Math 2215
Multivariable Calculus

Georgia State University
(*This paper consists of pages.*)

Test II **July 11, 2002** **Points: 91+ is A**

Last name: ______________________________

First name: ______________________________

POINTS

You have a choice of 14 problems ranked from 5 through 20 points. If your overall score exceeds 91 points your grade will be an "A".

Show all of your work. Calculators are not needed or permitted. Write neatly. Place answers in the space provided.

1 (15 points). Find $\mathbf{f}(t)$ from the following information

$$\mathbf{f}'(t) = t\sin(t)\mathbf{i} + \frac{2}{\sqrt{4-t^2}}\mathbf{j} - \frac{1}{t-2}\mathbf{k}, \qquad \text{and} \qquad \mathbf{f}(0) = \mathbf{j} - \mathbf{k}.$$

Multiple choice hint: $\cos^{-1}(u), \sin^{-1}(u), \tan^{-1}(u)$ and integration by parts.

2 (5 points). Find $\lim_{t\to 0} \mathbf{f}(\mathbf{t})$ if it exists

$$\mathbf{f}(\mathbf{t}) = \frac{2t^2+t}{\sin(t)}\mathbf{i} + t^2\left(1+\frac{1}{t^2}\right)\mathbf{k}$$

3 (15 points). Find

$$\frac{d}{dt}\left[\left(\ln t\,\mathbf{i} + \frac{t^2}{2}\mathbf{j} - (t^2-1)\mathbf{k}\right) \times \left(\frac{1}{t}\mathbf{i} + t\mathbf{j} - \mathbf{k}\right)\right] \quad \text{at} \quad t = 1.$$

4 (10 points). Find the tangent and acceleration vectors and the equation for the tangent line at $t = \pi/6$:

$$\mathbf{r}(t) = \cos(2t)\mathbf{i} - \sin(t)\mathbf{j} + \ln(t)\mathbf{k}$$

5 (10 points). Sketch the plane curve and indicate its orientation. Find the unit tangent and unit normal at the indicated point:

$$\mathbf{r}(t) = e^{-2t}\mathbf{i} + e^{2t}\mathbf{j} \quad \text{at} \quad t = 0.$$

6 (15 points). Find the length of the arc:

$$\mathbf{r}(t) = (t\sin t + \cos t)\mathbf{i} + (t\cos t - \sin t)\mathbf{j} + 102\mathbf{k} \quad \text{from} \quad t = 0 \quad \text{to} \quad t = 2\pi.$$

7 (10 points). Identify and sketch the surface

$$9y^2 - 4y^z - 36z^x = 36$$

$$(y-1)^2 + (z+1)^2 = 4$$

8 (10 points). What space curve(s) do the given two surfaces $4x^2 + 4y^2 + (z-2)^2$=16 and $x^2 + y^2 - z = 2$ intersect in?

9 (10 points). Identify the level curves of $f(x,y) = c$ and sketch them:

$$f(x,y) = \ln\left(\frac{x}{y^2}\right), \quad c = -1, 0, 1, 2$$

10 (10 points). Find the equation for the level surface of $f(x, y, z) = x^2 + y^2 - 4z$ that contains the given point $P(1, 1, 0.5)$ and identify and sketch it.

11 (10 points). Find $f_x(x, y)$ and $f_y(x, y)$ by forming the appropriate difference quotient and taking the limit as $h \to 0$: $f(x, y) = e^{3x}y^2$.

12 (5 points). Find $f_x(0, e)$ and $f_y(0, e)$ given that $f(x, y) = \ln(x/y) - ye^{2x}$.

13 (10 points). Let $f(x, y)$ be differentiable everywhere with

$$f_x(x, y) = y + x^2, \quad f_y(x, y) = x + e^y.$$

Does such f exist?

14 (Bonus: 15 points). Let $z = x^2 + y^2$, and C be the line of intersection of the surface with plane $y = 3$. Find the equation for the tangent line to C at the point $(1, 3, 10)$