Georgia State University
(*This paper consists of* **9** *pages.*)

Exam II **March 1, 2001**

Last name:	______________________
First name:	______________________

POINTS

Show all of your work. Calculators are not needed or permitted. Write neatly. Place answers in the space provided.

(15 points) An object is moving in three-space according to the parametric equations $x(t) = \sin(t), \quad y(t) = \cos(t), \quad z(t) = \sin(2t)$. Find

position vector $\mathbf{r}(t) =$

velocity vector $\mathbf{v}(t) =$

acceleration vector $\mathbf{a}(t) =$

speed $v(t) =$

equation for the tangent line at the moment $t = 0$ $\mathbf{R}(u) =$

the unit tangent vector at the moment $t = 0$ $\mathbf{T}(0) =$

(15 points) Identify the surface and find the traces. Then sketch the surface

$x^2 + 4z^2 = 4y$

$4x^2 + y^2 - z^2 = 0$

$4x^2 - y^2 - 4z^2 = 4$

(15 points) A quantity Q depends upon x and y according to $Q(x, y) = y\, e^{-xy}$.

Find the second partials of $Q(x, y)$

Let both x and y be changing with time t and at a certain instant you know that $x = 1, \quad y = 2, \quad x'(t) = 2, \quad y'(t) = -2.$

Use the chain rule to find $Q'(t)$ at this instant.

(15 points) Let $f(x,y) = 2x\sqrt{x+2y}$

Find the gradient vector of $f(x,y)$ $\quad \triangledown f(x,y) =$

Find the directional derivative of $f(x,y)$ at $(x,y) = (1,4)$ in the direction of the vector $\mathbf{a} = -\mathbf{i} + 4\mathbf{j}$

Find a unit vector in the direction in which $f(x,y)$ decreases most rapidly at $(x,y) = (1,4)$

Give the rate of change of $f(x,y)$ in that direction at $(x,y) = (1,4)$

(15 points) Find the length of the given curve

$$\mathbf{r}(t) = t\mathbf{i} + \left(\frac{1}{4}t^2 - \frac{1}{2}\ln t\right)\mathbf{j}, \quad t \in [1, 5]$$

(15 points) Find $\lim_{(x,y)\to(0,0)} \dfrac{x^2y^2}{x^4+y^4}$ or show that it does not exist.

(10 points) The temperature at (x, y) is given by $T(x, y) = x^2 + 2x - y$. Sketch the level curves $T(x, y) = 0$ and $T(x, y) = -1$.

(10 points) The equation

$$\frac{\partial T}{\partial t} = \delta \frac{\partial^2 T}{\partial x^2},$$

where $\delta \neq 0$ is some constant, is called the *heat* or *diffusion* equation. Verify that the function $T(t,x) = \frac{1}{\sqrt{t}} e^{-x^2/(4\delta t)}$ satisfies it.

(15 points) Let $f(x,y)$ be a function with everywhere continuous second partials. Is it possible that

$$\frac{\partial f}{\partial x} = y + x^2, \quad \text{and} \quad \frac{\partial f}{\partial y} = x + e^y\,?$$

If it is, can you reconstruct $f(x,y)$?