Math 2215
Multivariable Calculus

Georgia State University
(*This paper consists of* **12** *pages.*)

Test I **June 28, 2002** **Points: 91+ means A**

Last name: ______________________
First name: ______________________

POINTS

You have a choice of 14 problems ranked from 5 through 20 points. If your overall score exceeds 91 points your grade will be an "A".

Show all of your work. Calculators are not needed or permitted. Write neatly. Place answers in the space provided.

1 (5 points). Find the equation of the sphere satisfying the following conditions: the line segment joining $(0,4,2)$ and $(6,0,2)$.

2 (5 points). Find the vector of norm 2 in the direction of $\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$.

3 (5 points). Find all vectors $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$ that have norm 3 and the $\mathbf{i}$-component is twice $\mathbf{j}$-component. Picture the vectors.

4 (5 points). Find the angle between the vectors $\mathbf{a}(2, -3, 1)$ and $\mathbf{b}(-3, 1, 9)$

5 (5 points). Find all numbers x for which

$$(x\mathbf{i} + 11\mathbf{j} - 3\mathbf{k}) \quad \perp \quad (2x\mathbf{i} - x\mathbf{j} - 5\mathbf{k})$$

6 (20 points). Given three points $Q(1,0,2)$, $R(2,2,1)$ and $P(0,1,4)$, Find:

the coordinates of the forth vertex S of *parallelogram* $QPRS$.

the area of $PQRS$

the angle of ΔQPR at the vertex S

the equation of the plane containing the points S, Q and R

equations for the line through the point Q which is perpendicular to the plane containing ΔPQS.

7 (5 points). Let $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j}$. Show that $(\mathbf{a} \times \mathbf{b}) \parallel \mathbf{k}$.

8 (10 points). Show that the following 4 points: $A(-3,4,7)$, $B(2,-4,0)$, $C(1,2,-1)$, $D(1,1,2)$ are not in the same plane. If so, find the volume of parallelepiped determined by $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$.

9 (5 points). Find the symmetric equation of the line that passes through the point $(2, -2, 1)$ and is parallel to the (yz) plane.

10 (15 points). Find the vector parametric equation of the *line segment* connecting two points $P(6, 6, 1)$ and Q(-3,2,0). Find the symmetric equation of the line passing through these points.

11 (10 points). Find the distance from the point $P_1(0,0,2)$ to the line $l\ :\mathbf{r}(t) = 2\mathbf{i} - 3\mathbf{k}$.

12 (10 points). Find the equation of the plane that contains the lines l_1 and l_2 that pass through the point $(1,3,-2)$ and have the corresponding direction vectors:

$$\mathbf{d}_1 = \mathbf{i} + 2\mathbf{j} + 4\mathbf{k} \quad \text{and} \quad \mathbf{d}_2 = -\mathbf{i} - \mathbf{j} + 3\mathbf{k}.$$

13 (20 points). Find the scalar parametric equation of the line of intersection of two planes $2x - 3y + z - 1 = 0$ and $-x + y - z = 0$.

14 (10 points). Find the angle at the intersection point of the line $\mathbf{r}(t) = t\mathbf{d}$, where $\mathbf{d}(1, 0, 1)$ and the plane $x + y - z = 0$