know that the car averages 22 miles per gallon. You fill your 15.5-gallon gas tank and tell your friend, "I can travel 300 miles before I need to fill up the tank again." Explain why this is true.

**5.** The consumption of margarine (in pounds per person) increased from 2.4 in 1940 to 9.4 in 1960. What was the annual average rate of change? (*Source: www.census.gov*)

**6.** The percentage of people who own homes in the United States has gone from 62.9% in 1965 to 68% in 2003. What is the annual average rate of change in percentage points per year?

**7.** The accompanying table indicates females' SAT scores from 1997 to 2003.

| Year | Average Female Verbal SAT | Average Female Math SAT |
|---|---|---|
| 1997 | 503 | 494 |
| 1998 | 502 | 496 |
| 1999 | 502 | 495 |
| 2000 | 504 | 498 |
| 2001 | 502 | 498 |
| 2002 | 502 | 500 |
| 2003 | 503 | 503 |

*Source: www.collegeboard.com*

Find the average rate of change:

**a.** In the math scores from 1997 to 2003

**b.** In the verbal scores from 1997 to 2003

**8. a.** In 1990, aerospace industry net profits in the United States were \$4.49 billion, whereas in 1992, the aerospace industry showed a net loss (negative profit) of \$1.84 billion. Find the average annual rate of change in net profits from 1990 to 1992.

**b.** In 2002, aerospace industry net profits were \$8.97 billion. Find the average rate of change in net profits:

**i.** From 1990 to 2002

**ii.** From 1992 to 2002

**9.** According to the U.S. Bureau of the Census, in elementary and secondary schools, in the academic year ending in 1985 there were about 630,000 computers being used for student instruction, or about 84.1 students per computer. In the academic year ending in 2001, there were about 12,200,000 computers being used, or about 4.4 students per computer. Find the average rate of change from 1985 to 2000 in:

**a.** The number of computers being used

**b.** The number of students per computer

**10.** Using the information in the accompanying table on college completion:

**a.** Plot the data, labeling both axes and the coordinates of the points.

**b.** Calculate the average rate of change in percentage points per year.

**c.** Write a topic sentence summarizing what you think is the central idea to be drawn from these data.

| Year | Percentage of Persons 25 Years Old and Over Completing 4 or More Years of College |
|---|---|
| 1940 | 4.6 |
| 2000 | 24.4 |

*Source:* U.S. Bureau of the Census, *Statistical Abstract of the United States: 2002.*

**11.** Though reliable data about the number of African elephants are hard to come by, it is estimated that there were about 4,000,000 in 1930 and only 500,000 in 2000.

**a.** What is the average annual rate of change in elephants over time? Interpret your result.

**b.** During the 1980s it was estimated that 100,000 elephants were being killed each year due to systematic poaching for ivory. How does this compare with your answer in part (a)? What does this tell you about what was happening before or after the 1980s? (*Source: www.panda.org*)

**12. a.** According to the U.S. Census Bureau, between 1998 and 2001 total motor vehicle sales increased from \$2,644,951 million to \$3,153,315 million. Calculate the annual average rate of change.

**b.** During the same period the on-line sales of motor vehicles increased from \$4985 million to \$34,595 million. Calculate the annual average rate of change.

**c.** What do the two rates suggest?

**13.** Use the information in the accompanying table to answer the following questions.

**Percentage of Persons 25 Years Old and Over Who Have Completed 4 Years of High School or More**

| | 1940 | 2000 |
|---|---|---|
| All | 24.5 | 80.4 |
| White | 26.1 | 84.9 |
| Black | 7.3 | 78.5 |
| Asian/Pacific Islander | 22.6 | 85.7 |

*Source:* U.S. Bureau of the Census, *Statistical Abstract of the United States: 2002.*

**a.** What was the average rate of change (in percentage points per year) of completion of 4 years of high school from 1940 to 2000 for whites? For blacks? For Asian/Pacific Islanders? For all?

**b.** If these rates continue, what percentages of whites, of blacks, of Asian/Pacific Islanders, and of all will have finished 4 years of high school in the year 2004? Check the Internet to see if your predictions are accurate.

**c.** Write a 60-second summary describing the key elements in the high school completion data. Include rates of change and possible projections for the near future.

**d.** If these rates continue, in what year will 100% of whites have completed 4 years of high school or more? In what year 100% of blacks? In what year 100% of Asian/Pacific Islanders? Do these projections make sense?

**14.** The accompanying data show U.S. consumption and exports of cigarettes.

| Year | U.S. Consumption | Exports |
|---|---|---|
| 1960 | 484,400,000,000 | 20,200,000,000 |
| 1970 | 536,400,000,000 | 29,200,000,000 |
| 1980 | 631,000,000,000 | 82,000,000,000 |
| 1990 | 525,000,000,000 | 164,300,000,000 |
| 1995 | 487,000,000,000 | 231,100,000,000 |
| 2000 | 430,000,000,000 | 148,300,000,000 |
| 2001 | 422,000,000,000 | 133,900,000,000 |
| 2002 | 420,000,000,000 | 127,200,000,000 |

*Source:* U.S. Department of Agriculture.

**a.** Calculate the average rates of change in U.S. cigarette consumption from 1960 to 1980, from 1980 to 2002, and from 1960 to 2002.

**b.** Compute the average rate of change for cigarette exports from 1960 to 2002. Does this give an accurate image of cigarette exports?

**c.** The total number of cigarettes consumed in the United States in 1960 is very close to the number consumed in 1995. Does that mean smoking was as popular in 1995 as it was in 1960? Explain your answer.

**d.** Write a paragraph summarizing what the data tell you about the consumption and exports of cigarettes since 1960, using average rates of change.

DATA

**15.** Use the accompanying table on life expectancy to answer the following questions. (See also Excel or graph link file LIFEXPEC.)

**Average Number of Years of Life Expectancy in the United States by Race and Sex Since 1900**

| Life Expectancy at Birth by Year | White Males | White Females | Black Males | Black Females |
|---|---|---|---|---|
| 1900 | 46.6 | 48.7 | 32.5 | 33.5 |
| 1950 | 66.5 | 72.2 | 58.9 | 62.7 |
| 1960 | 67.4 | 74.1 | 60.7 | 65.9 |
| 1970 | 68.0 | 75.6 | 60.0 | 68.3 |
| 1980 | 70.7 | 78.1 | 63.8 | 72.5 |
| 1990 | 72.7 | 79.4 | 64.5 | 73.6 |
| 2000 | 74.8 | 80.0 | 68.2 | 74.9 |

*Source:* U.S. National Center for Health Statistics, *Statistical Abstract of the United States: 1995, 2000*.

**a.** What group had the highest life expectancy in 1900? In 2000? What group had the lowest life expectancy in 1900? In 2000?

**b.** Which group had the largest average rate of change in life expectancy between 1900 and 2000?

**c.** Write a short summary of the patterns in U.S. life expectancy over the last century using average rates of change to support your points.

**16.** The tables given show data on household TVs and movies.

**Number and Percentage of TV Households with Cable or VCR**

| Year | Cable (000s) | % of TV Households with Cable | VCR (000s) | % of TV Households with VCRs |
|---|---|---|---|---|
| 1960 | — | — | — | — |
| 1970 | 3,900 | 6.7 | — | — |
| 1975 | 8,600 | 12.6 | — | — |
| 1980 | 15,200 | 19.9 | 840 | 1.1 |
| 1985 | 36,340 | 42.8 | 17,740 | 20.9 |
| 1990 | 51,900 | 56.4 | 63,180 | 68.6 |
| 1995 | 60,460 | 63.4 | 77,270 | 81.0 |
| 1996 | 62,580 | 65.3 | 78,830 | 82.2 |
| 1997 | 64,470 | 66.5 | 81,670 | 84.2 |
| 1998 | 65,810 | 67.2 | 82,910 | 84.6 |
| 1999 | 67,120 | 67.5 | 84,140 | 84.6 |
| 2000 | 68,550 | 68.0 | 85,810 | 85.1 |
| 2003 | 74,430 | 69.8 | 97,630 | 91.5 |

*Source:* Television Bureau of Advertising.

**Household TV Viewing Time**

| Year | Time Spent per Day |
|---|---|
| 1970 | 5 hr 56 min |
| 1975 | 6 hr 7 min |
| 1980 | 6 hr 36 min |
| 1985 | 7 hr 10 min |
| 1990 | 6 hr 53 min |
| 1995 | 7 hr 17 min |
| 2000 | 7 hr 35 min |
| 2001 | 7 hr 40 min |

*Source:* Nielsen Media Research, NTI Annual Averages, Television Bureau of Advertising.

**Movie Box Office Gross and Attendance**

| Year | Gross (millions) | Attendance (millions) |
|---|---|---|
| 1950 | $1,379 | 3,018 |
| 1960 | 984 | 1,305 |
| 1970 | 1,429 | 921 |
| 1980 | 2,749 | 1,022 |
| 1985 | 3,749 | 1,056 |
| 1990 | 5,022 | 1,189 |
| 1995 | 5,494 | 1,263 |
| 1996 | 5,912 | 1,339 |
| 1997 | 6,366 | 1,388 |
| 2000 | 7,661 | 1,421 |

*Source:* U.S. Bureau of the Census, *www.census.gov*

**a.** What is the average rate of change in annual percentage points between 1980 and 2003 for TV households that have cable? That have VCRs? Which of these appears to have the greatest potential for further growth? Why?

**b.** By how much did household TV viewing time increase between 1970 and 2001? What does the data about TV sets and viewing time tell you about the TV industry?

**c.** Another side of the screen entertainment industry is represented by the movie data given on box office gross and movie attendance. From 1950 to 1970 what was the average rate of change in movie attendance? What factors do you think caused this change?

**d.** **i.** What was the average rate of change in movie attendance from 1970 to 2000?

**ii.** If the U.S. population was 203.3 million in 1970 and 281.4 million in 2000, what was the movie attendance per person in the United States for each of those years?

**iii.** What was the average rate of change from 1970 to 2000 in movie attendance per person?

**e.** From the information given, does it appear that the range of entertainment available on domestic TV has affected the growth of big-screen entertainment at movie theaters? Write a paragraph on your conclusions.

**17.** In each of the following, assume that the average rate of change is for $y$ with respect to $x$.

**a.** Find the average rate of change if the change in $y$ is 1275 and the change in $x$ is 15.

**b.** If the average rate of change is 5 and the change in $y$ is 117, what is the change in $x$?

**c.** If the average rate of change is 125 and the change in $x$ is 35, find the change in $y$.

**d.** If the change in $y$ is 65 and the average rate of change is 2.5, find the change in $x$.

**18.** The accompanying table gives the number of unmarried males and females over 15 in the United States.

**Marital Status of Population 15 Years Old and Older**

| Year | Number of unmarried males (in thousands) | Number of unmarried females (in thousands) |
|---|---|---|
| 1950 | 17,735 | 19,525 |
| 1960 | 18,492 | 22,024 |
| 1970 | 23,450 | 29,618 |
| 1980 | 30,134 | 36,950 |
| 1990 | 36,121 | 43,040 |
| 2000 | 43,429 | 50,133 |
| 2002 | 45,551 | 52,537 |

*Source:* U.S. Bureau of the Census, *www.census.gov*

**a.** Calculate the average rate of change in the number of unmarried males between 1950 and 2002. Interpret your results.

**b.** Calculate the average rate of change in the number of unmarried females between 1950 and 2002. Interpret your results.

**c.** Compare the two results.

**d.** What does this tell you, if anything, about the *percentages* of unmarried males and females?

## Exercises for Section 2.2

**19.** Calculate the average rate of change between adjacent points for the following function. (The first few are done for you.)

**a.** Is the function $f(x)$ increasing, decreasing, or constant throughout?

**b.** Is the average rate of change increasing, decreasing, or constant throughout?

| $x$ | $f(x)$ | Average Rate of Change |
|---|---|---|
| 0 | 0 | n.a. |
| 1 | 1 | 1 |
| 2 | 8 | 7 |
| 3 | 27 | |
| 4 | 64 | |
| 5 | 125 | |

**20.** Calculate the average rate of change between adjacent points for the following function. The first one is done for you.

**a.** Is the function $f(x)$ increasing, decreasing, or constant throughout?

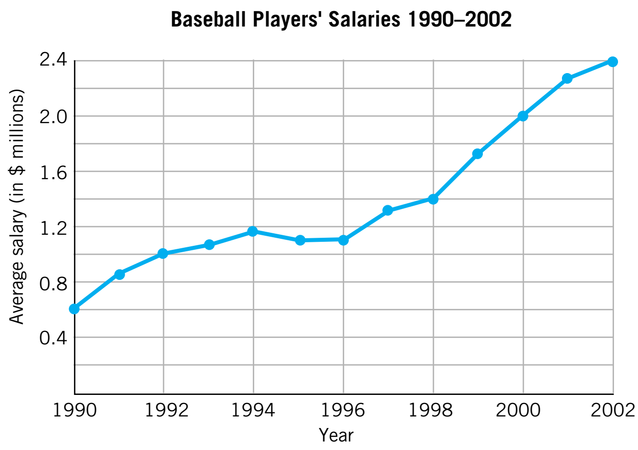
**b.** Is the average rate of change increasing, decreasing, or constant throughout?

| $x$ | $f(x)$ | Average Rate of Change |
|---|---|---|
| 0 | 0 | n.a. |
| 1 | 1 | 1 |
| 2 | 16 | |
| 3 | 81 | |
| 4 | 256 | |
| 5 | 625 | |

**21.** The accompanying table and graph illustrate the average annual salary of professional baseball players from 1990 to 2002.

| Year | Salary (millions) | Average Rate of Change over Prior Year |
|---|---|---|
| 1990 | \$0.59 | n.a. |
| 1991 | \$0.85 | |
| 1992 | \$1.01 | |
| 1993 | \$1.06 | |
| 1994 | \$1.15 | |
| 1995 | \$1.09 | |
| 1996 | \$1.10 | |
| 1997 | \$1.31 | |
| 1998 | \$1.38 | |
| 1999 | \$1.72 | |
| 2000 | \$1.98 | |
| 2001 | \$2.27 | |
| 2002 | \$2.38 | |

*Source: www.usatoday.com* and Major League Baseball.




**a.** Fill in the third column in the table.

**b.** During which 1-year interval was the average rate of change the smallest in absolute size?

**c.** During which 1-year interval was the average rate of change the largest?

**d.** Write a paragraph describing the change in professional baseball players' salaries between 1990 and 2002.

**22.** The accompanying table indicates the number of juvenile arrests (in thousands) in the United States for aggravated assault.

| Year | Juvenile Arrests (thousands) | Annual Average Rate of Change over Prior 5 Years |
|---|---|---|
| 1980 | 33.5 | n.a. |
| 1985 | 36.8 | |
| 1990 | 54.5 | |
| 1995 | 68.5 | |
| 2000 | 49.8 | |

**a.** Fill in the third column in the table by calculating the annual average rate of change.

**b.** Graph the annual average rate of change versus time.

**c.** During what 5-year period was the annual average rate of change the largest?

**d.** Describe the growth of aggravated assault cases during these years by referring both to the number and to the annual average rate of change.

**23.** Calculate the average rate of change between adjacent points for the following functions. (In each case the first one is done for you.)

**a.** Are the functions $f(x)$ and $g(x)$ increasing, decreasing, or constant throughout?

**b.** Is the average rate of change of each function increasing, decreasing, or constant throughout?

| $x$ | $f(x)$ | Average Rate of Change |
|---|---|---|
| 0 | 5 | n.a. |
| 10 | 25 | 2 |
| 20 | 45 | |
| 30 | 65 | |
| 40 | 85 | |
| 50 | 105 | |

| $x$ | $g(x)$ | Average Rate of Change |
|---|---|---|
| 0 | 270 | n.a. |
| 10 | 240 | $-3$ |
| 20 | 210 | |
| 30 | 180 | |
| 40 | 150 | |
| 50 | 120 | |

**24.** Calculate the average rate of change between adjacent points for each of the functions A–D. Then for each function decide which statement best describes it.

