Chapter 1 SUPPLEMENTARY EXERCISES

The supplementary exercises at the end of each chapter review material from the chapter, synthesize concepts from several chapters, or supplement the chapter material in some way. The text has solutions for most of the odd-numbered exercises. The *Study Guide* provides solutions for selected odd-numbered exercises that have only an answer or a *Hint*.

In each chapter, Exercise 1 consists of many true/false questions, whose level of difficulty varies. Some are similar to the ones that appear in many sections of the text, in which a word or phrase is sometimes missing or slightly misstated. Some follow fairly easily from a theorem: others may need careful reasoning. A few may require an argument that uses several ideas. In each case, think carefully about the statement and attempt to write a solution. The text provides the true/false answer, but you must supply the justification or counterexample. Careful work on Exercise 1 will help you prepare for an exam over the chapter material.

7. a. Set $v_1 = \begin{bmatrix} 2 \\ -5 \\ 7 \end{bmatrix}$, $v_2 = \begin{bmatrix} -4 \\ 1 \\ -5 \end{bmatrix}$, $v_3 = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. "Determine if $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ span $\mathbb{R}^3$." To do this, row reduce $[\mathbf{v}_1 \quad \mathbf{v}_2 \quad \mathbf{v}_3]$:

$$\begin{bmatrix} 1 & -2 & -1 \\ -5 & 1 & 1 \\ 7 & -5 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & -1 \\ 0 & -9 & -4 \\ 0 & 9 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & -1 \\ 0 & -9 & -4 \\ 0 & 0 & 0 \end{bmatrix}.$$ The matrix does not have a pivot in each row, so its columns do not span $\mathbb{R}^3$, by Theorem 4 in Section 1.4.

13. The reduced echelon form of A looks like $E = \begin{bmatrix} 1 & 0 & * \\ 0 & 1 & * \\ 0 & 0 & 0 \end{bmatrix}$. Since E is row equivalent to A, the equation $E\mathbf{x} = \mathbf{0}$ has the same solutions as $A\mathbf{x} = \mathbf{0}$. Thus $\begin{bmatrix} 1 & 0 & * \\ 0 & 1 & * \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. By inspection, $E = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$.

17. Here are two arguments. The first is a "direct" proof. The second is called a "proof by contradiction."

i. Since $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a linearly independent set, $\mathbf{v}_1 \neq \mathbf{0}$. Also, Theorem 7 shows that $\mathbf{v}_2$ cannot be a multiple of $\mathbf{v}_1$, and $\mathbf{v}_3$ cannot be a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$. By hypothesis, $\mathbf{v}_4$ is not a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$. Thus, by Theorem 7, $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ cannot be a linearly dependent set and so must be linearly independent.

ii. Since $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a linearly independent set, $\mathbf{v}_1 \neq \mathbf{0}$. Suppose that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly dependent. Then, by Theorem 7, one of the vectors in the set is a linear combination of the preceding vectors. This vector cannot be $\mathbf{v}_4$ because $\mathbf{v}_4$ is *not* in Span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$. Also, none of the vectors in $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a linear combinations of the preceding vectors, by Theorem 7. So the linear dependence of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is impossible, and $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly independent.

Chapter 1 GLOSSARY CHECKLIST

Check your knowledge by attempting to write definitions of the terms below. Then compare your work with the definitions given in the text's Glossary. Ask your instructor which definitions, if any, might appear on a test.

affine transformation: A mapping $T: \mathbb{R}^n \to \mathbb{R}^m$ of the form $T(\mathbf{x}) =$ . . . .

augmented matrix: A matrix made up of a . . . .

back-substitution (with matrix notation)**:** The . . . phase of row reduction of an . . . .

basic variable: A variable in a linear system that . . . .

codomain (of $T: \mathbb{R}^n \to \mathbb{R}^m$)**:** The set . . . that contains . . . .

coefficient matrix: A matrix whose entries are . . . .

consistent linear system: A linear system with . . . .

contraction: A mapping $\mathbf{x} \mapsto$ . . . .

difference equation (or **linear recurrence relation**): An equation of the form . . . whose solution is . . . .

dilation: A mapping $\mathbf{x} \mapsto$ . . . .

domain (of a transformation T)**:** The set of . . . .

