Georgia State University
(*This paper consists of* **11** *pages.*)

Final Exam **May 3, 2001**

Last name: _____
First name: _____

POINTS

Show all of your work. Calculators are not needed or permitted. Write neatly. Place answers in the space provided.

(10 pts) Show that the vectors $\overrightarrow{AB} = (1, 1, -1)$, $\overrightarrow{AC} = (2, 3, -2)$ and $\overrightarrow{AD} = (4, 5, -4)$ are coplanar.

(10 pts) Find the area of the triangle with the vertices $A(3, 0, -10)$, $B(4, 2, 5)$, $C(7, -2, 4)$.

(10 pts) Find the distance from the point $C(7, -2, 4)$ to the line through the points $A(3, 0, -10)$ and $B(4, 2, 5)$.

(20 pts) Find the equation for each of the following planes:

a) Plane containing the point $(2, -1, 3)$ and perpendicular to the line

$$x = 1 + 3t, \quad y = 4t, \quad z = 2 - t$$

b) Plane containing the points $P(1, 1, 1)$, $Q(2, 1, 3)$ and $R(1, -1, 2)$.

(25 pts) A surface is represented by the equation $F(x, y, z) = xy + 2xz^2 + 3yz = 56$. Find

a. (10 pts) the equation of the plane tangent to this surface at$(2, 1, 3)$;

b (10 pts) Find the directional derivative of $F(x, y, z)$ at the point $(2, 1, 3)$ in the direction of $\mathbf{v} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$;

c. (5 pts) Find $\frac{\partial z}{\partial y}$ on this surface at $(2, 1, 3)$.

(20 pts) Find and classify the stationary points of the function $f(x, y) = x^3 - xy^3 + 3xy$.

(25 pts) Use the method of Lagrange multipliers to find the largest value and the smallest value of $f(x, y, z) = xz + y^2$ on the sphere $x^2 + y^2 + z^2 = 4$.

(30 pts) Sketch the domain the area of which is given by the integral

$$\int_0^1 \int_{\sqrt{y}}^{\sqrt{2-y^2}} dx\, dy.$$

Change the order of integration and find the area.

(30 pts) Find the volume of the 3-D region enclosed by the surfaces $y = x^2$, $y = 4$, $z = 5 + x$, $z = 2$.

(20 pts) Find the volume of the solid T that is bounded by the paraboloid $z = 3(x^2 + y^2)$ and the plane $z = 12$ (Hint: use cylindrical coordinates).

Bonus (20 pts) Evaluate the repeated integral by changing to spherical coordinates

$$\int_0^2 \int_0^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} (x^2+y^2+z^2)\, dz\, dx\, dy$$