**a.** As $x$ increases, the function increases at a constant rate.

**b.** As $x$ increases, the function increases at an increasing rate.

**c.** As $x$ increases, the function decreases at a constant rate.

**d.** As $x$ increases, the function decreases at an increasing rate.

**A.**

| $x$ | $f(x)$ | Average Rate of Change |
|---|---|---|
| 0 | 1 | n.a. |
| 1 | 3 | |
| 2 | 9 | |
| 3 | 27 | |
| 4 | 81 | |
| 5 | 243 | |

**B.**

| $x$ | $g(x)$ | Average Rate of Change |
|---|---|---|
| 0 | 200 | n.a. |
| 15 | 155 | |
| 30 | 110 | |
| 45 | 65 | |
| 60 | 20 | |
| 75 | $-25$ | |

**C.**

| $x$ | $h(x)$ | Average Rate of Change |
|---|---|---|
| 0 | 50 | n.a. |
| 10 | 55 | |
| 20 | 60 | |
| 30 | 65 | |
| 40 | 70 | |
| 50 | 75 | |

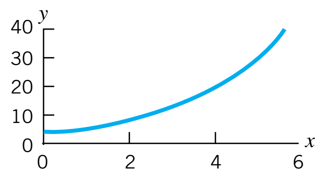
**D.**

| $x$ | $k(x)$ | Average Rate of Change |
|---|---|---|
| 0 | 40 | n.a. |
| 1 | 31 | |
| 2 | 24 | |
| 3 | 19 | |
| 4 | 16 | |
| 5 | 15 | |

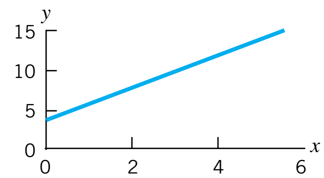
**25.** Each of the following functions has a graph that is increasing. If you calculated the average rate of change between sequential equal-size intervals, which function can be said to have an average rate of change that is:

**a.** Constant? **b.** Increasing? **c.** Decreasing?

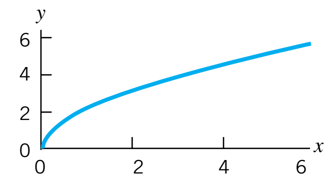
**A.**



**B.**



**C.**



**26.** Match the data table with its graph.

**A.**

| $x$ | $y$ |
|---|---|
| 0 | 2 |
| 1 | 5 |
| 2 | 8 |
| 3 | 11 |
| 4 | 14 |
| 5 | 17 |
| 6 | 20 |

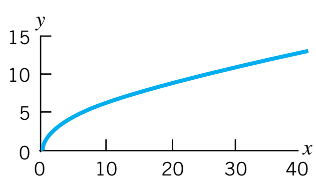
**B.**

| $x$ | $y$ |
|---|---|
| 0 | 0 |
| 1 | 0.5 |
| 2 | 2 |
| 3 | 4.5 |
| 4 | 8 |
| 5 | 12.5 |
| 6 | 18 |

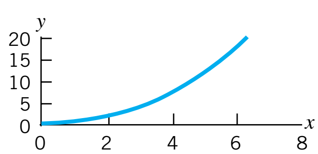
**C.**

| $x$ | $y$ |
|---|---|
| 0 | 0 |
| 1 | 2 |
| 4 | 4 |
| 9 | 6 |
| 16 | 8 |
| 25 | 10 |
| 36 | 12 |

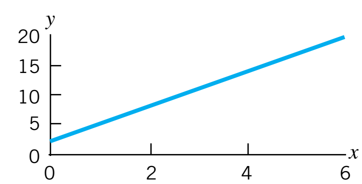
**D.**



**E.**



**F.**



**27.** Refer to the data tables in Exercise 26. Insert a third column in each table and label the column "average rate of change."

**a.** Calculate the average rate of change over adjacent data points.

**b.** Identify whether the table represents an average rate of change that is constant, increasing, or decreasing.

**c.** Explain how you could tell this by looking at the corresponding graph in Exercise 26.

**28.** Following are data on the U.S. population over the time period 1830–1930.

**U.S. Population**

| Year | Population (in millions) | Average Rate of Change (millions/yr) |
|---|---|---|
| 1830 | 12.9 | n.a. |
| 1850 | 23.2 | |
| 1870 | | 0.84 |
| 1890 | 63.0 | 0.147 |
| 1910 | 92.2 | |
| 1930 | | 1.55 |

*Source:* U.S. Bureau of the Census, *www.census.gov*

**a.** Fill in the missing parts of the chart.

**b.** Which 20-year interval experienced the largest average rate of change in population?

**c.** Which interval experienced the smallest average rate of change in population?

DATA

**29.** (Technology is recommended for parts (d) to (f).) The accompanying table and graph show the change in median age of the U.S. population from 1850 through the present and projected into the next century. (See also Excel or graph link file USMEDAGE.)

**a.** Using the given data, specify the longest time period over which the median age has been increasing. Calculate the average rate of change between the two end points of this time period.

**b.** Calculate the average rates of change between 1850 and 1900, 1900 and 1950, and 1950 and 2000, and the projected average rate of change between 2000 and 2050. Write a paragraph comparing these results.

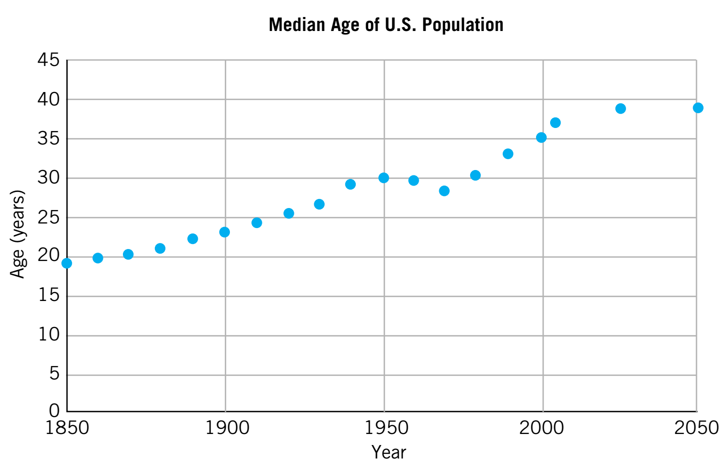
**c.** Does an increase in median age necessarily mean that more people are living longer? What changes in society could make this change in median age possible?

**Median Age of U.S. Population: 1850–2050***

| Year | Median Age (in years) | Year | Median Age (in years) |
|---|---|---|---|
| 1850 | 18.9 | 1950 | 30.2 |
| 1860 | 19.4 | 1960 | 29.5 |
| 1870 | 20.2 | 1970 | 28.0 |
| 1880 | 20.9 | 1980 | 30.0 |
| 1890 | 22.0 | 1990 | 32.8 |
| 1900 | 22.9 | 2000 | 35.3 |
| 1910 | 24.1 | 2005 | 36.7 |
| 1920 | 25.3 | 2025 | 38.5 |
| 1930 | 26.4 | 2050 | 38.8 |
| 1940 | 29.0 | | |

*Data for 2005–2050 are projected.

*Source:* U.S. Bureau of the Census, *Statistical Abstract of the United States: 1998, 2002*.




**d.** Starting in 1860 and ending in 2000, that calculate the annual average rate of change in median age over the previous decade.

**e.** Plot average rate of change versus years.

**f.** Identify periods with negative average rates of change and suggest reasons for the declining rates.

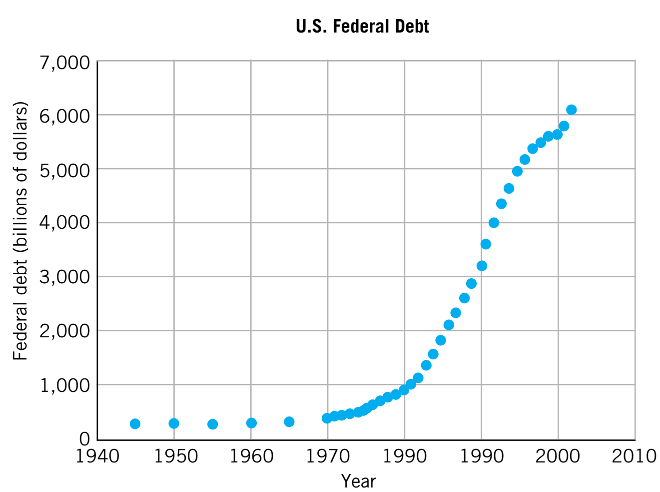
DATA FEDERAL

**30.** (Technology recommended) The accompanying table and graph are both representations of the accumulated gross debt of the federal government as a function of time. (See also Excel and graph link file FEDDEBT.)

**U.S. Federal Debt**

| Year | Debt (billions of dollars) | Year | Debt (billions of dollars) |
|---|---|---|---|
| 1945 | 260 | 1984 | 1565 |
| 1950 | 257 | 1985 | 1818 |
| 1955 | 274 | 1986 | 2121 |
| 1960 | 291 | 1987 | 2346 |
| 1965 | 322 | 1988 | 2601 |
| 1970 | 381 | 1989 | 2868 |
| 1971 | 408 | 1990 | 3207 |
| 1972 | 436 | 1991 | 3598 |
| 1973 | 466 | 1992 | 4002 |
| 1974 | 484 | 1993 | 4351 |
| 1975 | 542 | 1994 | 4644 |
| 1976 | 629 | 1995 | 4921 |
| 1977 | 706 | 1996 | 5182 |
| 1978 | 777 | 1997 | 5370 |
| 1979 | 829 | 1998 | 5479 |
| 1980 | 909 | 1999 | 5606 |
| 1981 | 995 | 2000 | 5629 |
| 1982 | 1137 | 2001 | 5770 |
| 1983 | 1372 | 2002 | 6137 (est.) |

*Source:* U.S. Bureau of the Census, *Statistical Abstract of the United States: 2002.*




**a.** What is the difference between the federal debt and the federal deficit?

**b.** Starting in 1950, calculate the average annual rate of change in the federal debt over the previous time interval.

**c.** Graph the average rates of change. Write a paragraph describing what your graph and the data show.

**DATA NEW PRINT**

**31.** (Technology recommended) The accompanying data and graphs give a picture of the two major methods of news communication in the United States. (See also Excel or graph link files NEWPRINT and ONAIRTV.)

**Number of U.S. Newspapers**

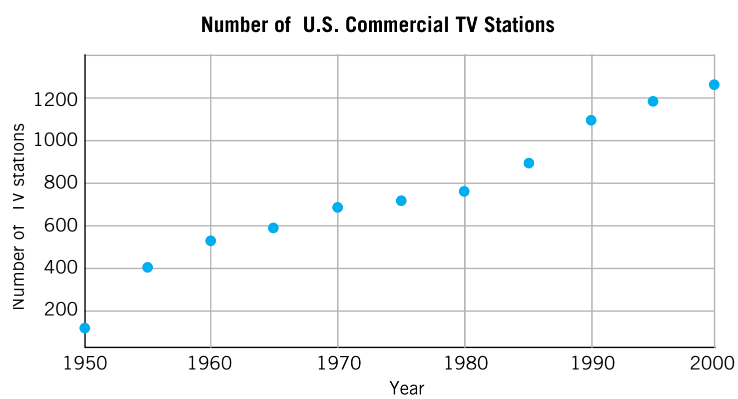
| Year | Newspapers (thousands of copies printed) | Number of Newspapers Published |
|---|---|---|
| 1915 | 28,777 | 2580 |
| 1920 | 27,791 | 2042 |
| 1925 | 33,739 | 2008 |
| 1930 | 39,589 | 1942 |
| 1935 | 38,156 | 1950 |
| 1940 | 41,132 | 1878 |
| 1945 | 48,384 | 1749 |
| 1950 | 53,829 | 1772 |
| 1955 | 56,147 | 1760 |
| 1960 | 58,882 | 1763 |
| 1965 | 60,358 | 1751 |
| 1970 | 62,108 | 1748 |
| 1975 | 60,655 | 1756 |
| 1980 | 62,202 | 1745 |
| 1985 | 62,766 | 1676 |
| 1990 | 62,324 | 1611 |
| 1995 | 58,200 | 1533 |
| 2000 | 55,800 | 1480 |

*Source:* U.S. Bureau of the Census, *www.census.gov*


**Number of U.S. Commercial TV Stations**

| Year | Number of Commercial TV Stations |
|---|---|
| 1950 | 98 |
| 1955 | 411 |
| 1960 | 515 |
| 1965 | 569 |
| 1970 | 677 |
| 1975 | 706 |
| 1980 | 734 |
| 1985 | 883 |
| 1990 | 1092 |
| 1995 | 1161 |
| 2000 | 1248 |

*Source:* U.S. Bureau of the Census, *www.census.gov*




**a.** Use the U.S. population numbers from Table 2.1 (at the beginning of this chapter) to calculate and compare the number of copies of newspapers *per person* in 1920 and in 2000.

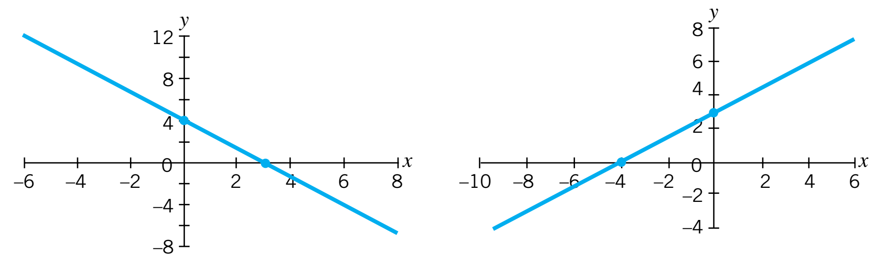
**b.** Create a table that displays the annual average rate of change in TV stations for each 5-year period since 1950. Create a similar table that displays the annual average rate of change in newspapers published for the same period. Graph the results.

**c.** If new TV stations continue to come into existence at the same rate as from 1990 to 2000, how many will there be by the year 2010? Do you think this is likely to be a reasonable projection, or is it overly large or small judging from past rates of growth? Explain.

**d.** What trends do you see in the dissemination of news as reflected in these data?

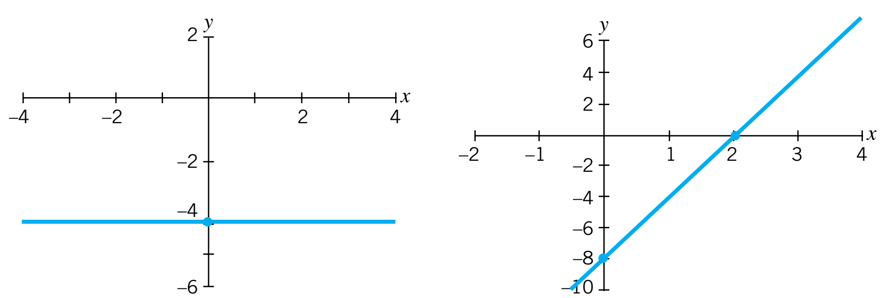
## Exercises for Section 2.3

**32.** Find the slope of each line using the $x$- and $y$-intercepts.




(*a*) (*b*)

**33.** Find the slope of each line using the $x$- and $y$-intercepts.

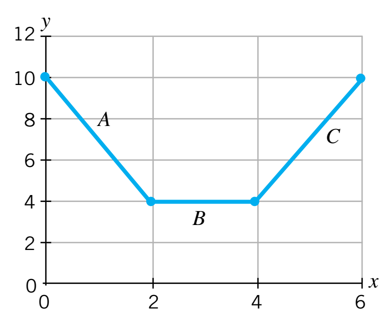



(*a*) (*b*)

**34.** Examine the line segments $A$, $B$, and $C$.

**a.** Which line segment has a slope that is positive? That is negative? That is zero?

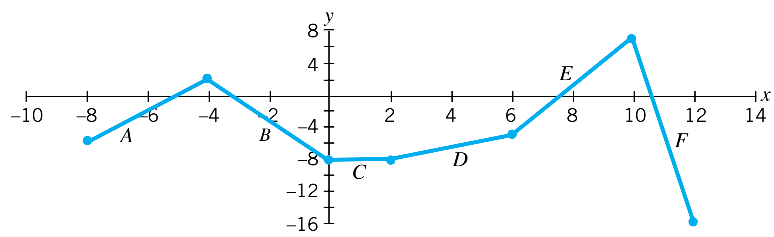
**b.** Calculate the exact slope for each line segment $A$, $B$, and $C$.



**35.** **a.** Estimate the slope for each line segment *A–F*. (*Note:* Each letter refers to a line segment, not a point.)

**b.** Which line segment has the steepest slope?

**c.** Which line segment has a slope closest to zero?




**36.** Plot each pair of points and calculate the slope of the line that passes through them.

**a.** (3, 5) and (8, 15)

**b.** (−1, 4) and (7, 0)

**c.** (5, 9) and (−5, 9)

**d.** (−2, 6) and (2, −6)

**e.** (−4, −3) and (2, −3)

**37.** The following problems represent calculations of the slopes of different lines. Solve for the variable in each equation.