echelon form (or **row echelon form**, of a matrix)**:** An echelon matrix that . . . .

echelon matrix (or **row echelon matrix**): A rectangular matrix that has three properties: (1) . . . (2) . . . (3) . . . .

elementary row operations: (1) . . . (2) . . . (3) . . .

equal vectors: Vectors in $\mathbb{R}^n$ whose . . . .

equivalent (linear) systems: Linear systems with the . . . .

existence question: Asks, "Does . . . exist?" or "Is . . .?" Also, "Does . . . exist for . . .?"

floating point arithmetic: Arithmetic with numbers represented as . . . .

flop: One arithmetic operation . . . .

free variable: Any variable in a linear system that . . . .

Gaussian elimination: *See* row reduction algorithm.

general solution (of a linear system)**:** A . . . description of a solution set that expresses . . . .

homogenous equation: An equation of the . . . .

identity matrix (denoted by I or I_n)**:** A square matrix . . . .

image (of a vector **x** under a transformation T)**:** The vector . . . . (Use symbols)

inconsistent linear system: A linear system with . . . .

leading entry: The . . . entry in a row of a matrix.

linear combination: A sum of . . . .

linear dependence relation: A . . . equation where . . . .

linear equation (in the variables $x_1, \ldots, x_n$)**:** An equation that can be written in the form . . . .

linearly dependent (vectors): An indexed set $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ with the property that . . . .

linearly independent (vectors): An indexed set $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ with the property . . . .

linear system: A collection of one or more . . . . equations involving . . . .

linear transformation: A transformation $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$ is linear if (i) . . . , and (ii) . . . .

line through p parallel to v: The set . . . . (Use symbols)

matrix: A rectangular . . . .

matrix equation: An equation that . . . .

matrix transformation: A mapping **x** $\mapsto$ . . . .

migration matrix: A matrix that gives the . . . movement between different locations, from . . . .

***m ´ n* matrix:** A matrix with . . . .

nontrivial solution: A nonzero solution of . . . .

one-to-one (mapping)**:** A mapping T: $\mathbb{R}^n \to \mathbb{R}^m$ such that . . . .

onto (mapping)**:** A mapping T: $\mathbb{R}^n \to \mathbb{R}^m$ such that. . . .

overdetermined system: A system of equations with . . . .

parallelogram rule for addition: A geometric interpretation of . . . .

parametric equation of a line: An equation of the form . . . .

parametric equation of a plane: An equation of the form . . . .

pivot: A . . . number that either is used . . . or is . . . .

pivot column: A column that . . . .

pivot position: A position in a matrix A that corresponds . . . .

plane through u, v, and the origin: A set whose parametric equation is . . . .

product *A*x: . . . .

range (of a linear transformation T)**:** The set of . . . .

reduced echelon form (or **reduced row echelon form,**) of a matrix**:** A rectangular matrix in echelon format that has these additional properties . . . .

roundoff error: Error in floating point arithmetic caused when . . . .

row-column rule for computing *A*x: . . . .

row equivalent (matrices)**:** Two matrices for which there exists . . . .

row reduction algorithm: A systematic method using . . . .

row replacement: An elementary row operation that . . . .

scalar: . . . .

scalar multiple of u by *c*: The vector . . . .

set spanned by {$\mathbf{v}_1$, ... , $\mathbf{v}_p$}: . . . .

size (of a matrix)**:** Two numbers . . . .

solution (of a linear system)**:** . . . .

solution set: The set of . . . .

span {$\mathbf{v}_1$, ..., $\mathbf{v}_p$}: The set . . . .

standard matrix (for a linear transformation *T*)**:** The matrix . . . .

system of linear equations (or a linear system)**:** A collection of . . . .

transformation (or **function** or **mapping**) ***T* from** $\mathbb{R}^n$ ***to*** $\mathbb{R}^m$**:** A rule that assigns to each vector x in $\mathbb{R}^n$ a . . . . Notation: T: $\mathbb{R}^n \rightarrow \mathbb{R}^m$.

translation (by a vector **p**)**:** The operation of . . . .

trivial solution: The solution . . . of a . . . .

underdetermined system: A system of equations with . . . .

uniqueness question: Asks, “If a solution of a system . . . ?”

vector: . . . .

vector equation: An equation involving . . . .

weights: . . . .