Georgia State University
(*This paper consists of pages.*)

Test III **April 7, 2003** **Points: 91 & + is A**

Last name: ____________________
First name: ____________________

POINTS

Show all of your work. Calculators are not needed or permitted. Write neatly. Place answers in the space provided.

1 (10 points). Find the domain and the range of the following function

$$f(x, y, z) = \frac{\sqrt{1 - x^2} + \sqrt{y^2 - 4}}{1 - \sqrt{9 - z^2}}$$

2 (15 points). Identify the given surfaces and sketch them

$$x - y^2 - 6z^2 = 0, \qquad \frac{x^2}{4} - \frac{y^2}{9} = 1$$

3 (15 points). Find $f_x(x,y,z)$ and $f_z(x,y,z)$ by forming the appropriate *difference quotient* and taking the limit as $h \to 0$.

4 (20 points). Let $z = x^2 - y^2$ and let C be the curve of intersection of the surface with the plane $y = 3$. Find the equation for the tangent line to the graph of C at the point $(-3, 3, 0)$.

5 (15 points). Show that

$$\lim_{(x,y)\to(0,0)} \frac{x^2 - y^2}{x^2 + y^2}$$

does not exist.

6 (20 points). Determine whether a function $z = f(x, y)$ with the following gradient $\bigtriangledown f(z, y) = (x^2 + y)\mathbf{i} + (y^3 + x)\mathbf{j}$ may exist. If so, find such a function. Hint: the mixed partials test.

7 (30 points). Find the directional derivative of $f(x, y, z) = x^2 + yz$ at $(1, -3, 2)$ in the direction of the path $\mathbf{r}(t) = t^2\mathbf{i} + 3t\mathbf{j} + (1 - t^3)\mathbf{k}$. Hint: firstly, find $\mathbf{r}'(t)$ at the moment of $t =?$ when $\mathbf{r}(t)$ goes through the given point.

8 (25 points). The radius r of a right circular cylinder decreases at the rate of 2 centimeters per second. At what rate the height h of the cylinder should change in order for its volume not to change at the instant when $r = 10$ and $h = 5$.

9 (30 points). Let $f(x, y) = x^2 + y^2 - 1$ be C^1 everywhere. Let $\mathbf{a}(0, 0)$ and $\mathbf{b}(1, 1)$. Find the point $\mathbf{c}$ on the line segment connecting $\mathbf{a}$ and $\mathbf{b}$ where $f(\mathbf{b}) - f(\mathbf{a}) = \triangledown f(\mathbf{c}) \cdot (\mathbf{b} - \mathbf{a})$.

10 (Bonus 15 points). Let $f(x, y) = 2xy + \sin(xe^y)$. Find dy/dx.

11 (20 points). Show that the sphere $x^2 + y^2 + z^2 - 8x - 8y - 6z + 24$ is tangent to the ellipsoid $x^2 + 3y^2 + 2z^2 = 9$ at the point $(2, 1, 1)$. Find the equations of the tangents planes at this point. What is the equation of the normal line at the point of tangency?