**a.** $\dfrac{150 - 75}{20 - 10} = m$

**b.** $\dfrac{70 - y}{0 - 8} = 0.5$

**c.** $\dfrac{182 - 150}{28 - x} = 4$

**d.** $\dfrac{6 - 0}{x - 10} = 0.6$

**38.** Use the slope formula to find the slope $m$ of the line through the points $(0, b)$ and $(x, y)$, then solve the equation for $y$.

**39.** Find the value of $t$ if $m$ is the slope of the line that passes through the given points.

**a.** $(3, t)$ and $(-2, 1)$, $m = -4$

**b.** $(5, 6)$ and $(t, 9)$, $m = \frac{2}{3}$

**40.** **a.** Find the value of $x$ so that the slope of the line through $(x, 5)$ and $(4, 2)$ is $\frac{1}{3}$.

**b.** Find the value of $y$ so that the slope of the line through $(1, -3)$ and $(-4, y)$ is $-2$.

**c.** Find the value of $y$ so that the slope of the line through $(-2, 3)$ and $(5, y)$ is 0.

**41.** Points that lie on the same line are said to be *collinear*. Determine if the following points are collinear.

**a.** (2, 3), (4, 7), and (8, 15)

**b.** (−3, 1), (2, 4), and (7, 8)

**42.** **a.** Find the slopes of the lines through each of the following pairs of points.

**i.** (−1, 4) and (−2, 4)

**ii.** (7, −3) and (−7, −3)

**iii.** (−2, −6) and (5, −6)

**b.** Summarize your findings.

**43.** Graph a line through each pair of points and then calculate its slope.

**a.** The origin and (6, −2)

**b.** The origin and (−4, 7)

**44.** Find some possible values of the $y$-coordinates for the points $(-3, y_1)$ and $(6, y_2)$, such that the slope $m = 0$.

**45.** Calculate the slope of the line passing through each of the following pairs of points.

**a.** $(0, \sqrt{2})$ and $(\sqrt{2}, 0)$

**b.** $\left(0, -\frac{3}{2}\right)$ and $\left(-\frac{3}{2}, 0\right)$

**c.** $(0, b)$ and $(b, 0)$

**d.** What do these pairs of points and slopes all have in common?

**46.** Which pairs of points produce a line with a negative slope?

**a.** $(-5, -5)$ and $(-3, -3)$

**b.** $(-2, 6)$ and $(-1, 4)$

**c.** $(3, 7)$ and $(-3, -7)$

**d.** $(4, 3)$ and $(12, 0)$

**e.** $(0, 3)$ and $(4, -10)$

**f.** $(4, 2)$ and $(6, 2)$

**47.** In the previous exercise, which pairs of points produce a line with a positive slope?

**48.** A study on numerous streams examined the effects of a warming climate. It found an increase in water temperature of about 0.7°C for every 1°C increase in air temperature.

**a.** Find the rate of change in water temperature with respect to air temperature. What are the units?

**b.** If the air temperature increased by 5°C, by how much would you expect the stream temperature to increase?

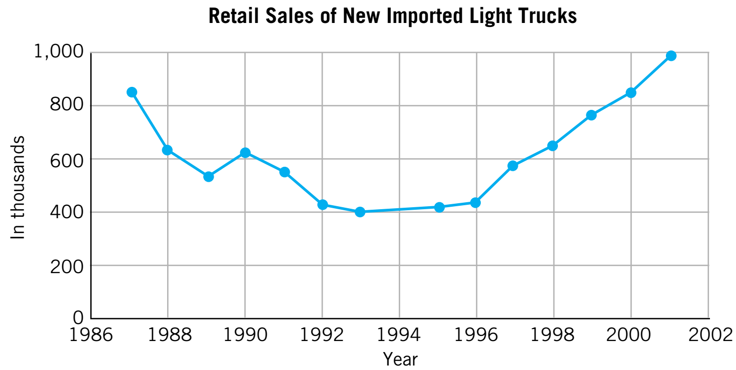
**49.** Handicapped Vietnam veterans successfully lobbied for improvements in the architectural standards for wheelchair access to public buildings.

**a.** The old standard was a 1-foot rise for every 10 horizontal feet. What would the slope be for a ramp built under this standard?

**b.** The new standard is a 1-foot rise for every 12 horizontal feet. What would the slope of a ramp be under this standard?

**c.** If the front door is 3 feet above the ground, how long would the handicapped ramp be using the old standard? Using the new?

**50.** The accompanying graph gives sales information about imported light trucks. Specify the intervals on the accompanying graph for which the slope of the line segment between adjacent data points appears positive. For which does it appear negative? For which zero?

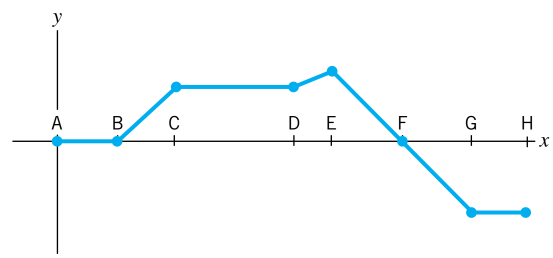


**51.** Given the accompanying graph of a function, specify the intervals over which:

**a.** The *function* is positive, negative, or zero.

**b.** The *rate of change between any two points in the interval* is positive, is negative, or is zero.

(*Note:* A, B, C, etc. represent points on the graph on the *x*-axis.)



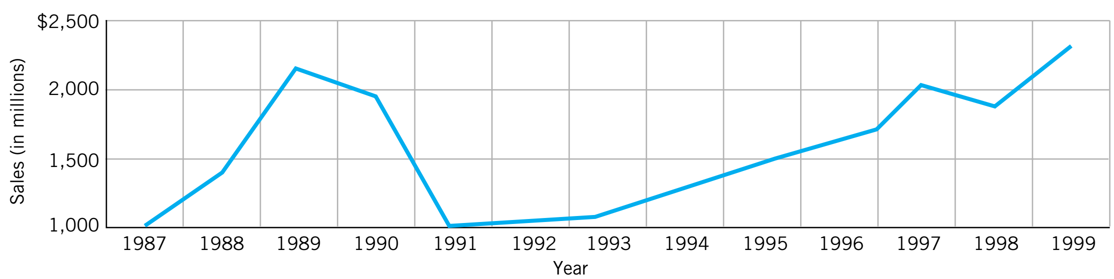

**52.** Examine the given graphs of Christie's and Sotheby's auction sales showing their annual sales in the art market.

**a.** Identify a period of negative slope on the Christie's graph. Describe in words what negative slope means in this art market. Estimate the slope, giving units.

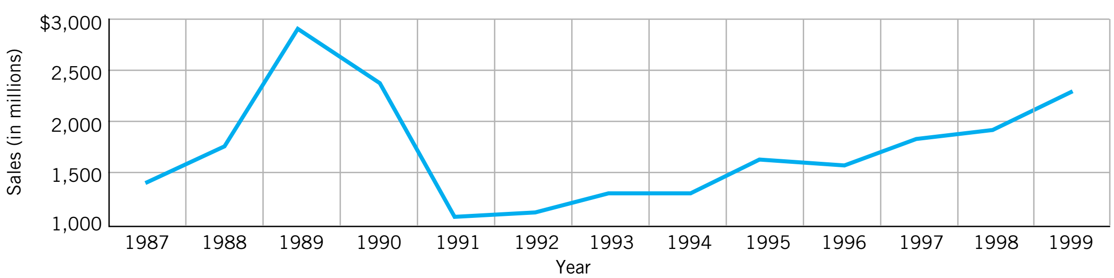
**b.** Identify a period of positive slope on the Sotheby's graph. Describe in words what positive slope means in this art market. Estimate the slope, giving units.

**c.** Write a paragraph about these art markets using numerical descriptions or comparisons and mention both discouraging and encouraging features.

**Christie's Annual Auction Sales (in millions)**




**Sotheby's Annual Auction Sales (in millions)**




*Sources:* Sotheby's Holdings Inc. and Christie's International PLC.

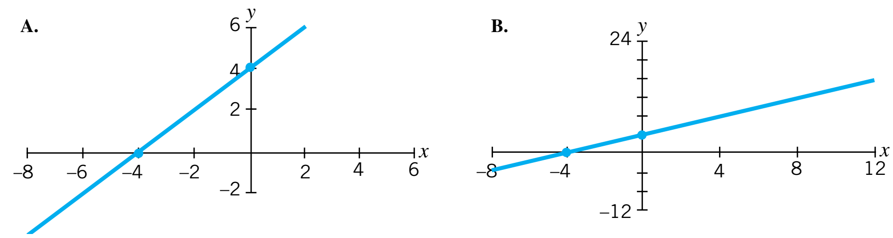
**53.** Read the Anthology article "Slopes" and describe two practical applications of slopes, one of which is from your own experience.

## Exercises for Section 2.4

**54.** Compare the accompanying graphs.

**a.** Which line appears to have the steeper slope?

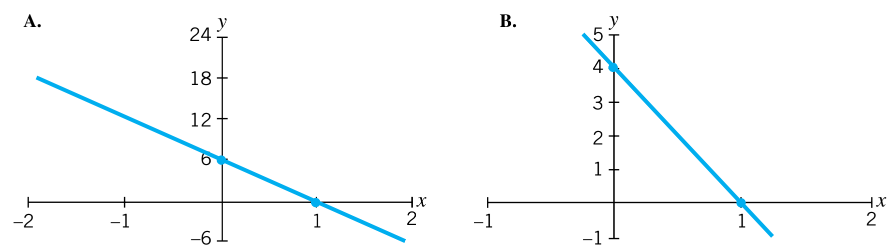
**b.** Use the intercepts to calculate the slope. Which graph actually does have the steeper slope?




**55.** Compare the accompanying graphs.

**a.** Which line appears to have the steeper slope?

**b.** Use the intercepts to calculate the slope. Which graph has the more negative (and hence steeper) slope?

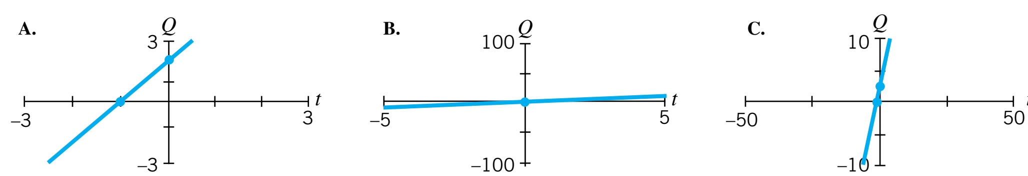



**56.** The following graphs show the same function graphed on different scales.

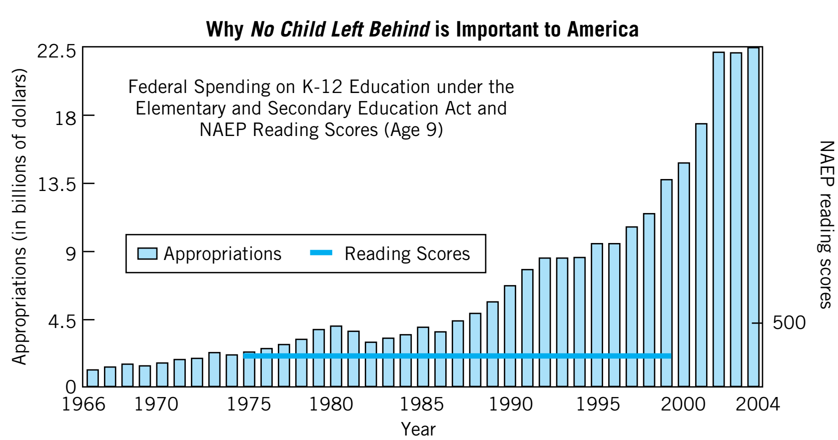
**a.** In which graph does $Q$ appear to be growing at the fastest rate?

**b.** In which graph does $Q$ appear to be growing at a near zero rate?

**c.** Explain why the graphs give different impressions.




**57.** The following chart appeared in July 2003 on the U.S. Department of Education home page at *www.ed.gov.*



**a.** Summarize the message that this chart conveys concerning NAEP reading scores and federal spending.

**b.** What additional information would be beneficial to have to verify the impression of the graph?

**58.** Generate two graphs and on each draw a line through the points (0, 3) and (4, 6), choosing $x$ and $y$ scales such that:

**a.** The first line appears to have a slope of almost zero.

**b.** The second line appears to have a very large positive slope.

**59.** **a.** Generate a graph of a line through the points $(0, -2)$ and $(5, -2)$.

**b.** On a new grid, choose different scales so that the line through the same points appears to have a large positive slope.

**c.** What have you discovered?

**60.** What are three adjectives (like "explosive") that would imply rapid growth?

**61.** What are three adjectives (like "severe") that would imply rapid decline?

**62.** Examine the data given on women in the U.S. military forces.

**Women in Uniform: Female Active-Duty Military Personnel**

| Year | Total | Army | Navy | Marine Corps | Air Force |
|---|---|---|---|---|---|
| 1965 | 30,610 | 12,326 | 7,862 | 1,581 | 8,841 |
| 1970 | 41,479 | 16,724 | 8,683 | 2,418 | 13,654 |
| 1975 | 96,868 | 42,295 | 21,174 | 3,186 | 30,213 |
| 1980 | 171,418 | 69,338 | 34,980 | 6,706 | 60,394 |
| 1985 | 211,606 | 79,247 | 52,603 | 9,695 | 70,061 |
| 1990 | 227,018 | 83,621 | 59,907 | 9,356 | 74,134 |
| 1995 | 196,116 | 68,046 | 55,830 | 8,093 | 64,147 |
| 2000 | 204,498 | 72,021 | 53,920 | 9,742 | 68,815 |

*Source:* U.S. Defense Department.

**a.** Make the case with graphs and numbers that women are a growing presence in the U.S. military.

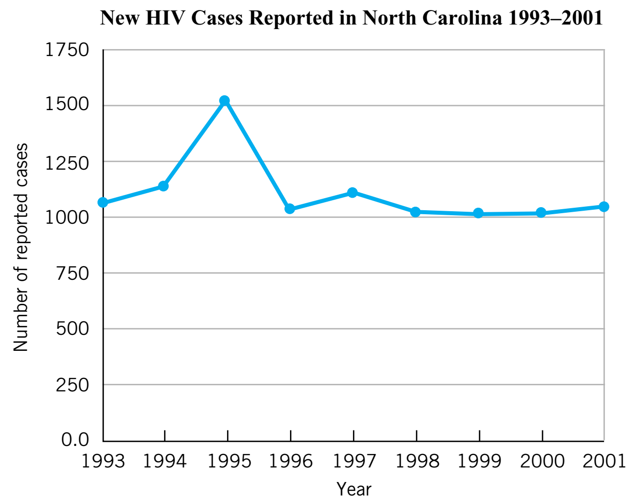
**b.** Make the case with graphs and numbers that women are a declining presence in the U.S. military.

**c.** Write a paragraph that gives a balanced picture of the changing presence of women in the military using appropriate statistics to make your points. What additional data would be helpful?

**63.** The accompanying table and graph show the number of new cases of HIV infection reported in North Carolina from 1993 to 2001.

| Year | Number of New HIV Cases |
|---|---|
| 1993 | 1064 |
| 1994 | 1141 |
| 1995 | 1526 |
| 1996 | 1039 |
| 1997 | 1111 |
| 1998 | 1021 |
| 1999 | 1017 |
| 2000 | 1011 |
| 2001 | 1046 |

*Source:* N.C. Division of Public Health, *www.epi.state.ncus*

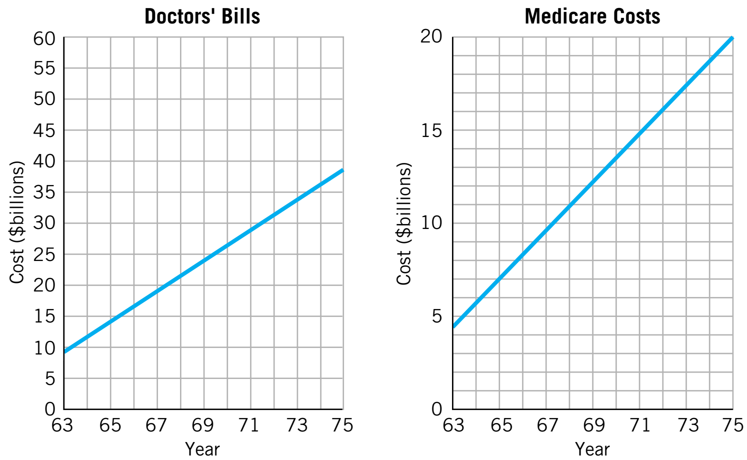



**a.** Find something encouraging to say about these data by using numerical evidence, including average rates of change.

