**Georgia State University**
(*This paper consists of* **10** *pages.*)

**Exam I** **February 06, 2001** **Points: 100 — A**

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**Last name:** ____________________
**First name:** ____________________

| POINTS |
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You have a choice of 12 different problems ranked from 5 through 25 points. If your overall score is, at least, 100 points you will get an "A". The points above 100 go as an extra credit towards the final grade.

Show all of your work. Calculators are not needed or permitted. Write neatly. Place answers in the space provided.

**1 (10 points).** Find the distance between the center of the sphere

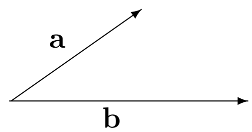
$$x^2 + y^2 + z^2 - 2x - 6z = 0$$

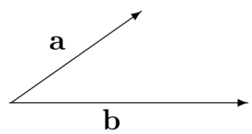
and the line

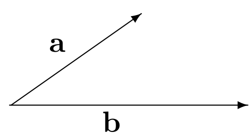
$$\frac{x+2}{2} = \frac{y}{1} = \frac{z+3}{2}.$$

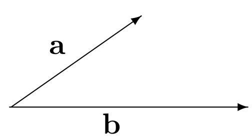
**2 (5 points).** Where does the line $\frac{x+2}{2} = \frac{y}{1} = \frac{z+3}{2}$ intersect the plane $x - y + z - 1 = 0$?

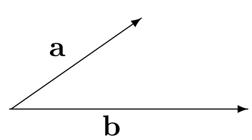
**3 (10 points).** Draw $\mathbf{a}+\mathbf{b}$, $\mathbf{a}-\mathbf{b}$, $\mathbf{b}-\mathbf{a}$, $-\mathbf{a}-\mathbf{b}$, $\frac{1}{2}(\mathbf{a}+\mathbf{b})$:



**4 (20 points).** Given the three points $P(1,0,2)$, $Q(2,2,1)$ and $R(0,1,4)$, let $\Delta PQR$ denote the triangle having $P$, $Q$ and $R$ as vertices. Find:

the area of $PQR$

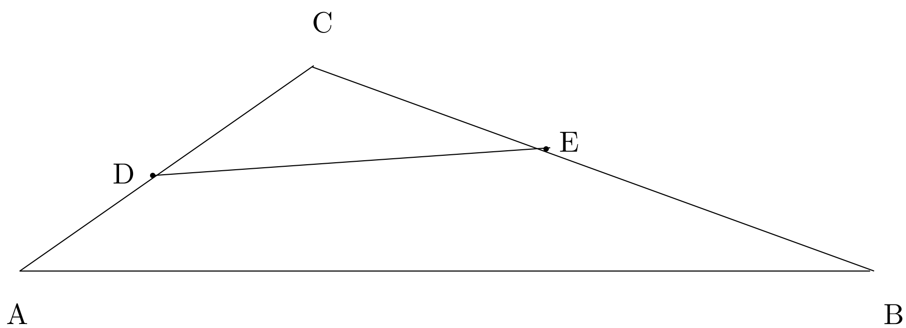
the angle of $\Delta PQR$ at the vertex $P$

the equation of the plane containing the points $P$, $Q$ and $R$

equations for the line through the point $P$ which is perpendicular to the plane containing $\Delta PQR$.

**5 (10 points).** Find the volume of the parallelepiped whose edges are $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$ with $A(1,2,3)$, $B(1,-1,2)$, $C(3,4,7)$, $D(2,4,0)$.

**6 (25 points).** The vertices of the triangle $ABC$ are $A(1,1)$, $B(1,8)$, and $C(4,3)$. Points $D$ and $E$ are located on $AC$ and $BC$, and $\frac{d(CD)}{d(CA)} = \frac{3}{5}$, $\frac{d(CE)}{d(CB)} = \frac{1}{2}$. Using the cross product find the area of the triangle $DCE$.



**7 (5 points).** Determine whether the vectors are coplanar

$$\mathbf{i}, \quad \mathbf{i} - 2\mathbf{j}, \quad 3\mathbf{j} + \mathbf{k}.$$

**8 (15 points).** Let $l_1$ and $l_2$ be lines that pass though the origin and have direction vectors

$$\mathbf{d}_1 = \mathbf{i} + 2\mathbf{j} + 4\mathbf{k} \quad \text{and} \quad \mathbf{d}_2 = -\mathbf{i} - \mathbf{j} + 3\mathbf{k}.$$

Find an equation of the plane that contains $l_1$ and $l_2$.

**9 (15 points).** Find $\mathbf{f}(t)$ from the following information

$$\mathbf{f}'(t) = \mathbf{i} + t^2\mathbf{j} \quad \text{and} \quad \mathbf{f}(0) = \mathbf{j} - \mathbf{k}$$

**10 (15 points).** Calculate $\mathbf{r}(t)\cdot\mathbf{r}'(t)$ and $\mathbf{r}(t)\times\mathbf{r}'(t)$ given that $\mathbf{r}(t)=\cos t\mathbf{i}+\sin t\mathbf{j}$

**11 (20 points).** Find the point at which the curves

$$\begin{aligned}\mathbf{r_1}(t) &= e^t\mathbf{i}+2\sin(t+\tfrac{1}{2}\pi)\mathbf{j}+(t^2-2)\mathbf{k}\\ \mathbf{r_2}(t) &= u\mathbf{i}+2\mathbf{j}+(u^2-3)\mathbf{k}\end{aligned}$$

intersect and find the angle.

**12 (5 points).** Find an equation in $x$ and $y$ for the curve $\mathbf{r}(t) = t^2\mathbf{i} + t^3\mathbf{j}$. Does it have the tangent vector in the origin? If so, what is the unit tangent vector?