Georgia State University
(*This paper consists of* **11** *pages.*)

Test III **July 26, 2002** **Points: 91+ means A**

Last name: _____
First name: _____

POINTS

You have a choice of 8 problems ranked from 10 through 25 points. If your overall score exceeds 91 points your grade will be an "A".

Show all of your work. Calculators are not needed or permitted. Write neatly. Place answers in the space provided.

1 (10 points). Show that the following function has no limit at $(0, 0)$:

$$f(x, y) = \frac{x^2 - y^2}{x^2 + y^2}.$$

2 (10 points). Is there a function $f(x, y)$ with

$$\frac{\partial f}{\partial x} = \sin(y) - y \quad \text{and} \quad \frac{\partial f}{\partial x} = x\cos(y) - x?$$

3 (20 points). Find the directional derivative of $f(x, y, z) = z \ln(\frac{x}{y})$ at $(1, 1, 2)$ toward the point $(2, 2, 1)$ [10 points]. Find the unit vector in direction the function descends most rapidly and the direction the function does NOT change in at $(1, 1, 2)$ [10 points].

4 (20 points). Find a point **c** on the segment connecting $(0, 1, 1)$ and $(1, 3, 2)$ at which $f(x, y, z) = 4xz - y^2 + z^2$ satisfies the mean value theorem.

5 (15 points). Find the rate of change of $f(x, y) = xe^y + ye^{-x}$ with respect to t along the path $\mathbf{r}(t) = (\ln t)\mathbf{i} + t(\ln t)\mathbf{j}$ at $t = 1$

6 (20 points). Show that the sphere $x^2 + y^2 + z^2 - 8x - 8y - 6z + 24$ is tangent to the ellipsoid $x^2 + 3y^2 + 2z^2 = 9$ at the point $(2, 1, 1)$. Find the equations of the tangents planes at this point. What is the equation of the normal line at the point of tangency?

7 (20 points). Classify the stationary points of $f(x, y) = x^4 - 2x^2 + y^2 - 2$. The second derivative test is rather useful here. Find the absolute max and min values of the function within the disk $x^2 + y^2 \leq 25$

8 (15 points). Find the critical points of $f(x, y) = (4 - x - y)xy$ and determine their nature.

9 (25 points). Find the point of the plane $3x - 4y + 2z + 32$ that is closest to the point $P(-1, 2, 4)$ and and distance between the points. Estimate also the distance from the point P to the plane. Hint: $\text{dist}(P; \text{plane}) = \frac{|Ax_p + By_p + Cz_p + D|}{\sqrt{A^2 + B^2 + C^2}}$.

10 (10 points). Find dy/dx of y implicitly defined as a differentiable function of x:

$$x^2 - 2xy + y^4 = 4.$$

Hint: Apply either the chain rule from Cacl III or implicit differentiation from Calc I, which is the same indeed.

11 (BONUS 10+15 points). The temperature near a heater at the origin is given by

$$T(x, y) = 115F + e^{-y}sin(x).$$

In what direction should not a heat-*avoiding* beetle run? Can you find its escaping path from the origin?