**b.** Find numerical support for something discouraging to say about the data.

**c.** How might we explain the enormous jump in new HIV cases reported from 1994 to 1995 and the drop-off the following year? Is there anything potentially misleading about the data?

**64.** The accompanying graphs are an approximation of the costs of doctors' bills and Medicare between 1963 and 1975.




*Source:* Adapted with permission from L. Ferleger and L. Horowitz, *Statistics for Social Change* (Boston: South End Press, 1980).

**a.** At first glance, which *appears* to have been growing at a faster average annual rate of change, doctors' bills or Medicare costs? Why?

**b.** Which actually grew at a faster average annual rate of change? How can you tell?

**65.** The accompanying graphs show the same data on the income of black people as a percentage of the income of white people over a 30-year period. Describe the impression each graph gives and how the axes have been altered in each graph to convey these impressions.


*Source:* Adapted from L. Ferleger and L. Horowitz, *Statistics for Social Change* (Boston: South End Press, 1980).

**66.** Laws to regulate environmental pollution in America are a relatively recent phenomenon, with the first federal regulations appearing in the 1950s. The Clean Air Act, passed in 1963 and amended in 1970, established for the first time uniform national air pollution standards. The act placed national limits and a timetable on three classes of automotive pollutants: hydrocarbons (HC), carbon monoxide (CO), and nitrogen oxides (NO). All new cars for each particular year were restricted from exceeding these standards. The accompanying tables show the results for the early years of its enforcement. (See also Excel or graph link files POLLUTE for emission standards per mile and EMISSION for total annual automobile emission estimates.) You may want to go onto the web to find current automobile emission estimates.

**a.** Make a convincing argument that the Clean Air Act was a success.

**b.** Make a convincing argument that the Clean Air Act was a failure. (*Hint:* How is it possible that the emissions per vehicle mile were down but the total amount of emissions did not improve?)

**National Automobile Emission Control Standards**

| Car Model Year Applicable | HC (grams/mile) | CO (grams/mile) | $NO_2$* (grams/mile) |
|---|---|---|---|
| Pre-1968 | 8.7 | 87 | 4.4 |
| 1968 | 6.2 | 51 | n.r.† |
| 1970 | 4.1 | 34 | n.r. |
| 1972 | 3.0 | 28 | n.r. |
| 1973 | 3.0 | 28 | 3.1 |
| 1975 | 1.5 | 15 | 3.1 |
| 1975‡ | 0.9 | 9 | 2.0 |
| 1976 | 1.5 | 15 | 3.1 |
| 1977 | 1.5 | 15 | 2.0 |
| 1978 | 1.5 | 15 | 2.0 |
| 1979 | 1.5 | 15 | 2.0 |
| 1980 | 0.41 | 7 | 2.0 |
| 1981 and beyond | 0.41 | 3.4 | 1.0 |

*$NO_2$, nitrogen dioxide, is a form of nitrogen oxide.
†No requirement.
‡California standards.
*Source:* P. Portney (ed.), *Current Issues in U.S. Environmental Policy* (Baltimore, MD: Johns Hopkins University Press, 1978), p. 76.

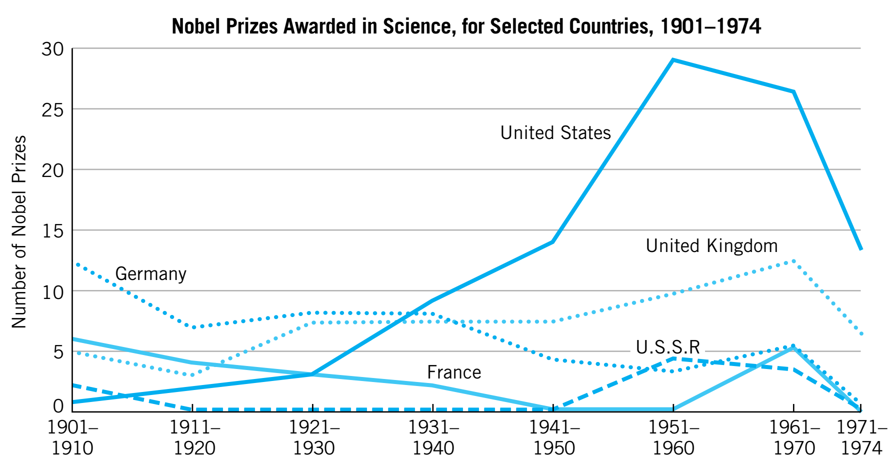
**National Automobile Emissions Estimates (million metric tons per year)**

| Year | HC | CO | $NO_2$ |
|---|---|---|---|
| 1970 | 28.3 | 102.6 | 19.9 |
| 1971 | 27.8 | 103.1 | 20.6 |
| 1972 | 28.3 | 104.4 | 21.6 |
| 1973 | 28.4 | 103.5 | 22.4 |
| 1974 | 27.1 | 99.6 | 21.8 |
| 1975 | 25.3 | 97.2 | 20.9 |
| 1976 | 27.0 | 102.9 | 22.5 |
| 1977 | 27.1 | 102.4 | 23.4 |
| 1978 | 27.8 | 102.1 | 23.3 |

*Source: Environmental Quality—1980: The Eleventh Annual Report of the Council on Environmental Quality,* p. 170.

**67.** (Computer required) Open "L6: Changing Axis Scales" in *Linear Functions*. Generate a line in the upper left-hand box. The same line will appear graphed in the three other boxes but with the axes scaled differently. Describe how the axes are rescaled in order to create such different impressions.

**68.** The accompanying graph shows the number of Nobel Prizes awarded in science for various countries between 1901 and 1974. It contains accurate information but gives the impression that the number of prize winners declined drastically in the 1970s, which was not the case. What flaw in the construction of the graph leads to this impression?




*Source:* E. R. Tufte, *The Visual Display of Quantitative Information* (Cheshire, Conn.: Graphics Press, 1983).

## Exercises for Section 2.5

You might wish to hone your algebraic mechanical skills with three programs in *Linear Functions:* "L1: Finding $m$ & $b$", "L3: Finding a Line through 2 Points", and "L4: Finding 2 Points on a Line." They offer practice in predicting values for $m$ and $b$, generating linear equations and finding corresponding solutions.

**69.** Consider the equation $E = 5000 + 100n$.

**a.** Find the value of $E$ for $n = 0, 1, 20$.

**b.** Express your answers to part (a) as points with coordinates $(n, E)$.

**70.** Consider the equation $G = 12{,}000 + 800n$.

**a.** Find the value of $G$ for $n = 0, 1, 20$.

**b.** Express your answers to part (a) as points with coordinates $(n, G)$.

**71.** Determine if any of the following points satisfy one or both of the equations in Exercises 69 and 70.

**a.** (5000, 0) **b.** (15, 24000) **c.** (35, 40000)

**72.** Suppose during a 5-year period the profit $P$ (in billions of dollars) for a large corporation was given by $P = 7 + 2Y$, where $Y$ represents the year.

**a.** Fill in the chart.

| $Y$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P$ | | | | | |

**b.** What are the units of $P$?

**c.** What does the 2 in the equation represent, and what are its units?

**d.** What was the initial profit?

**73.** Consider the equation $D = 3.40 + 0.11n$

**a.** Find the values of $D$ for $n = 0, 1, 2, 3, 4$.

**b.** If $D$ represents the average consumer debt in thousands of dollars, over $n$ years, what does 0.11 represent? What are its units?

**c.** What does 3.40 represent? What are its units?

**74.** Suppose the equations $E = 5000 + 1000n$ and $G = 12{,}000 + 800n$ give the total cost of operating an electrical ($E$) versus a gas ($G$) heating/cooling system in a home for $n$ years.

**a.** Find the cost of heating a home using electricity for 10 years.

**b.** Find the cost of heating a home using gas for 10 years.

**c.** Find the initial (or installation) cost for each system.

**d.** Determine how many years it will take before \$40,000 has been spent in heating/cooling a home that uses:

**i.** Electricity **ii.** Gas

**75.** If the equation $E = 5000 + 1000n$ gives the total cost of heating/cooling a home after $n$ years, rewrite the equation using only units of measure.

**76.** Over a 5-month period at Acadia National Park in Maine, the average night temperature increased on average 5 degrees Fahrenheit per month. If the initial temperature is 25 degrees, create a formula for the night temperature $N$ for month $t$, where $0 \le t \le 4$.

**77.** A residential customer who purchases gas from a utility company is charged according to the formula $C(g) = 11 + 5.26g$, where $C(g)$ is the cost, in dollars, for $g$ thousand cubic feet of gas.

**a.** Find $C(0)$, $C(1.5)$, and $C(15.6)$.

**b.** What is the cost if the customer uses no gas?

**c.** What is the rate per thousand cubic feet charged for using the gas?

**d.** How much would it cost if the customer uses 2500 $ft^3$ of gas?

**78.** Create the formula for converting degrees centigrade, $C$, to degrees Fahrenheit, $F$, if for every increase of 5 degrees centigrade the Fahrenheit temperature increases by 9 degrees, with an initial point of $(C, F) = (0, 32)$.

**79.** Determine the vertical intercept and the rate of change for each of these formulas:

**a.** $P = 4s$ **b.** $C = \pi d$ **c.** $C = 2\pi r$ **d.** $C = \frac{5}{9}F - 17.78$

**80.** A hiker can walk 2 miles in 45 minutes.

**a.** What is his average speed in miles per hour?

**b.** What formula can be used to find the distance traveled, $d$, in $t$ hours?

**81.** The cost $C(x)$ of producing $x$ items is determined by adding fixed costs to the product of the cost per item and the number of items produced, $x$. Below are several possible cost functions $C(x)$, measured in dollars. Match each description with the most likely cost function.

**a.** $C(x) = 125{,}000 + 42.5x$ **e.** The cost of producing a computer

**b.** $C(x) = 400{,}000 + 0.30x$ **f.** The cost of producing a college algebra text

**c.** $C(x) = 250{,}000 + 800x$ **g.** The cost of producing a CD

**82.** A new \$25,000 car depreciates in value \$5000 per year. Construct a linear function for the value $V$ of the car after $t$ years.

**83.** The state of Pennsylvania has a 6% sales tax on taxable items. (*Note:* Clothes, food, and certain pharmaceuticals are not taxed in Pennsylvania.)

**a.** Create a formula for the total cost (in dollars) of an item $C(p)$ with a price tag $p$. (Be sure to include the sales tax.)

**b.** Find $C(9.50)$, $C(115.25)$, and $C(1899)$. What are the units?

**84. a.** If $S(x) = 20{,}000 + 1000x$ describes the annual salary in dollars for a person who has worked for $x$ years for the Acme Corporation, what is the unit of measure for 20,000? For 1000?

**b.** Rewrite $S(x)$ as an equation using only units of measure.

**c.** Evaluate $S(x)$ for $x$ values of 0, 5, and 10 years.

**d.** How many years will it take for a person to earn an annual salary of \$43,000?

**85.** The following represent linear equations written using only units of measure. In each case supply the missing unit.

**a.** inches = inches + (inches/hour) · (?)

**b.** miles = miles + (?) · (gallons)

**c.** calories = calories + (?) · (grams of fat)

**86.** Identify the slopes and the vertical intercepts of the lines with the given equations:

**a.** $y = 3 + 5x$ **c.** $y = 4$ **e.** $f(E) = 10{,}000 + 3000E$

**b.** $f(t) = -t$ **d.** $Q = 35t - 10$

**87.** For each of the following, find the slope and the vertical intercept. Sketch a graph.

**a.** $y = 0.4x - 20$ **b.** $P = 4000 - 200C$

**88.** Construct an equation and sketch the graph of its line with the given slope, $m$, and vertical intercept, $b$.

**a.** $m = 2, b = -3$ **b.** $m = -\frac{3}{4}, b = 1$ **c.** $m = 0, b = 50$

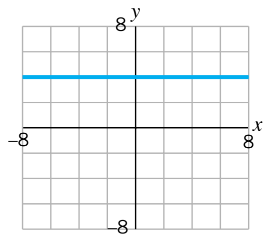
In Exercises 89 to 91 find an equation, generate a small table of solutions, and sketch the graph of a line with the indicated attributes.

**89.** A line that has a vertical intercept of $-2$ and a slope of 3.

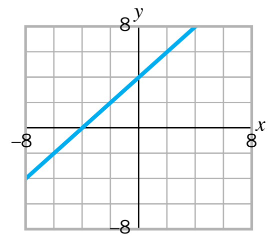
**90.** A line that crosses the vertical axis at 3.0 and has a rate of change of $-2.5$.

**91.** A line that has a vertical intercept of 1.5 and a slope of 0.

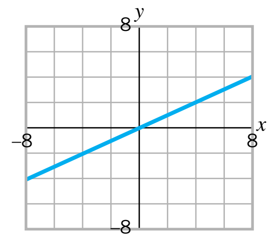
**92.** Estimate $b$ (the $y$-intercept) and $m$ (the slope) for each of the accompanying graphs. Then, for each graph, write the corresponding linear function. Be sure to note the scales on the axes.



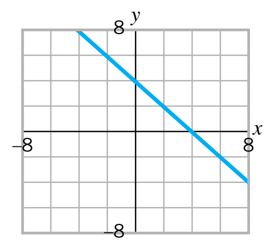

Graph *A*



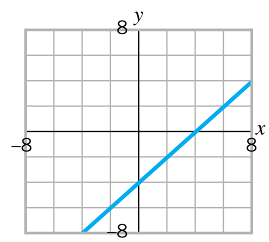

Graph *B*



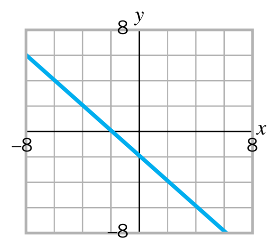

Graph *C*




Graph *D*




Graph *E*



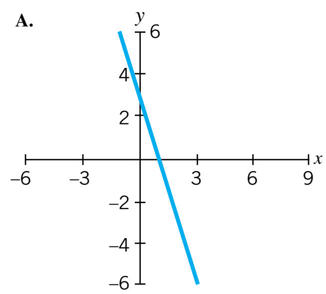

Graph *F*

## Exercises for Section 2.6

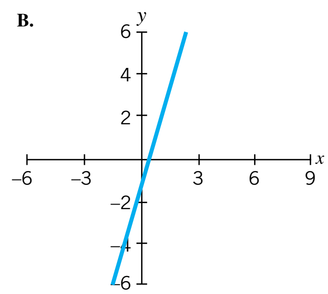
**93.** Assuming $m$ is the slope, for which graph is:

**a.** $m = 3$? **b.** $|m| = 3$? **c.** $m = 0$? **d.** $m = -3$?

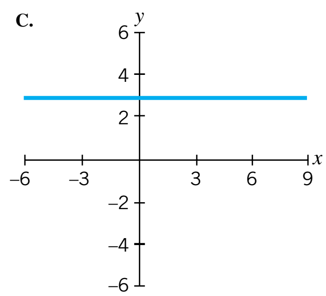
**A.**



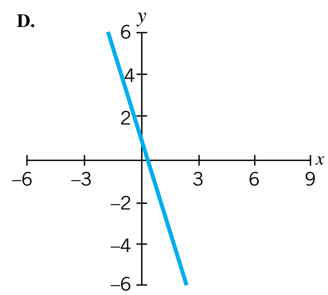

**B.**




**C.**

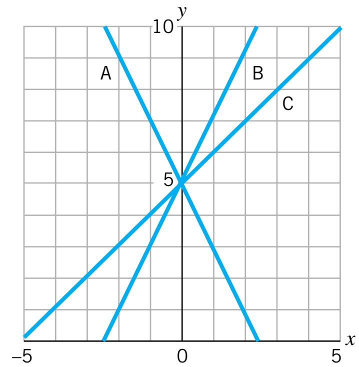



**D.**



**94.** Which line(s) have a slope $m$ such that:

**a.** $|m| = 2$? **b.** $m = \frac{1}{2}$?




**95.** For each set of conditions, construct a linear equation and draw its graph.

**a.** A slope of zero and a $y$-intercept of $-3$

**b.** A positive slope and a vertical intercept of $-3$

**c.** A slope of $-3$ and a vertical intercept that is positive

**96.** Construct a linear equation for each of the following conditions.

**a.** A negative slope and a positive $y$-intercept

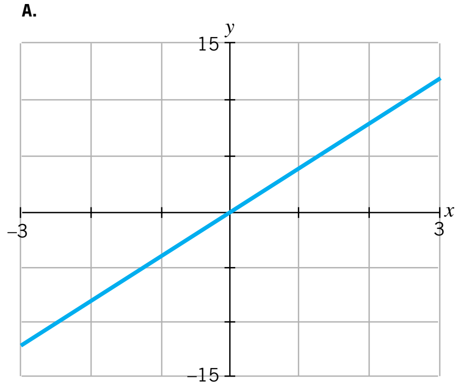
**b.** A positive slope and a vertical intercept of $-10.3$

**c.** A constant rate of change of \$1300/year

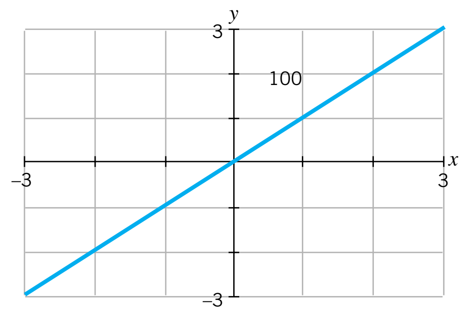
**97.** Match the graph with the correct equation.

