

Georgia State University
(This paper consists of 10 pages.)

Exam III

April 5, 2001

Last name: _____
First name: _____

POINTS

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Show all of your work. Calculators are not needed or permitted. Write neatly. Place answers in the space provided.

(10 pts) Write an equation of the plane tangent to the ellipsoid $2x^2 + 4y^2 + z^2 = 45$ at the point $(2, -3, -1)$

(10 pts) Write an equation of the plane tangent to the paraboloid $z = 5 - 2x^2 - y^2$ at the point $(1,1,2)$

(20 points) Let $f(x, y) = 2x^2 + 3y^2 - 4x - 5$.

- (a) (8 pts) Find and classify the critical points of f .
- (b) (12 pts) Find the absolute maximum of f on the elliptical region $x^2 + 2y^2 \leq 18$.

(15 pts) Using the method of Lagrange multipliers find the points of the hyperbola $xy = 1$ that are closest to the origin $(0, 0)$.

(15 pts) Find the most general function, if any!, with the given gradient

$$\left(2 \ln(3y) + \frac{1}{x}\right) \mathbf{i} + \left(\frac{2x}{y} + y^2\right) \mathbf{j}$$

(15 pts) Evaluate

$$\int_{\Omega} \int (6x + 2y^2) dx dy,$$

where Ω is the region bounded by the parabola $x = y^2$ and the straight line $x + y = 2$

(15 pts) **Bonus.** Change the order of integration *only* in

$$\int_0^1 \int_{x^2}^{2x} f(x, y) dy dx.$$

Hint: sketch the domain in the (x, y) -plane.