**a.** $y = x$ **b.** $y = 2x$ **c.** $y = \frac{x}{2}$ **d.** $y = 4x$

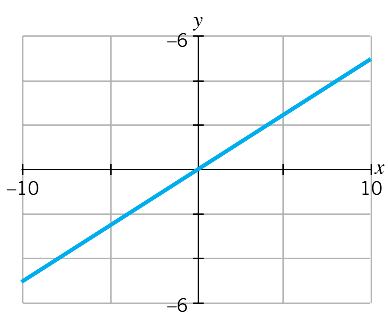
**A.**



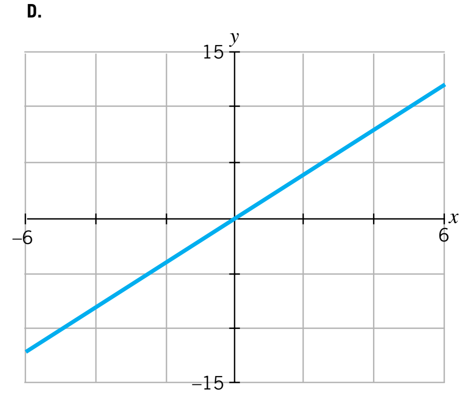

**B.**




**C.**

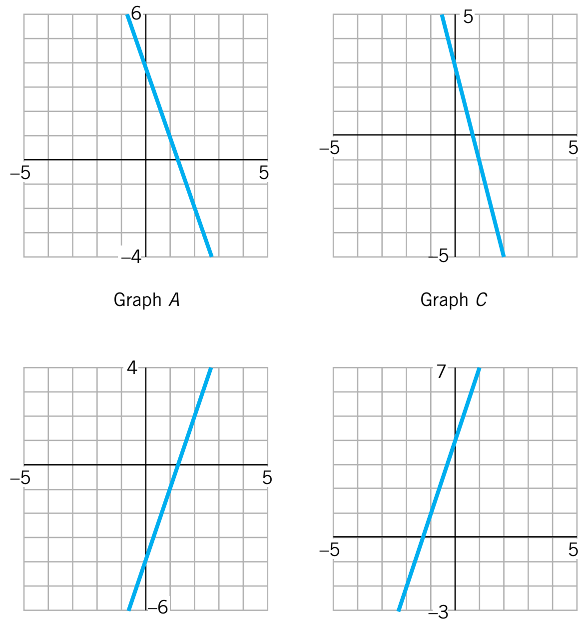



**D.**



**98.** Match the function with its graph. (*Note:* There is one graph that has no match.)

**a.** $y = -4 + 3x$ **b.** $y = -3x + 4$ **c.** $y = 4 + 3x$




Graph *A* Graph *C*

Graph *B* Graph *D*

**99.** In each part construct three different linear equations that all have:

**a.** The same slope

**b.** The same vertical intercept

**100.** Which equation has the steepest slope?

**a.** $y = 2 - 7x$ **b.** $y = 2x + 7$ **c.** $y = -2 + 7x$

**101.** Given the function $Q(t) = 13 - 5t$, construct a related function whose graph:

**a.** Lies five units above the graph of $Q(t)$

**b.** Lies three units below the graph of $Q(t)$

**c.** Has the same vertical intercept

**d.** Has the same slope

**e.** Has the same steepness, but the slope is positive

**102.** Given the equation $C(n) = 30 + 15n$, construct a related equation whose graph:

**a.** Is steeper

**b.** Is flatter (less steep)

**c.** Has the same steepness, but the slope is negative

**103.** On the same axes, graph (and label with the correct equation) three lines that go through the point (0, 2) and have the following slopes:

**a.** $m = \frac{1}{2}$ **b.** $m = 2$ **c.** $m = \frac{5}{6}$

**104.** On the same axes, graph (and label with the correct equation) three lines that go through the point (0, 2) and have the following slopes:

**a.** $m = -\frac{1}{2}$ **b.** $m = -2$ **c.** $m = -\frac{5}{6}$

## Exercises for Section 2.7

**105.** Write an equation for the line through $(-2, 3)$ that has slope:

**a.** 5 **b.** $-\frac{3}{4}$ **c.** 0

**106.** Write an equation for the line through (0, 50) that has slope:

**a.** $-20$ **b.** 5.1 **c.** 0

**107.** Calculate the slope and write an equation for the linear function represented by each of the given tables.

**a.**

| $x$ | $y$ |
|---|---|
| 2 | 7.6 |
| 4 | 5.1 |

**b.**

| $A$ | $W$ |
|---|---|
| 5 | 12 |
| 7 | 16 |

**108.** Determine which of the following tables represent a linear function. If it is linear, write the equation for the linear function.

**a.**

| $x$ | $y$ |
|---|---|
| 0 | 3 |
| 1 | 8 |
| 2 | 13 |
| 3 | 18 |
| 4 | 23 |

**b.**

| $q$ | $R$ |
|---|---|
| 0 | 0.0 |
| 1 | 2.5 |
| 2 | 5.0 |
| 3 | 7.5 |
| 4 | 10.0 |

**c.**

| $x$ | $g(x)$ |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |
| 4 | 16 |

**d.**

| $t$ | $r$ |
|---|---|
| 10 | 5.00 |
| 20 | 2.50 |
| 30 | 1.67 |
| 40 | 1.25 |
| 50 | 1.00 |

**e.**

| $x$ | $h(x)$ |
|---|---|
| 20 | 20 |
| 40 | $-60$ |
| 60 | $-140$ |
| 80 | $-220$ |
| 100 | $-300$ |

**f.**

| $p$ | $T$ |
|---|---|
| 5 | 0.25 |
| 10 | 0.50 |
| 15 | 0.75 |
| 20 | 1.00 |
| 25 | 1.25 |

**109.** Plot each pair of points, then determine the equation of the line that goes through the points.

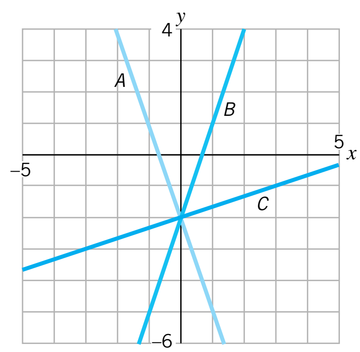
**a.** (2, 3), (4, 0)

**b.** $(-2, 3)$, (2, 1)

**c.** (2, 0), (0, 2)

**d.** (4, 2), $(-5, 2)$

**110.** Find the equation for each of the lines A–C on the accompanying graph.




**111.** Put the following equations in $y = mx + b$ form, then identify the slope and the vertical intercept.

**a.** $2x - 3y = 6$

**b.** $3x + 2y = 6$

**c.** $\frac{1}{3}x + \frac{1}{2}y = 6$

**d.** $2y - 3x = 0$

**e.** $6y - 9x = 0$

**f.** $\frac{1}{2}x - \frac{2}{3}y = -\frac{1}{6}$

**112.** Solve each equation for $y$ in terms of $x$, then identify the slope and the $y$-intercept. Graph each line by hand. Verify your answers with a graphing utility if available.

**a.** $-4y - x - 8 = 0$

**b.** $\frac{1}{2}x - \frac{1}{4}y = 3$

**c.** $-4x - 3y = 9$

**d.** $6x - 5y = 15$

**113.** Complete the table for each of the linear functions, and then sketch a graph of each function. Make sure to choose an appropriate scale and label the axes.

**a.**

| $x$ | $f(x) = 0.10x + 10$ |
|---|---|
| $-100$ | |
| 0 | |
| 100 | |

**b.**

| $x$ | $g(x) = 20 - 5x$ |
|---|---|
| $-25$ | |
| 0 | |
| 25 | |

**c.**

| $x$ | $h(x) = 50x + 100$ |
|---|---|
| $-0.5$ | |
| 0 | |
| 0.5 | |

**114.** The equation $K = 4F - 160$ models the relationship between $F$, the temperature in degrees Fahrenheit, and $K$, the number of chirps per minute for the snow tree cricket.

**a.** Assuming $F$ is the independent variable and $K$ is the dependent variable, identify the slope and vertical intercept in the given equation.

**b.** Identify the units for $K$, 4, $F$, and $-160$.

**c.** What is a reasonable domain for this model?

**d.** Generate a small table of points that satisfy the equation. Be sure to choose realistic values for $F$ from the domain of your model.

**e.** Calculate the slope directly from two data points. Is this value what you expected? Why?

**f.** Graph the equation, indicating the domain.

**115.** Find an equation to represent the cost of attending college classes if application and registration fees are \$150 and classes cost \$120 per credit.

**116. a.** Write an equation that describes the total cost to produce $x$ items if the startup cost is \$200,000 and the production cost per item is \$15.

**b.** Why is the total average cost per item less if the item is produced in large quantities?

**117.** Your bank charges you a \$2.50 monthly maintenance fee on your checking account and an additional \$0.10 for each check you cash. Write an equation to describe your monthly checking account costs.

**118.** If a town starts with a population of 63,500 that declines by 700 people each year, construct an equation to model its population size over time. How long would it take for the population to drop to 53,000?

**119.** A teacher's union has negotiated a uniform salary increase for each year of service up to 20 years. If a teacher started at \$26,000 and 4 years later had a salary of \$32,000:

**a.** What was the annual increase?

**b.** What function would describe the teacher's salary over time?

**c.** What would be the domain for the function?

**120.** Your favorite aunt put money in a savings account for you. The account earns simple interest; that is, it increases by a fixed amount each year. After 2 years your account has \$8250 in it and after 5 years it has \$9375.

**a.** Construct an equation to model the amount of money in your account.

**b.** How much did your aunt put in initially?

**c.** How much will your account have after 10 years?

**121.** You read in the newspaper that the river is polluted with 285 parts per million (ppm) of a toxic substance, and local officials estimate they can reduce the pollution by 15 ppm each year.

**a.** Derive an equation that represents the amount of pollution, $P$, as a function of time, $t$.

**b.** The article states the river will not be safe for swimming until pollution is reduced to 40 ppm. If the cleanup proceeds as estimated, in how many years will it be safe to swim in the river?

**122.** The women's recommended weight formula from Harvard Pilgrim Healthcare says, "Give yourself 100 lb for the first 5 ft plus 5 lb for every inch over 5 ft tall."

**a.** Find a mathematical model for this relationship. Be sure you clearly identify your variables.

**b.** Specify a reasonable domain for the function and then graph the function.

**c.** Use your model to calculate the recommended weight for a woman 5 feet, 4 inches tall; and for one 5 feet, 8 inches tall.

**123.** In 1973 a math professor bought her house in Cambridge, Massachusetts, for \$20,000. The value of the house has risen steadily so that in 2003 real estate agents tell her the house is now worth \$250,000.

**a.** Find a formula to represent these facts about the value of the house, $V(t)$, as a function of time, $t$.

**b.** If she retires in 2010, what does your formula predict her house will be worth then?

**c.** If she turns 57 in 2005, and the house continues to gain value at the same rate, how old will she be when her house is worth half a million dollars?

**124.** The $y$-axis, the $x$-axis, the line $x = 6$, and the line $y = 12$ determine the four sides of a 6-by-12 rectangle in the first quadrant (where $x > 0$ and $y > 0$) of the $xy$ plane. Imagine that this rectangle is a pool table. There are pockets at the four corners and at the points (0, 6) and (6, 6) in the middle of each of the longer sides. When a ball bounces off one of the sides of the table, it obeys the "pool rule": The slope of the path after the bounce is the negative of the slope before the bounce. (*Hint:* It helps to sketch the pool table on a piece of graph paper first.)

**a.** Your pool ball is at (3, 8). You hit it toward the $y$-axis, along the line with slope 2.

**i.** Where does it hit the $y$-axis?

**ii.** If the ball is hit hard enough, where does it hit the side of the table next? And after that? And after that?

**iii.** Show that the ball ultimately returns to (3, 8). Would it do this if the slope had been different from 2? What is special about the slope 2 for this table?

**b.** A ball at (3, 8) is hit toward the $y$-axis and bounces off it at $\left(0, \frac{16}{3}\right)$. Does it end up in one of the pockets? If so, what are the coordinates of that pocket?

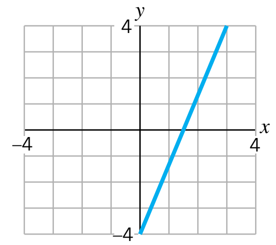
**c.** Your pool ball is at (2, 9). You want to shoot it into the pocket at (6, 0). Unfortunately, there is another ball at (4, 4.5) that may be in the way.

**i.** Can you shoot directly into the pocket at (6, 0)?

**ii.** You want to get around the other ball by bouncing yours off the $y$-axis. If you hit the $y$-axis at (0, 7), do you end up in the pocket? Where do you hit the line $x = 6$?

**iii.** If bouncing off the $y$-axis at (0, 7) didn't work, perhaps there is some point (0, $b$) on the $y$-axis from which the ball would bounce into the pocket at (6, 0). Try to find that point.

**125.** Find the equation of the line shown on the accompanying graph. Use this equation to create two new graphs, taking care to label the scales on your new axes. For one of your graphs, choose scales that make the line appear steeper than in the original graph. For your second graph, choose scales that make the line appear less steep than in the original graph.




**126.** The exchange rate that a bank gave for euros in June 2004 was 0.82 euros for $1 U.S. They also charged a constant fee of $5 per transaction. The bank's exchange rate from euros to British pounds was 0.66 pounds for 1 euro, with a transaction fee of 4.1 euros.

**a.** Write a general equation for how many euros you got when changing dollars. Use $E$ for euros and $D$ for dollars being exchanged. Draw a graph of $E$ versus $D$.

**b.** Would it have made any sense to exchange $10 for euros?

**c.** Find a general expression for the percentage of the total euros converted from dollars that the bank kept for the transaction fee.

**d.** Write a general equation for how many pounds you would get when changing euros. Use $P$ for British pounds and $E$ for the euros being exchanged. Draw a graph of $P$ versus $E$.

**127.** Suppose that:

- For 8 years of education, the mean salary for women is approximately $6800.
- For 12 years of education, the mean salary for women is approximately $11,600.
- For 16 years of education, the mean salary for women is approximately $16,400.

**a.** Plot this information on a graph.

**b.** What sort of relationship does this information suggest between mean salary for women and education? Justify your answer.

**c.** Generate an equation that could be used to model the data from the limited information given (letting $E$ = years of education and $S$ = mean salary). Show your work.

**128.** **a.** Fill in the third column in the Tables 1 and 2.

| $t$ | $d$ | Average Rate of Change |
|---|---|---|
| 0 | 400 | n.a. |
| 1 | 370 | |
| 2 | 340 | |
| 3 | 310 | |
| 4 | 280 | |
| 5 | 250 | |

**Table 1**

| $t$ | $d$ | Average Rate of Change |
|---|---|---|
| 0 | 1.2 | n.a. |
| 1 | 2.1 | |
| 2 | 3.2 | |
| 3 | 4.1 | |
| 4 | 5.2 | |
| 5 | 6.1 | |

**Table 2**

**b.** In either table, is $d$ a linear function of $t$? If so, construct a linear equation relating $d$ and $t$ for that table.

**129.** Adding minerals or organic compounds to water lowers its freezing point. Antifreeze for car radiators contains glycol (an organic compound) for this purpose. The accompanying table shows the effect of salinity (dissolved salts) on the freezing point of water. Salinity is measured in the number of grams of salts dissolved in 1000 grams of water. So our units for salinity are in parts per thousand, abbreviated ppt. Is the relationship between the freezing point and salinity linear? If so, construct an equation that models the relationship. If not, explain why.

**Relationship between Salinity and Freezing Point**

| Salinity (ppt) | Freezing Point (°C) |
|---|---|
| 0 | 0.00 |
| 5 | −0.27 |
| 10 | −0.54 |
| 15 | −0.81 |
| 20 | −1.08 |
| 25 | −1.35 |

*Source:* Data adapted from P.R. Pinel, *Oceanography: An Introduction to the Planet Oceanus* (St. Paul, MN: West, 1992), p. 522.

**130.** The accompanying data show rounded average values for blood alcohol concentration (BAC) for people of different weights, according to how many drinks (5 oz wine, 1.25 oz 80-proof liquor, or 12 oz beer) they have consumed.

**Blood Alcohol Concentration for Selected Weights**

| Number of Drinks | 100 lb | 120 lb | 140 lb | 160 lb | 180 lb | 200 lb |
|---|---|---|---|---|---|---|
| 2 | 0.075 | 0.063 | 0.054 | 0.047 | 0.042 | 0.038 |
| 4 | 0.150 | 0.125 | 0.107 | 0.094 | 0.083 | 0.075 |
| 6 | 0.225 | 0.188 | 0.161 | 0.141 | 0.125 | 0.113 |
| 8 | 0.300 | 0.250 | 0.214 | 0.188 | 0.180 | 0.167 |
| 10 | 0.375 | 0.313 | 0.268 | 0.235 | 0.208 | 0.188 |

**a.** Examine the data on BAC for a 100-pound person. Is this linear data? If so, find a formula to express blood alcohol concentration, $A$, as a function of the number of drinks, $D$, for a 100-pound person.

**b.** Examine the data on BAC for a 140-pound person. Is this linear data? If it's not precisely linear, what might be a reasonable estimate for the average rate of change of blood alcohol concentration, $A$, with respect to number of drinks, $D$? Find a formula to estimate blood alcohol concentration, $A$, as a function of number of drinks, $D$, for a 140-pound person. Can you make any general conclusions about BAC as a function of number of drinks for all of the weight categories?

**c.** Examine the data on BAC for people who consume two drinks. Is this linear data? If so, find a formula to express blood alcohol concentration, $A$, as a function of weight, $W$, for people who consume two drinks. Can you make any general conclusions about BAC as a function of weight for any particular number of drinks?

## *Exercises for Section 2.8*

**131.** Using the general formula $y = mx$ that describes direct proportionality, find the value of $m$ if:

**a.** $y$ is directly proportional to $x$ and $y = 2$ when $x = 10$.

**b.** $y$ is directly proportional to $x$ and $y = 0.1$ when $x = 0.2$.

**c.** $y$ is directly proportional to $x$ and $y = 1$ when $x = \frac{1}{4}$.

**132.** For each part, construct an equation and then use it to solve the problem.

**a.** Pressure $P$ is directly proportional to temperature $T$, and $P$ is 20 lb per square inch when $T$ is 60 degrees Fahrenheit. What is the pressure when the temperature is 80 degrees Fahrenheit?

**b.** Earnings $E$ are directly proportional to the time $T$ worked, and $E$ is \$46 when $T$ is 2 hours. How long has a person worked if she earned \$471.50?

**c.** The number of centimeters of water depth $W$ produced by melting snow is directly proportional to the number of centimeters of snow depth $S$. If $W$ is 15.9 cm when $S$ is 150 cm, then how many centimeters of water depth are produced by a 100-cm depth of melting snow?

**133.** In the accompanying table $y$ is directly proportional to $x$.

| | | | |
|---|---|---|---|
| Number of CDs purchased ($x$) | 3 | 4 | 5 |
| Cost of CDs ($y$) | 42.69 | 56.92 | |

**a.** Find the formula relating $y$ and $x$, then determine the missing value in the table.

**b.** Interpret the coefficient of $x$ in this situation.

**134.** The electrical resistance $R$ (in ohms) of a wire is directly proportional to its length $l$ (in feet).

**a.** If 250 feet of wire has a resistance of 1.2 ohms, find the resistance for 150 ft of wire.

**b.** Interpret the coefficient of $l$ in this context.

**135.** For each of the following linear functions, determine the independent and dependent variables and then construct an equation for each function.

**a.** Sales tax is 6.5% of the purchase price.

**b.** The height of a tree is directly proportional to the amount of sunlight it receives.

**c.** The average salary for full-time employees of American domestic industries has been growing at an annual rate of \$1300/year since 1985, when the average salary was \$25,000.

**136.** On the scale of a map 1 inch represents a distance of 35 miles.

**a.** What is the distance between two places that are 4.5 inches apart on the map?

**b.** Construct an equation that converts inches on the map to miles in the real world.

**137.** Find a function that represents the relationship between distance, $d$, and time, $t$, of a moving object using the data in the accompanying table. Is $d$ directly proportional to $t$? Which is a more likely choice for the object, a person jogging or a moving car?

| $t$ (hours) | $d$ (miles) |
|---|---|
| 0 | 0 |
| 1 | 5 |
| 2 | 10 |
| 3 | 15 |
| 4 | 20 |

**138.** Determine which (if any) of the following variables ($w$, $y$, or $z$) is directly proportional to $x$:

| $x$ | $w$ | $y$ | $z$ |
|---|---|---|---|
| 0 | 1 | 0.0 | 0 |
| 1 | 2 | 2.5 | $-\frac{1}{3}$ |
| 2 | 5 | 5.0 | $-\frac{2}{3}$ |
| 3 | 10 | 7.5 | $-1$ |
| 4 | 17 | 10.0 | $-\frac{4}{3}$ |

**139.** Find the slope of the line through the pair of points, then determine the equation.

**a.** (2, 3) and (5, 3)

**b.** $(-4, -7)$ and $(12, -7)$

**c.** $(-3, 8)$ and $(-3, 4)$

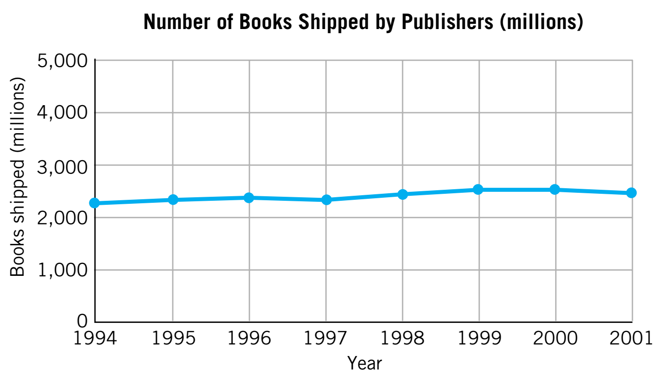
**d.** $(2, -3)$ and $(2, -1)$

**140.** Describe the graphs of the following equations.

**a.** $y = -2$ **c.** $x = \frac{2}{5}$ **e.** $y = 324$

**b.** $x = -2$ **d.** $y = \frac{x}{4}$ **f.** $y = \frac{2}{3}$

**141.** The accompanying figure shows the quantity of books (in millions) shipped by publishers in the United States between 1994 and 2001. Construct the equation of a horizontal line that would be a reasonable model for these data.




*Source:* U.S. Bureau of the Census. *Statistical Abstract of the United States:* 2002.

**142.** An employee for an aeronautical corporation has a starting salary of \$25,000/year. After working there for 10 years and not receiving any raises, he decides to seek employment elsewhere. Graph the employee's salary as a function of time for the time he was employed with this corporation. What is the domain? What is the range?

**143.** For each of the given points write equations for three lines such that one of the three lines is horizontal, one is vertical, and one has slope 2.

**a.** $(1, -4)$ **b.** $(2, 0)$ **c.** $(8, 50)$

**144.** Consider the function $f(x) = 4$.

**a.** What is $f(0)$? $f(30)$? $f(-12.6)$? **c.** Describe the slope of this function's graph.

**b.** Describe the graph of this function.

**145.** A football player who weighs 175 pounds is instructed at the end of spring training that he has to put on 30 pounds before reporting for fall training.

**a.** If fall training begins 3 months later, at what (monthly) rate must he gain weight?

**b.** Suppose that he eats a lot and takes several nutritional supplements to gain weight, but due to his metabolism he still weighs 175 pounds throughout the summer and at the beginning of fall training. Sketch a graph of his weight versus time for those 3 months.

**c.** Describe the graph you could draw in part (b), if you plotted the player's daily weight over the 3 months.

**146. a.** Write an equation for the line parallel to $y = 2 + 4x$ that passes through the point (3, 7).

**b.** Find an equation for the line perpendicular to $y = 2 + 4x$ that passes through the point (3, 7).

**147. a.** Write an equation for the line parallel to $y = 4 - x$ that passes through the point (3, 7).

**b.** Find an equation for the line perpendicular to $y = 4 - x$ that passes through the point (3, 7).

**148.** Construct the equation of a line that goes through the origin and is parallel to the graph of given equation.

**a.** $y = 6$ **b.** $x = -3$ **c.** $y = -x + 3$

**149.** Construct the equation of a line that goes through the origin and is perpendicular to the given equation.

**a.** $y = 6$ **b.** $x = -3$ **c.** $y = -x + 3$

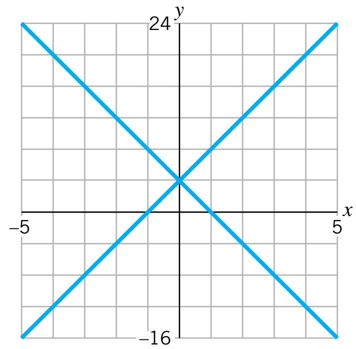
**150.** Which lines are parallel to each other? Which lines are perpendicular to each other?

**a.** $y = \frac{1}{3}x + 2$ **c.** $y = -2x + 10$ **e.** $2y + 4x = -12$

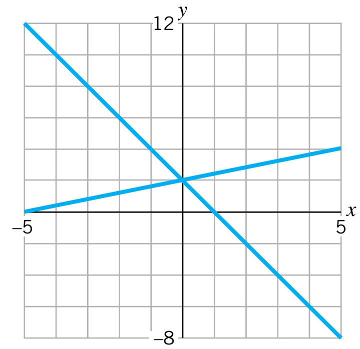
**b.** $y = 3x - 4$ **d.** $y = -3x - 2$ **f.** $y - 3x = 7$

**151.** Because of the scale that is chosen on graphing utilities, it is often difficult to tell if lines are perpendicular to each other. Using the intercepts in parts (a) and (b), determine the equation of the two lines and determine if they are perpendicular to each other.

**a.**




**b.**




**152.** Construct the equations of two lines that:

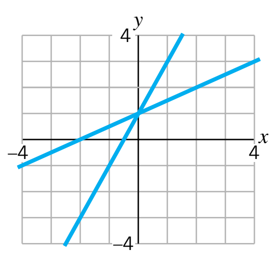
**a.** Are parallel to each other

**b.** Intersect at the same point on the $y$-axis

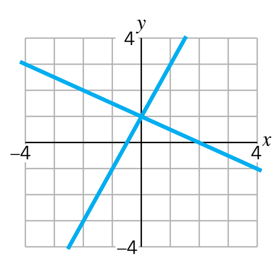
**c.** Both go through the origin

**d.** Are perpendicular to each other

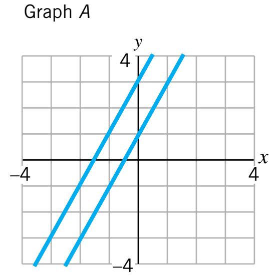
**153.** For each of the accompanying graphs you don't need to do any calculations or determine the actual equations. Rather, using just the graphs, determine if each pair of lines has the same slope. Are the slopes both positive or both negative, or is one negative and one positive? Do they have the same $y$-intercept?



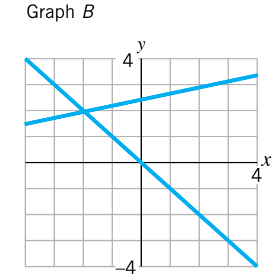

Graph *A*




Graph *B*




Graph *C*




Graph *D*

**154.** Find the equation of the line in the form $y = mx + b$ for each of the following sets of conditions. Show your work.

**a.** Slope is \$1400/year and line passes through the point (10 yr, \$12,000).

**b.** Line is parallel to $2y - 7x = y + 4$ and passes through the point $(-1, 2)$.

**c.** Equation is $1.48x - 2.00y + 4.36 = 0$.

**d.** Line is horizontal and passes through (1.0, 7.2).

**e.** Line is vertical and passes through (275, 1029).

**f.** Line is perpendicular to $y = -2x + 7$ and passes through (5, 2).

**155.** In the equation $Ax + By = C$:

**a.** Solve for $y$ so as to rewrite the equation in the form $y = mx + b$.

**b.** Identify the slope.

**c.** What is the slope of any line parallel to $Ax + By = C$?

**d.** What is the slope of any line perpendicular to $Ax + By = C$?

**156.** Use the results of Exercise 155, parts (c) and (d), to find the slope of any line that is parallel and then one that is perpendicular to the given lines.

**a.** $5x + 8y = 37$ **b.** $7x + 16y = -14$ **c.** $30x + 47y = 0$

## *Exercises for Section 2.9*

"Extended Exploration: Looking for Links between Education and Earnings," which follows this chapter, has many additional exercises that involve finding best-fit lines using technology.

**157.** Match each equation with the appropriate table and graph.

**a.** $y = 3x + 2$ **b.** $y = \frac{1}{2}x + 2$ **c.** $y = 1.5x + 2$

**d.**

| $x$ | $y$ |
|---|---|
| 0 | 2 |
| 2 | 3 |
| 4 | 4 |
| 6 | 5 |
| 8 | 6 |

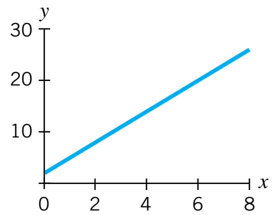
**e.**

| $x$ | $y$ |
|---|---|
| 0 | 2 |
| 2 | 5 |
| 4 | 8 |
| 6 | 11 |
| 8 | 14 |

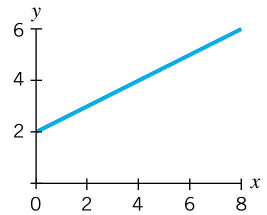
**f.**

| $x$ | $y$ |
|---|---|
| 0 | 2 |
| 2 | 8 |
| 4 | 14 |
| 6 | 20 |
| 8 | 26 |

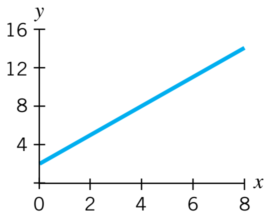
**g.**




**h.**




**i.**




**158.** Match each of the equations with the appropriate table and graph.

**a.** $y = 10 - 2x$ **b.** $y = 10 - 5x$ **c.** $y = 10 - 0.5x$

**d.**

| $x$ | $y$ |
|---|---|
| 0 | 10 |
| 2 | 0 |
| 4 | −10 |
| 6 | −20 |
| 8 | −30 |

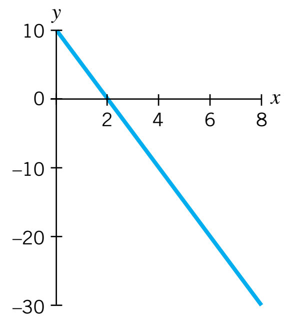
**e.**

| $x$ | $y$ |
|---|---|
| 0 | 10 |
| 10 | 5 |
| 20 | 0 |
| 30 | −5 |
| 40 | −10 |

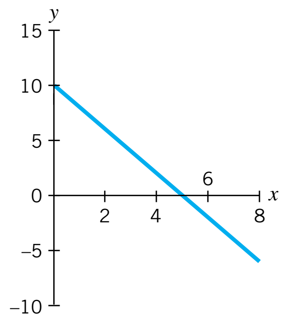
**f.**

| $x$ | $y$ |
|---|---|
| 0 | 10 |
| 2 | 6 |
| 4 | 2 |
| 6 | −2 |
| 8 | −6 |

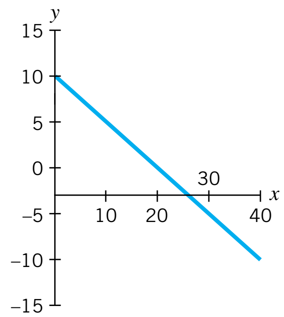
**g.**




**h.**




**i.**




**159.** Identify which of the following data tables represent exact and which approximate linear relationships. For the one(s) that are exactly linear, construct the corresponding equation(s). For the one(s) that are approximately linear, generate the equation of a best-fit line. [That is, plot the points, draw in a line approximating the data, pick two points on the line (not necessarily from your data) to generate the slope, and then construct the equation.]

**a.**

| | | | | | | |
|---|---|---|---|---|---|---|
| $x$ | −2 | −1 | 0 | 1 | 2 | 3 |
| $y$ | −6.5 | −5.0 | −3.5 | −2.0 | −0.5 | 1.0 |

**b.**

| | | | | | | |
|---|---|---|---|---|---|---|
| $t$ | −1 | 0 | 1 | 2 | 3 | 4 |
| $Q$ | 8.5 | 6.5 | 3.0 | 1.2 | −1.5 | −2 |

**c.**

| | | | | | | |
|---|---|---|---|---|---|---|
| $N$ | 0 | 15 | 23 | 45 | 56 | 79 |
| $P$ | 35 | 80 | 104 | 170 | 203 | 272 |

**160.** (Technology recommended) Plot the data in each of the following data tables. Determine which data are exactly linear and which are approximately linear. For those that are approximately linear, sketch a line that looks like a best fit to the data. In each case generate the equation of a line that you think would best model the data.

**a.** The number of cocaine-related emergency room episodes

| Year | 1994 | 1996 | 1998 | 2000 |
|---|---|---|---|---|
| Cocaine-related emergency room episodes (in thousands) | 143 | 152 | 172 | 174 |

*Source*: Centers for Disease Control, National Center for Health-Related Statistics.

**b.** The amount of tax owed on a purchase price.

| | | | | |
|---|---|---|---|---|
| Price | \$2.00 | \$5.00 | \$10.00 | \$12.00 |
| Tax | \$0.12 | \$0.30 | \$0.60 | \$0.72 |

**c.** The number of pounds in a number of kilograms.

| | | | | |
|---|---|---|---|---|
| Kilograms | 1 | 5 | 10 | 20 |
| Pounds | 2.2046 | 11.023 | 22.046 | 44.092 |

Use the linear equations found in parts (a), (b), and (c) to approximate the values for:

**d.** The number of cocaine-related emergency room episodes in 1997 and 2004

**e.** The amount of tax owed on \$7.79 and \$25.75 purchase prices

**f.** The number of pounds in 15 kg and 150 kg

**161.** Determine which data represent exactly linear and which approximately linear relationships. For the approximately linear data, sketch a line that looks like a best fit to the data. In each case generate the equation of a line that you think would best model the data.

**a.** The number of solar energy units consumed (in quadrillions of British thermal units, called Btus)

| | | | | |
|---|---|---|---|---|
| Year | 1994 | 1995 | 1997 | 1998 |
| Solar consumption in quadrillions of Btus | 0.07 | 0.07 | 0.07 | 0.07 |

*Source:* U.S. Census Bureau, *Statistical Abstract 2001.*

**b.** Gross farm output (billions of dollars)

| | | | | |
|---|---|---|---|---|
| Year | 1990 | 1994 | 1996 | 1998 |
| Gross farm output in billions \$ | 185.3 | 203.3 | 222.6 | 214.6 |

*Source:* U.S. Census Bureau, *Statistical Abstract 2001.*

Use the linear equations found in parts (a) and (b) to predict values for:

**c.** The number of solar energy units consumed in 1996 and 2004

**d.** The gross farm output in 1997 and 2003

**162.** In 1990, the United States imported \$917 million worth of wine. In 2000, the United States imported \$2209 million worth of wine.

**a.** Assuming the growth was linear, create a function that could model the trend in imported wines.

**b.** Estimate the dollar amount of imported wine for 1998. The actual amount was \$1876 million worth of wine. How accurate was your approximation?

**c.** If this trend continues, predict the dollar amount of imported wines in 2005.

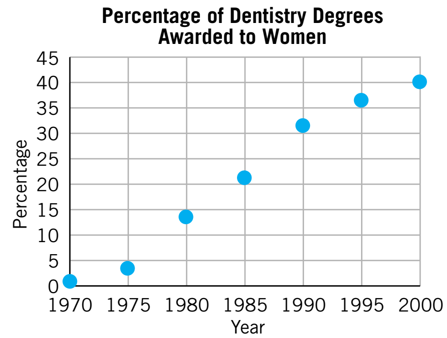
(*Source*: U.S. Department of Agriculture, Economic Research Service, February 2001)

**163.** The percentage of dentistry degrees awarded to women in the United States between 1970 and 2000 is shown in the accompanying table and graph.

**Percentage of Dentistry Degrees Awarded to Women**

| | | | | | | |
|---|---|---|---|---|---|---|
| 1970 | 1975 | 1980 | 1985 | 1990 | 1995 | 2000 |
| 0.9 | 3.1 | 13.3 | 20.7 | 30.9 | 36.4 | 40.1 |

*Source:* U.S. National Center for Education Statistics, "Digest of Education Statistics," annual, *Statistical Abstract of the United States,* 2002.




The data show that since 1970 the percentage of dentists who are women has been rising.

**a.** Sketch a line that best represents the data points. What is the rate of change of percentage of dentistry degrees awarded to women according to your line?

**b.** If you extrapolate your line, estimate when 100% of dentistry degrees will be awarded to women.

**c.** Since it seems extremely unlikely that 100% of dentistry degrees will ever be granted to women, comment on what is likely to happen to the rate of growth of women's degrees in dentistry; sketch a likely graph for the continuation of the data into this new century.

**164.** The accompanying table and graph show the mortality rates (in deaths per 1000) for male and female infants in the United States from 1980 to 2000.

**a.** Sketch a line through the graph of the data that best represents female infant mortality rates. Does the line seem to be a reasonable model for the data? What is the approximate slope of the line through these points? Show your work.

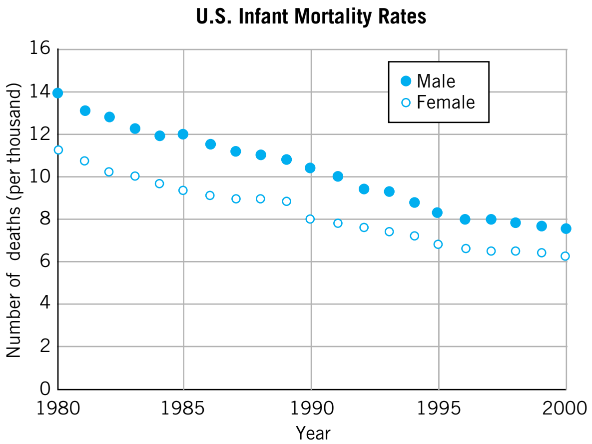
**b.** Sketch a similar line through the male mortality rates. Is it a reasonable approximation? Estimate its slope. Show your work.

**c.** List at least two important conclusions from the data set.

**U.S. Infant Mortauty Rates**

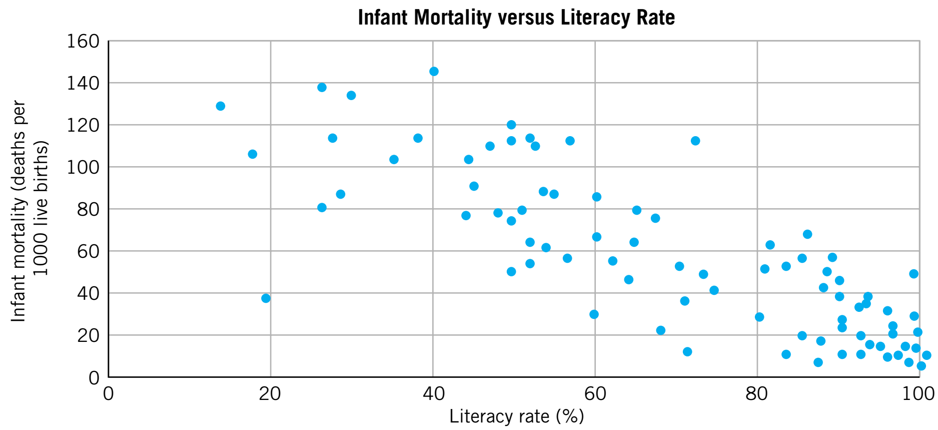
| Year | Male | Female |
|---|---|---|
| 1980 | 13.9 | 11.2 |
| 1981 | 13.1 | 10.7 |
| 1982 | 12.8 | 10.2 |
| 1983 | 12.3 | 10.0 |
| 1984 | 11.9 | 9.6 |
| 1985 | 11.9 | 9.3 |
| 1986 | 11.5 | 9.1 |
| 1987 | 11.2 | 8.9 |
| 1988 | 11.0 | 8.9 |
| 1989 | 10.8 | 8.8 |
| 1990 | 10.3 | 8.1 |
| 1991 | 10.0 | 7.8 |
| 1992 | 9.4 | 7.6 |
| 1993 | 9.3 | 7.4 |
| 1994 | 8.8 | 7.2 |
| 1995 | 8.3 | 6.8 |
| 1996 | 8.0 | 6.6 |
| 1997 | 8.0 | 6.5 |
| 1998 | 7.8 | 6.5 |
| 1999 | 7.7 | 6.4 |
| 2000 | 7.6 | 6.2 |

*Source:* Centers for Disease Control and Prevention, *www.cdc.gov*



DATA

**165.** The accompanying scatter plot shows the relationship between literacy rate (the percentage of the population who can read and write) and infant mortality rate (infant deaths per 1000 live births) for 91 countries. The raw data are contained in the Excel or graph link file NATIONS and are described at the end of the Excel file. (You might wish to identify the outlier, the country with about a 20% literacy rate and a low infant mortality rate of about 40 per 1000 live births.) Construct a linear model. Show all your work and clearly identify the variables and units. Interpret your results.

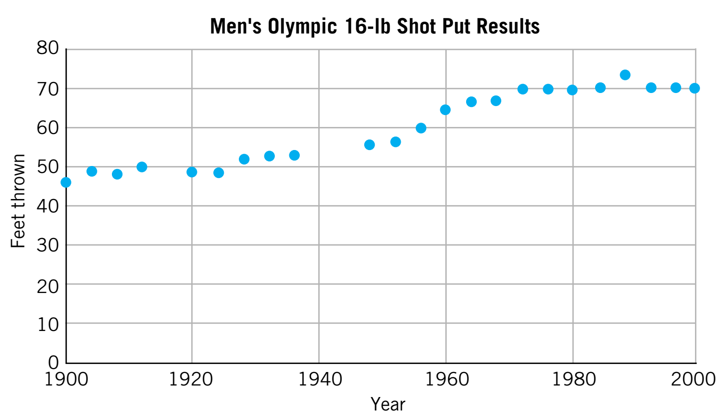



**166.** The accompanying table and graph show data for the men's Olympic 16-pound shot put.

**Olympic Shot Put**

| Year | Feet Thrown | Year | Feet Thrown |
|---|---|---|---|
| 1900 | 46 | 1960 | 65 |
| 1904 | 49 | 1964 | 67 |
| 1908 | 48 | 1968 | 67 |
| 1912 | 50 | 1972 | 70 |
| 1920 | 49 | 1976 | 70 |
| 1924 | 49 | 1980 | 70 |
| 1928 | 52 | 1984 | 70 |
| 1932 | 53 | 1988 | 74 |
| 1936 | 53 | 1992 | 71 |
| 1948 | 56 | 1996 | 71 |
| 1952 | 57 | 2000 | 70 |
| 1956 | 60 | | |

*Source:* Reprinted with permission from *The 1996 Universal Almanac,* Universal Press Syndicate. 2000 result from *www.cnnsi.com.*



**a.** Draw a line approximating the data and find its equation. Show your work and interpret your results. For ease of calculation you may wish to think of 1900 as year 0 and let your $x$-coordinate measure the number of years since 1900.

**b.** If the shot put results continued to change at the same rate, in what year would you predict that the winner will put the shot a distance of 80 feet? Does this seem like a realistic estimate? Why or why not?

**167.** The accompanying table compares the trends in home computer ownership in the United States, Japan, and Europe. It would be a typical part of a presentation by an outside information specialist to a high-technology corporation to help in formulating a business strategy.

**Home Computer Ownership**

| | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 |
|---|---|---|---|---|---|---|
| **United States** | | | | | | |
| Home computers in use (millions) | 33.1 | 37.7 | 42.3 | 46.9 | 51.4 | 56.2 |
| Household penetration (%) | 32 | 36 | 40 | 44 | 47 | 51 |
| **Japan** | | | | | | |
| Home computers in use (millions) | 5.4 | 6.9 | 8.5 | 10.1 | 12.1 | 14.8 |
| Household penetration (%) | 13 | 16 | 20 | 24 | 28 | 34 |
| **Europe** | | | | | | |
| Home computers in use (millions) | 22.6 | 26.1 | 30.0 | 34.3 | 39.0 | 44.3 |
| Household penetration (%) | 15 | 17 | 20 | 22 | 25 | 28 |

Numbers for 2000 and 2001 are projections.
*Source: www.infoplease.com*

**a.** On a single graph, plot the number of home computers (in millions) in use in the United States, Japan, and Europe between 1996 and 2001. For ease of use, connect the dots for each separate region.

**b.** Does the growth in number of home computers seem roughly linear for each of these regions? If so, construct a linear model for:

**i.** The United States **ii.** Japan **iii.** Europe

In each case, interpret the average rate of change of your model.

**iv.** If the current average rates of change continue, what would be the number of home computers in use in 2005 in the United States? In Japan? In Europe?

**c.** On a single new graph, plot the household penetration (in %) of computers in the United States, Japan, and Europe between 1996 and 2001. Again connect the points for each region.

**d.** Does the growth in household penetration seem roughly linear for each of these regions? If so, construct a linear model for:

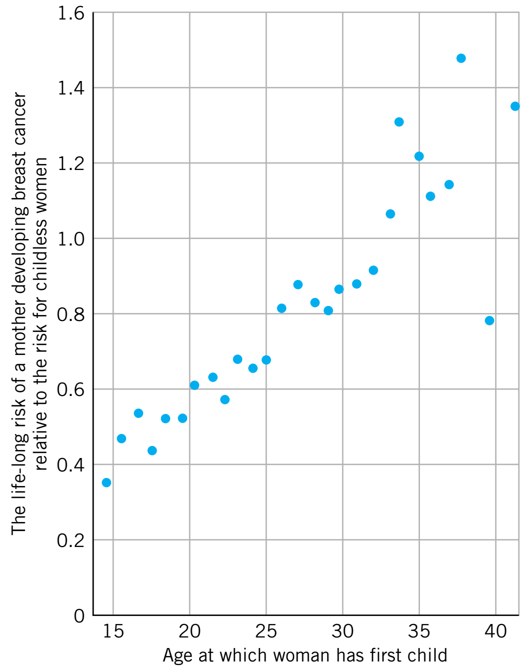
**i.** The United States **ii.** Japan **iii.** Europe

Again in each case, interpret the average rate of change of your model.

**iv.** If the current average rates of change continue, what would be the percentage of household penetration in 2005 in the United States? In Japan? In Europe?

**e.** Write a 60-second summary describing the growth in home computer ownership.

**168.** The accompanying graph shows the relationship between the age of a woman when she has her first child and her life-long risk of getting breast cancer relative to a childless woman.




*Source:* J. Cairns, *Cancer: Science and Society* (San Francisco: W.H. Freeman, 1978), p. 49.

**a.** If a woman has her first child at age 18, approximately what is her risk of developing cancer relative to a woman who has never born a child?

**b.** At roughly what age are the chances the same that a woman will develop breast cancer whether or not she has a child?

**c.** If a mother is beyond the age you specified in part (b), is she more or less likely to develop breast cancer than a childless woman?

**d.** Sketch a line that looks like a best fit to the data, estimate the coordinates of two points on the line, and use them to calculate the slope.

**e.** Interpret the slope in this context.

**f.** Construct a linear model for these data, identifying your independent and dependent variables.

**169.** (Technology recommended) The given data show that health care is becoming more expensive and is taking a bigger share of the U.S. gross domestic product (GDP). The GDP is the market value of all goods and services that have been bought for final use.

| Year | 1960 | 1965 | 1970 | 1975 | 1980 | 1985 | 1990 | 1995 | 2000 |
|---|---|---|---|---|---|---|---|---|---|
| U.S. health care costs as a percentage of GDP | 5.1 | 5.7 | 7.1 | 8.0 | 8.9 | 10.2 | 12.2 | 13.6 | 13.3 |
| Amount per person, $ | 141 | 202 | 341 | 582 | 1052 | 1733 | 2691 | 3633 | 4675 |

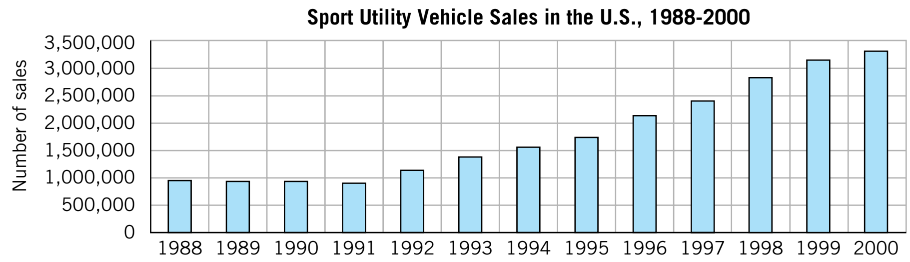
*Source:* U.S. Health Care Financing Administration and Centers for Medicare and Medicaid Services.

**a.** Graph health care costs as a percentage of GDP versus year with time on the horizontal axis. Measure time in years since 1960. Draw a straight line by eye that appears to be the closest fit to the data. Figure out the slope of your line and write an equation for $H$, health care percentage of the GDP, as a function of $t$, year's since 1960.

**b.** What does your formula predict for health care as a percentage of GDP for the year 2010?

**c.** Why do you think the health care cost per person has gone up so much more dramatically than the health care percentage of the GDP?

**170. a.** From the accompanying chart showing sport utility vehicle (SUV) sales, estimate what the rate of increase in sales has been from 1991 to 2000. (*Hint:* Convert the chart into an equivalent scatter plot.)




*Source*: American Automobile Manufacturers Association and Office of Transportation Technologies, *www.ott.doe.gov.*

**b.** Estimate a linear formula to represent sales in years since 1991.

**c.** If the popularity of SUVs continues to grow at the same rate, how many would be sold in 2005?

**171.** The Gas Guzzler Tax is imposed on manufacturers on the sale of new-model cars (*not* minivans, sport utility vehicles, or pickup trucks) whose fuel economy fails to meet certain statutory regulations, to discourage the production of fuel-inefficient vehicles. The tax is collected by the IRS and paid by the manufacture. The table shows the amount of tax that the manufacturer must pay for a vehicle's miles per gallon fuel efficiency.

**a.** Assuming that the data are linear, plot the data and add a line of best fit.

**b.** Choose two points on the line, find the slope, and then form a linear equation with $x$ as the fuel efficiency in mpg and $y$ as the tax in dollars.

**c.** What is the rate of change of the amount of tax imposed on fuel-inefficient vehicles? Interpret the units.

**Gas Guzzler Tax**

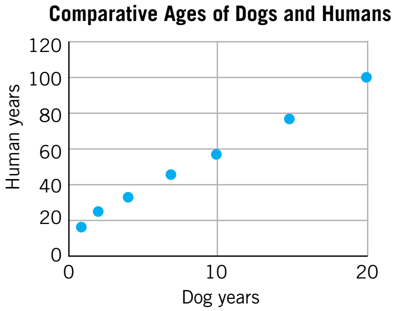
| MPG | Tax per Car |
|---|---|
| 12.5 | \$6400 |
| 13.5 | \$5400 |
| 14.5 | \$4500 |
| 15.5 | \$3700 |
| 16.5 | \$3700 |
| 17.5 | \$2600 |
| 18.5 | \$2100 |
| 19.5 | \$1700 |
| 20.5 | \$1300 |
| 21.5 | \$1000 |
| 22.5 | \$0 |

*Source: http://www.epa.gov/otaq/cert/factshts/fefact 0.1.pdf*

**172.** A veterinarian's office displayed the following table comparing dog age (in dog years) to human age (in human years). The chart shows that the relationship is fairly linear.

**Comparative Ages of Dogs and Humans**

| Dog Years | Human Years |
|---|---|
| 1 | 15 |
| 2 | 24 |
| 4 | 32 |
| 7 | 45 |
| 10 | 56 |
| 15 | 76 |
| 20 | 98 |




**a.** Draw a line that looks like a best fit to the data.

**b.** Estimate the coordinates and label two points on the line. Use them to find the slope. Interpret the slope in this context.

**c.** Using $H$ for human age and $D$ for dog age, identify which you are using as the independent and which you are using for the dependent variable.

**d.** Generate the equation of your line.

**e.** Use the linear model to determine the "human age" of a dog who is 17 dog years old.

**f.** Middle age in humans is 45-59 years. Use your model equation to find the corresponding middle age in dog years.

**g.** What is the domain for this data?

DATA

**173.** The American Automobile Association published these cost-of-driving estimates for the decade up to 2003: (See Excel and graph link file DRIVCOST.)

**Cost-of-Driving Estimates**

| Year | 1993 | 1994 | 1995 | 1996 | 1997 | 1998 | 1999 | 2000 | 2001 | 2002 | 2003 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $/year | 5804 | 5916 | 6185 | 6389 | 6723 | 6908 | 7050 | 7363 | 7654 | 7533 | 7754 |
| ¢/mile | 38.7 | 39.4 | 41.2 | 42.6 | 44.8 | 46.1 | 47.0 | 49.1 | 51.0 | 50.2 | 51.7 |

**a.** Make a plot of cents per mile versus year, putting year on the horizontal axis.

**b.** On your plot draw a straight line by eye that is your best approximation of the data. Estimate the slope of the line. What is the practical meaning of this slope?

**c.** To compute dollars per year of driving, the people who gathered this cost data had to estimate how many miles the average driver goes in a year. From these data figure out how many miles the estimate was.

**d.** Write a paragraph giving a summary of what has happened to the cost of driving over this decade, and mention what factors are major influences on the rise or fall of driving cost.

# EXPLORATION 2.1

## Having It Your Way

### Objective

- construct arguments supporting opposing points of view from the same data

### Material/Equipment

- excerpts from the *Student Statistical Portrait* of the University of Massachusetts, Boston, or from the equivalent for the student body at your institution
- computer with spreadsheet program and printer or graphing calculator with projection system (all optional)
- graph paper and/or overhead transparencies (and overhead projector)

### Procedure

#### Working in Small Groups

Examine the data and graphs from the Student Statistical Portrait of the University of Massachusetts, Boston, or from your own institution. Explore how you would use the data to construct arguments that support at least two different points of view. Decide on the arguments you are going to make and divide up tasks among your team members.

#### Rules of the Game

- Your arguments needn't be lengthy, but you need to use graphs and numbers to support your position. You may use only legitimate numbers, but you are free to pick and choose those that support your case. If you construct your own graphs, you may, of course, use whatever scaling you wish on the axes.
- For any data that represent a time series, as part of your argument, pick two appropriate end points and calculate the associated average rate of change.
- Use "loaded" vocabulary (e.g., "surged ahead," "declined drastically"). This is your chance to be outrageously biased, write absurdly flamboyant prose, and commit egregious sins of omission.
- Decide as a group how to present your results to the class. Some students enjoy realistic "role playing" in their presentations and have added creative touches such as mock protesters complete with picket signs.

#### Suggested Topics

Your instructor might ask your group to construct one or both sides of the arguments on one topic. If you're using data from your own institution, answer the questions provided by your instructor.

### Using the *Student Statistical Portrait* from the University of Massachusetts, Boston

**1.** You are the Dean of the College of Management. Use the data on "SAT Scores of New Freshmen by College/Program" to make the case that:

**a.** The freshmen admitted to the College of Management are not as prepared as the students in the College of Arts and Sciences, and therefore you need more resources to support the freshmen in your program.

**b.** The freshmen admitted to the College of Management are better prepared than the students in the College of Arts and Sciences, and therefore you would like to expand your program.

**2.** You are an Associate Provost lobbying the state legislature. Use the data on "Undergraduate Admissions" to present a convincing argument that:

**a.** UMass is becoming less desirable as an institution for undergraduates, and so more funds are needed to strengthen the undergraduate program.

**b.** UMass is becoming more desirable as an institution for undergraduates, and so more funds are needed to support the undergraduate program.

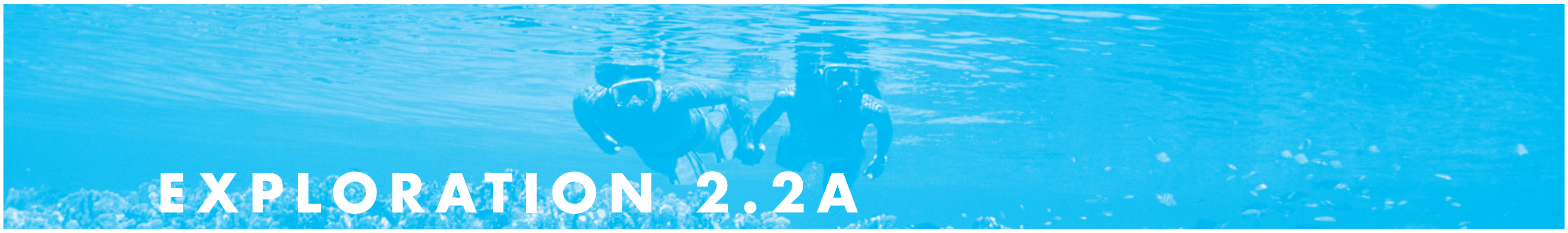
**3.** Use the data on "Distribution of High School Rank and SAT Scores" to support each of the following:

**a.** You are the president of the student body, arguing that UMass/Boston is becoming a more elite institution and hence is turning its back on its urban mission.

**b.** You are the head of the Honors Society at UMass/Boston, writing a letter to the student newspaper proclaiming that the university is lowering its academic standards and hence is in danger of compromising its academic credibility.

### Exploration-Linked Homework

With your partner or group prepare a short class presentation of your arguments, using, if possible, overhead transparencies or a projection panel. Then write individual 60-second summaries to hand in.




# Looking at Lines with the Course Software

## Objective

- find patterns in the graphs of linear equations of the form $y = mx + b$

## Equipment

- computer with course software "L1: *m* & *b* Sliders" in *Linear Functions*

## Procedure

In each part try working first in pairs, comparing your observations and taking notes. Your instructor may then wish to bring the whole class back together to discuss everyone's results.

### Part I: Exploring the Effect of $m$ and $b$ on the Graph of $y = mx + b$

Open the program *Linear Functions* and click on the button "L1: *m* & *b* Sliders."

**1.** What is the effect of $m$ on the graph of the equation?

Fix a value for $b$. Construct four graphs with the same value for $b$ but with different values for $m$. Continue to vary $m$, jotting down your observations about the effect on the line when $m$ is positive, negative, or equal to zero. Do you think your conclusions work for values of $m$ that are not on the slider?

Choose a new value for $b$ and repeat your experiment. Are your observations still valid? Compare your observations with those of your partner.

**2.** What is the effect of $b$ on the graph of the equation?

Fix a value for $m$. Construct four graphs with the same value for $m$ but with different values for $b$. What is the effect on the graph of changing $b$? Record your observations. Would your conclusions still hold for values of $b$ that are not on the slider?

Choose a new value for $m$ and repeat your experiment. Are your observations still valid? Compare your observations with those of your partner.

**3.** Write a 60-second summary on the effect of $m$ and $b$ on the graph of $y = mx + b$.

### Part II: Constructing Lines under Certain Constraints

**1.** Construct the following sets of lines still using "L1: *m* & *b* Sliders." Be sure to write down the equations for the lines you construct. What generalizations can you make about the lines in each case? Are the slopes of the $y$-intercepts of the lines related in some way?

Construct any line. Then construct another line that has a steeper slope, and then construct one that has a shallower slope.

Construct three *parallel lines.*

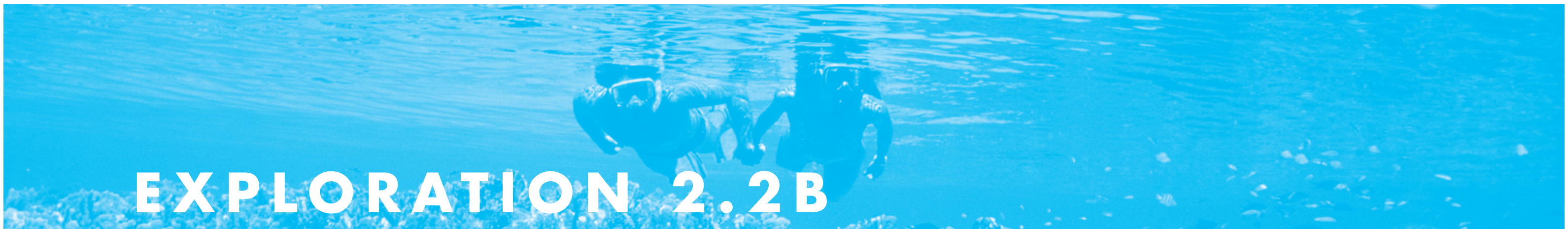
Construct three lines with the *same* $y$*-intercept*, the point where the line crosses the $y$-axis.

Construct a pair of lines that are *horizontal*.

Construct a pair of lines that go *through the origin*.

Construct a pair of lines that are *perpendicular* to each other.

**2.** Write a 60-second summary of what you have learned about the equations of lines.




## Looking at Lines with a Graphing Calculator

### Objective

- find patterns in the graphs of linear equations of the form $y = mx + b$

### Material/Equipment

- graphing calculator (instructions for the T1-82, T1-83, and TI-84 are available in the Graphing Calculator Workbook)

### Procedure

#### Getting Started

Set your calculator to the integer window setting. For the T1-82 or T1-83, do the following:

```
WINDOW FORMAT
Xmin=–47
Xmax=47
Xscl=10
Ymin=–31
Ymax=31
Yscl=10
```

**1.** Press ZOOM and select [6:ZStandard].
**2.** Press ZOOM and select [8:ZInteger], ENTER.
**3.** Press WINDOW to see whether the settings are the same as the duplicated screen image.

#### Working in Pairs

In each part try working first in pairs, comparing your observations and taking notes. Your instructor may then wish to bring the whole class back together to discuss everyone's results.

#### Part I: Exploring the Effect of $m$ and $b$ on the Graph of $y = mx + b$

**1.** What is the effect of $m$ on the graph of the equation $y = mx$?

**a.** Case 1: $m > 0$

Enter the following functions into your calculator and then sketch the graphs by hand. To get started, try $m = 1, 2, 5$. Try a few other values of $m$ where $m > 0$.

Y1 $= x$
Y2 $= 2x$
Y3 $= 5x$
Y4 $= \cdots$
Y5 $=$
Y6 $=$

Compare your observations with those of your partner. In your notebook describe the effect of multiplying $x$ by a positive value for $m$ in the equation $y = mx$.

**b.** Case 2: $m < 0$

Begin by comparing the graphs of the lines when $m = 1$ and $m = -1$. Then experiment with other negative values for $m$ and compare the graphs of the equations.

$Y1 = x$
$Y2 = -x$
$Y3 = \cdots$
$Y4 =$
$Y5 =$
$Y6 =$

Alter your description in part 1(a) to describe the effect of multiplying $x$ by any real number $m$ for $y = mx$ (remember to also explore what happens when $m = 0$).

**2.** What is the effect of $b$ on the graph of an equation $y = mx + b$?

**a.** Enter the following into your calculator and then sketch the graphs by hand. To get started, try $m = 1$ and $b = 0, 20, -20$. Try other values for $b$ as well.

$Y1 = x$
$Y2 = x + 20$
$Y3 = x - 20$
$Y4 = \cdots$
$Y5 =$
$Y6 =$

**b.** Discuss with your partner the effect of adding any number $b$ to $x$ for $y = x + b$. (*Hint:* Use "trace" to find where the graph crosses the $y$-axis.) Record your comments in your notebook.

**c.** Choose another value for $m$ and repeat the exercise. Are your observations still valid?

**3.** Write a 60-second summary on the effect of $m$ and $b$ on the graph of $y = mx + b$.

## Part II: Constructing Lines under Certain Constraints

**1.** Construct the following sets of lines using your graphing calculator. Be sure to write down the equations for the lines you construct. What generalizations can you make about the lines in each case? Are the slopes of the lines related in any way? Are the vertical intercepts related? Which graph is the steepest?

Construct three *parallel lines*.

Construct three lines with the *same y-intercept*.

Construct a pair of lines that are *horizontal.*

Construct a pair of lines that go *through the origin*.

Construct a pair of lines that are *perpendicular* to each other.

**2.** Write a 60-second summary of what you have learned about the equations of lines.